DEVELOPING INSTRUCTION BASED ON OPEN APPROACH AND ITS IMPACT ON LEVELS OF GEOMETRIC THINKING AND GEOMETRIC ACHIEVEMENT OF EIGHTH GRADE STUDENTS

A DISSERTATION

BY

TIPPARAT NOPARIT

Presented in Partial Fulfillment of the Requirements for the

Doctor of Education Degree in Mathematics Education

at Srinakharinwirot University

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The purposes of this study were:

1. to develop instruction based on the open approach and to study its impact on levels of geometric thinking of eighth grade students,
2. to evaluate the effect of the instruction based on the open approach on geometric achievement of eighth grade students,
3. to trace changes in target students' levels of geometric thinking,
4. to examine sociomathematical norms and classroom mathematical practices emerged during the instruction.

Sample of this study were randomized two classes from five classes of eighth grade students in Chiang Mai Demonstration School, one class with 40 students was randomly assigned to be the experimental group and another class with 42 students was randomly assigned to be the control group. The experimental group was taught by the instruction developed by the researcher according to Taba Model and the open approach. The control group was taught by another mathematics teacher in conventional method. Before the instruction, all students in the experimental group were administered the van Hiele Geometric Thinking Test adapted from Usiskin (1982) to assign the level of geometric thinking. Six students, three from Level 2 of geometric thinking and three from Level 3 of geometric thinking, were purposefully chosen to be target students. Target students were traced their changes in levels of geometric thinking according to the constructs suggested by Gutierrez & Jaime (1998). The constructs were recognition, definition, classification and proof. During the instruction, the researcher taught three days a week for eight weeks (twelve 100 minute-sessions). One witness observed the classes. After the instruction, all students in the experimental group were administered the van Hiele Geometric Thinking Test again. The experimental group and the control group were administered the Geometric Achievement Test.

Data of this study were from the following sources: (a) the van Hiele Geometric Thinking Test, (b) the Geometric Achievement Test, (c) videotapes and audiotapes of

classroom events, (d) students' written work, and (e) witness' reflections. There were two parts of the data analyses. The first part was concerned with students' responses in both tests. The second part was concerned with analysis of target students' geometric thinking during the instruction.

The results of this study showed as follows:

1. The instruction developed according to Taba Model and based on the open approach could be applicable. For the impact of the instruction on students' levels of geometric thinking, there was significant difference (χ^2 (df = 2) = 14.10, p< .05) on levels of geometric thinking and the instruction based on the open approach. This showed that after the instruction, there were the increase in the number of students' exhibiting Level 3 of geometric thinking and the corresponding decrease in the number of students' exhibiting Level 2 of geometric thinking.

2. The effect of the instruction on students' geometric achievement; there was no significant difference (p = .432) on geometric achievement between the experimental and control group.

3. Changes of all Level 2 target students to Level 3 of geometric thinking; they were traced changes in the following constructs:

Recognition. All three students coherently and consistently recognized figures from their physical attributes which indicated Level 1 of geometric thinking to the use of mathematical properties which indicated Level 2 of geometric thinking in this construct.

Definition. Students showed that they were able to know sets of properties associated with definitions of figures and the necessary conditions of congruent triangles which indicated Level 2 of geometric thinking in this construct. They exhibited Level 3 of geometric thinking, interrelate properties between classes of figures, understand the sufficient conditions, and necessary and sufficient conditions of congruent triangles.

Classification. All students showed that they improved to Level 3 of geometric thinking in this construct. Students exhibited the idea of inclusive relationships among classes of triangles and also classes of quadrilaterals.

Proof. Students exhibited this construct at Level 2 of geometric thinking. They often used several examples or measurements to verify the truth of statements. They also used the patty paper to verify the congruence of triangles. However, students were able to show some aspects of proof in Level 3 of geometric thinking, for example, they were able to understand if-then statements, apply the properties to solve problems, follow the proof but not be able to construct informal proof.

4. The emergence of sociomathematical norms and classroom mathematical practices. Two sociomathematical norms emerged accompanied with their classrooms mathematical practices. The first sociomathematical norm was "what count as a valid way of showing that triangles are congruent". The classrooms mathematical practices accompanied with this sociomathematical norm were the use of measurements, the use of fit on top, and the use of reasoning. The second sociomathemacal norm was "what constitutes a valid proof" accompanied with the classroom mathematical practices were the use of drawing and examples, and chain of reasoning.

การพัฒนาการสอนโดยใช้การสอนแบบเปิด และผลของการสอนที่มีต่อระดับการคิดทางเรขาคณิตและผลสัมฤทธิ์ทางการเรียนเรขาคณิตของนักเรียนชั้นมัธยมศึกษาปีที่สอง

บทคัดย่อ

ของ

ทิพย์รัตน์ นพฤทธิ์

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ของการศึกษาตามหลักสูตรปริญญาการศึกษาดุษฎีบัณฑิต

สาขาวิชาคณิตศาสตรศึกษา

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จุดประสงค์ของการศึกษา มีดังนี้

1. เพื่อพัฒนาการสอนโดยใช้การสอนแบบเปิด และศึกษาผลของการสอนที่มีต่อระดับการคิดทางเรขาคณิตของนักเรียนชั้นมัธยมศึกษาปีที่สอง
2. เพื่อประเมินผลของการสอนโดยใช้การสอนแบบเปิดที่มีต่อผลสัมฤทธิ์ทางเรขาคณิตของนักเรียนชั้นมัธยมศึกษาปีที่สอง
3. เพื่อสืบค้นการเปลี่ยนแปลงระดับการคิดทางเรขาคณิตของนักเรียนเป้าหมาย
4. เพื่อตรวจสอบบรรทัดฐานของห้องเรียนในการทำกิจกรรมทางคณิตศาสตร์ และวิธีปฏิบัติทางคณิตศาสตร์ในห้องเรียนซึ่งเกิดขึ้นระหว่างการสอน

กลุ่มตัวอย่างในครั้งนี้เป็นนักเรียนชั้นมัธยมศึกษาปีที่ 2 ของโรงเรียนสาธิตมหาวิทยาลัยเชียงใหม่ ซึ่งได้มาจากการสุ่มแบบเกาะกลุ่ม จำนวน 2 ห้องเรียนจากทั้งหมด 5 ห้องเรียนที่คละความสามารถกัน ห้องหนึ่งซึ่งมีนักเรียนจำนวน 40 คน กำหนดให้เป็นกลุ่มทดลอง และอีกห้องหนึ่งซึ่งมีนักเรียนจำนวน 42 คน กำหนดให้เป็นกลุ่มควบคุม กลุ่มทดลองได้รับการสอนโดยใช้แผนการสอนซึ่งพัฒนาขึ้นโดยผู้วิจัย โดยยึดแบบรูปของทาบา (Taba Model) และการสอนแบบเปิด จำนวน 12 แผนการสอน แต่ละแผนใช้เวลา 100 นาที กลุ่มควบคุมได้รับการสอนแบบปกติ ซึ่งสอนตามแบบเรียนและคู่มือครูวิชาคณิตศาสตร์ของสสวท. ก่อนการสอน นักเรียนทุกคนในกลุ่มทดลองได้รับการทดสอบเพื่อวัดระดับการคิดทางเรขาคณิตโดยใช้แบบวัดระดับการคิดทางเรขาคณิตของแวนฮีลี (van Hiele Geometric Thinking Test) ซึ่งพัฒนาโดย ยูซิสกิน นักเรียน 6 คนถูกเลือกอย่างเฉพาะเจาะจงเพื่อเป็นนักเรียนเป้าหมายในการศึกษารายกรณี เพื่อสืบค้นการเปลี่ยนแปลงระดับการคิดทางเรขาคณิตโดยยึดตามตัวบ่งชี้ซึ่งเสนอโดยกูเธอเรสและเจมี (Gutierrez & Jaime) ดังนี้ 1. การตระหนักเกี่ยวกับรูปเรขาคณิต 2. การนิยาม 3. การจำแนกกลุ่มรูปเรขาคณิต และ 4. การพิสูจน์ โดยเป็นนักเรียนที่มีระดับการคิดทางเรขาคณิตในระดับ 2 และ 3 ระดับละ 3 คน ระหว่างการสอน ผู้วิจัยสอน 3 คาบต่อสัปดาห์ จำนวน 8 สัปดาห์ และมีผู้ช่วยวิจัย 1 คนช่วยสังเกตการสอน หลังการสอน นักเรียนทุกคนในกลุ่มทดลองได้รับการทดสอบเพื่อวัดระดับการคิดทาง

เรขาคณิตโดยใช้แบบวัดระดับการคิดทางเรขาคณิตของแวนฮีลีอีกครั้งหนึ่ง กลุ่มทดลองและกลุ่มควบคุมรับการทดสอบโดยใช้แบบทดสอบวัดผลสัมฤทธิ์ทางการเรียนเรขาคณิต

ข้อมูลในการศึกษาครั้งนี้ได้มาจาก 1. การทดสอบโดยใช้แบบวัดระดับการคิดทางเรขาคณิตของแวนฮีลี 2. แบบทดสอบวัดผลสัมฤทธิ์ทางการเรียนเรขาคณิต 3. วีดีทัศน์บันทึกเหตุการณ์ในห้องเรียน 4. ใบงานและแบบฝึกหัดของนักเรียน และ 5. ความคิดเห็นของผู้ช่วยวิจัย การวิเคราะห์ข้อมูลแบ่งเป็นสองส่วน ส่วนแรก เป็นการวิเคราะห์ผลการตอบของนักเรียนจากทั้งสองแบบทดสอบ และส่วนที่สองเป็นการวิเคราะห์การคิดของนักเรียนเป้าหมายระหว่างการสอน

ผลการวิจัย มีดังนี้

1. การสอนโดยใช้การสอนแบบเปิด ทำให้นักเรียนมีระดับการคิดทางเรขาคณิตสูงขึ้น (χ^2 (df = 2) = 14.10, p< .05)

2. การสอนโดยใช้การสอนแบบเปิด ให้ผลสัมฤทธิ์ทางการเรียนเรขาคณิตไม่แตกต่างกับการสอนแบบปกติ (p = .432)

3. นักเรียนเป้าหมายที่มีระดับการคิดทางเรขาคณิตระดับที่ 2 พัฒนาระดับการคิดทางเรขาคณิตไปสู่ระดับที่ 3 ผู้วิจัยจึงสืบค้นการเปลี่ยนแปลงระดับการคิดทางเรขาคณิตของนักเรียนเป้าหมายกลุ่มนี้ ตามตัวบ่งชี้ ต่อไปนี้

การตระหนักเกี่ยวกับรูปเรขาคณิต นักเรียนเป้าหมายทุกคนตระหนักเกี่ยวกับรูปเรขาคณิตโดยมองภาพรวมและรูปลักษณ์ภายนอกของรูปเรขาคณิต ซึ่งแสดงถึงระดับการคิดทางเรขาคณิตระดับที่ 1 และตระหนักเกี่ยวกับรูปเรขาคณิตโดยอาศัยสมบัติของรูปเรขาคณิต ซึ่งแสดงถึงระดับการคิดในระดับทางเรขาคณิตระดับที่ 2

การนิยาม นักเรียนสามารถใช้บทนิยามของรูปเรขาคณิต และรู้สมบัติของรูปเรขาคณิตนั้นพร้อมกับเข้าใจเงื่อนไขที่จำเป็นของรูปสามเหลี่ยมที่เท่ากันทุกประการ ซึ่งแสดงถึงระดับการคิดทางเรขาคณิตระดับที่ 2 ในตัวบ่งชี้นี้ และสามารถเห็นและเข้าใจความสัมพันธ์ระหว่างสมบัติของรูปเรขาคณิต เงื่อนไขที่เพียงพอ และเงื่อนไขที่จำเป็นและเพียงพอที่ทำให้รูปสามเหลี่ยมเท่ากันทุกประการ ซึ่งแสดงถึงระดับการคิดทางเรขาคณิตระดับที่ 3

การจำแนกกลุ่มรูปเรขาคณิต นักเรียนทุกคนแสดงถึงความเข้าใจเกี่ยวกับความสัมพันธ์ที่เกี่ยวโยงกันในกลุ่มของรูปสามเหลี่ยม และในกลุ่มของรูปสี่เหลี่ยม ซึ่งแสดงถึงระดับการคิดทางเรขาคณิตระดับที่ 3

การพิสูจน์ นักเรียนแสดงระดับการคิดทางเรขาคณิตระดับ 2 ในตัวบ่งชี้นี้ นักเรียนมักใช้การยกตัวอย่างหลาย ๆ ตัวอย่าง หรือใช้การวัด ในการพิสูจน์ นักเรียนใช้กระดาษลอกลายในการตรวจสอบความเท่ากันทุกประการของรูปสามเหลี่ยม อย่างไรก็ตาม นักเรียนสามารถแสดงถึงการเข้าใจบางแง่มุมของตัวบ่งชี้นี้ที่อยู่ในระดับการคิดทางเรขาคณิตระดับที่ 3 กล่าวคือ นักเรียนสามารถ

เข้าใจประโยคเงื่อนไข สามารถประยุกต์สมบัติต่าง ๆ ใช้ในการแก้ปัญหา สามารถติดตามการพิสูจน์ แต่ไม่สามารถเขียนการพิสูจน์ด้วยตนเองได้

4. ระหว่างการสอนพบบรรทัดฐานของห้องเรียนในการทำกิจกรรมทางคณิตศาสตร์ 2 แบบ คือ 1. วิธีการใดที่นับว่าเป็นวิธีที่ดีในการแสดงว่ารูปสามเหลี่ยมสองรูปเท่ากันทุกประการ และ 2. วิธีการใดที่นับได้ว่าเป็นวิธีการที่ดีในการพิสูจน์ วิธีปฏิบัติทางคณิตศาสตร์ในห้องเรียนสำหรับบรรทัดฐานที่หนึ่ง คือ (1) ใช้การวัด (2) ใช้การเคลื่อนที่รูปให้ทับกันสนิท (3) ใช้การให้เหตุผล วิธีปฏิบัติทางคณิตศาสตร์ในห้องเรียนสำหรับบรรทัดฐานที่สอง คือ (1) ใช้การวาดรูปและยกตัวอย่าง (2) ใช้การให้เหตุผลแบบนิรนัย

The dissertation titled

"Developing Instruction Based on Open Approach and Its Impact on Levels of Geometric Thinking and Geometric Achievement of Eighth Grade Students"

by

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has approved by the Graduate School as partial fulfillment of the requirements for the Doctor of Education Degree in Mathematics Education of Srinakharinwirot University.

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CHAPTER 1

INTRODUCTION

Problems and Background

Geometry is referred as the mathematics of space. It is a way of connecting mathematics and the real world. In mathematics, we use geometry as a vehicle for representing mathematical or other concepts which are abstract. For example, we use various geometric theorems in mathematics and use a number line to picture real numbers. Circles and other graphs display numerical information. Geometry is also a prime example of studying axiomatic systems. The study of logic and deduction in geometry encourages students to think and justify logically (Geddes; & Fortunato. 1993 : 199; Burger; & Culpepper. 1993 : 140; Usiskin. 1987: 26-27). That is why geometry still has a solid footing in school mathematics.

Despite the solid footing in the school curriculum, many students in various parts of the world have been facing difficulties in learning geometry. Many studies have made serious investigations into why students have such difficulties. In 1959, two mathematics teachers from the Netherlands, Pierre van Hiele and Dina van Hiele-Geldof, tried to explain why students had those difficulties. The van Hieles proposed five levels of geometric thinking by which to understand students' learning. The levels were visualization, analysis, informal deduction, deduction and rigor (Crowley. 1987 : 2-3). After their discovery, various studies have been conducted to verify and elaborate the theory including on of Gutierrez and Jaime (1998 : 32). They analyzed the lists of decriptors of van Hiele Theory from many publications. They proposed a framework of geometric thinking with different key constructs across the levels : recognition, definition, classification and proof.

According to van Hiele model, student thinking in geometry progresses sequentially through levels (Crowley. 1987 : 1). Several studies (Burger; & Shaughnessy. 1986; Fuys; Geddes; & Tischler. 1988 ; Battista. 1998: 7; Hoffer; & Hoffer. 1992: 257) and also a study in Thailand (Chaiyasang. 1987) found that most middle school students were functioning at lower levels of geometric thinking than they should be. According to Piaget's study (Piaget; Inhelder; & Szeminska. 1960), students in middle school who are the formal operations stage begun at about age 12, should be capable of logical deduction. They should be consistent with van Hiele level 3, informal deduction, but the research cited earlier suggests that they have not reached that level. The van Hiele claims that higher levels can be achieved through instruction. Therefore, the possible cause that impedes students' learning moving through to the higher level of thinking may come from ineffective teaching.

The Geometry Standard in *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM]. 2000 : 41) suggests that the study of geometry should focus on the various aspects of geometry such as figures, relationships between figures and the connection between geometric figures and the physical world. As students learn to make sense of those aspects, they will develop thinking skills that will serve them well in understanding geometry. Teaching students to develop reasoning competencies is essential because it helps them go beyond the level of merely memorizing facts, rules, and procedures.

In traditional teaching especially in Thailand, most of the teachers spend much time on teaching students to remember facts, definitions, rules, and theorems rather than to understand or to solve problems (Inprasitha. 2003 : 2-3). Nodha (Inprasitha. 1997 : 239 ; citing Nodha. 1995. *International Reviews on Mathematical Education*) pointed out that students' deepen their understanding of mathematical ideas and their thinking will be evolved through problem solving. Therefore, an alternative way to promote thinking skill is teaching by problem solving. He suggested a teaching method called the "open approach" (Nodha. 2000 : 40). This approach engages students in mathematical inquiry through the use of non-routine "open-ended problems"; problems that can be solved with various processes or correct answers. Students are encouraged to use their own mathematical thinking abilities to solve these open-ended problems. They provides different solutions or different ways of solving (Conway. 1996 : 3). In addition to solving problems, this approach provides an opportunity for the students to see a variety of their peers' solutions. It serves individual differences among students in mathematical thinking.

A way to develop mathematical thinking and understanding is students' participation in the mathematics community (Baroody; & Caslick. 1998 : 1-21). While engaging to work in small groups, they have the opportunity to propose ideas and test their own and other's ideas through discussion with each other. When students discuss with the whole class, they have an opportunity to explain their reasoning, to offer alternative explanations, and to make sense of other student's reasoning. In essence, they negotiate with one another. These negotiations create sociomathematical norms and classroom mathematical practices. With this environment, students' thinking develops.

From the van Hiele levels of geometric thinking and the teaching method suggested by Nodha, open approach, the researcher conducted this study by developing an instruction based on the open approach and studying its impact on geometric thinking of students and also evaluating the effect of the instruction on geometric achievement of middle school students. The researcher also used Gutierrez and Jaime's Framework to trace changes in levels of geometric thinking according to the constructs and examined the sociomathematical norms and classroom mathematical practices emerged during the instruction.

Purposes of the Study

The purposes of the study are as follows:

1. to develop instruction based on the open approach and to study its impact on levels of geometric thinking of eighth grade students

2. to evaluate the effect of the instruction based on the open approach on geometric achievement of eighth grade students

3. to trace changes in target students' levels of geometric thinking

4. to examine sociomathematical norms and classroom mathematical practices emerged during the instruction.

Scope of the Study

Population. The population in this study was eighth grade students in the Demonstration School, Faculty of Education, Chiang Mai University, Thailand. There were five classes with mixed ability students.

Samples. Samples of this study were two classes that were randomly sampled from five classes. One class with 40 students was assigned as an experimental group and another class with 42 students was assigned as a control group.

Contents. The contents used in this study were congruence of triangles, parallel lines and parallelograms. They are divided into twelve sessions.

Variables. There were two variables in this study.

Independent Variable. The instruction based on the open approach.

Dependent Variable. Students' levels of geometric thinking and students' geometric achievement.

Definition of Terms

1. Open-ended problems. Open-ended problems refer to non-routine problems with multiple correct answers or multiple ways to solve.

2. Open approach. Open approach refers to a teaching method that engages students in using their own mathematical thinking ability to solve open-ended problems.

3. Instruction based on the open approach. Instruction based on the open approach refers to instruction that is sequenced as follows:

1. Review the previous lesson
2. Present and understand the problems
3. Solve the problems by students
4. Present and discuss the solutions
5. Summarize the lessons
6. Work individually

4. Geometric Thinking. Geometric thinking refers to how students consider and use the following key constructs according to Gutierrez & Jaime Framework (1998):

4.1 Recognition. Recognition refers to recognition of types and families of geometric figures, identification of components and properties of the figures.

4.2 Definition. Definition refers to definition of geometric concept. This construct was viewed as two ways: as students' use of a given definition read in a textbook, heard from the teacher or other students, and as students' formulation of definitions of the concept they are learning.

4.3 Classification. Classification refers to classification of geometric figures or concepts into different families or classes.

4.4 Proof. Proof refers to proving of properties or statements by explaining in some convincing way why such a property or statement is true.

5. Geometric Achievement. Geometric Achievement refers to scores of eighth grade students both the experimental group and the control group from the Geometric Achievement Test constructed by the researcher. The test contents are congruence of triangles, parallel lines and parallelograms.

6. Sociomathematical norms. Sociomathematical norms refer to participation structure within the classroom and the discourse related to student expectations and questions of alternative perspectives about mathematics (Cobb. 2000a : 322-324).

7. Classroom mathematical practices. Classroom mathematical practices refer to taken-as-shared, understood by all members of a classroom through mutual negotiation, ways of discussing, disagreeing, and symbolizing while discussing particular ideas (Cobb. 1999 : 9).

Hypotheses of the Study

1. The number of students exhibiting in one level of geometric thinking was changeable to the other level of geometric thinking.
2. The geometric achievement of students in the experimental group was different from that of students in the control group.
3. Target students' levels of geometric thinking according to the construct were changed.

Significance of the Study

This study could be one example of assisting teachers in developing instructional activities that help students develop their geometric thinking. This study also provided knowledge about students' geometric thinking and instruction of which has a few research in Thailand.

CHAPTER 2

REVIEW OF THE LITERATURE

The purpose of this chapter is to provide a review of the literature relevant to this study. The researcher provides an overview of the van Hiele levels of geometric thinking, the open approach, some overview of qualitative analysis and the meaning and examples of sociomathematical norms and classroom mathematical practices. These topics are sequenced as follows:

1. The van Hiele Theory of Geometric Thinking
   1.1 The levels of geometric thinking
   1.2 Characteristics of the theory
   1.3 Phases of instruction
   1.4 History of the van Hiele Geometric Thinking Test
   1.5 The framework used to analyze geometric thinking
   1.6 Implications for instruction to develop geometric thinking
2. Model for Developing Instruction
3. The Open Approach
   3.1 Teaching objectives of the open approach
   3.2 The principles of teaching by the open approach
   3.3 Open-ended problems
      3.3.1 Types of problem used in the open approach
      3.3.2 How to construct a problem
      3.3.3 Advantages of open-ended problems
   3.4 How to develop a teaching plan
   3.5 Ideas on teaching the problem
   3.6 Teaching situation by the open approach
   3.7 Patterns of teaching
   3.8 Research on the open approach and open-ended problems
4. An Overview of Qualitative Research
   4.1 Data collection of qualitative research
   4.2 Data analysis of qualitative research

5. Meaning and Examples of Sociomathematical Norms and Classroom Mathematical Practices

1. The van Hiele Theory of Geometric Thinking

In 1959, there were two mathematics teachers from the Netherlands, Pierre van Hiele and Dina van Hiele-Geldof who were interested in finding ways to help their students think and analyze problems in geometry. The van Hieles formulated explanations for why students had such difficulty in geometry. From their study, they proposed five levels of geometric thinking (van Hiele. 1959/1985; Clements & Battista, 1992 : 426-428).

1.1 The levels of geometric thinking

In this research, the researcher numbered the van Hiele Levels from 1 to 5.

Level 1 (Visual). Initially students identify and operate on shapes and other geometric configurations according to their appearance. They recognize figures as visual *gestalts*, and, thus, they are able to mentally represent these figures as visual images. In identifying figures, they often use visual prototypes; students say that a given figure is a rectangle, for instance, because "it looks like a door." They do not, however, attend to geometric properties or to characteristic traits of the class of figures represented. At this level, students' reasoning is dominated by perception. For example, they might distinguish one figure from another without being able to name a single property of either figure or they might judge that two figures are congruent because they look the same; "There is no why, one just sees it." During students' transition from the visual to the descriptive level, classes of visual objects begin to be associated with their characteristic properties.

At the visual level, the objects about which students' reason are classes of figures recognized visually as "the same shape." The end product of this reasoning is the creation of conceptualizations of figures that are based on the explicit recognition of their properties. After this conceptual construction, the student is at Level 2.

Level 2 (Descriptive/Analytic). At this level, students recognize and can characterize shapes by their properties. For instance, a student might think of a rhombus as a figure with four equal sides, so the term "rhombus" refers to a collection of "properties that he has learned to call rhombus". Students see figures as a whole, but now as collections of properties rather than as visual *gestalts*; at this level, the image begins to fall into the background. Properties are established experimentally by observing, measuring, drawing, and modeling. Students discover that some combinations of properties signal a class of figures and some do not. Students at this level do not, however, see relationships between classes of figures (for example, a student might contend that a figure is not a rectangle because it is a square).

At this level, the objects about which students reason are classes of figures thought about in terms of the sets of properties that the students associate with those figures. The product of this reasoning is the establishment of relationships between and the ordering of properties and classes of figures.

Level 3 (Abstract/Relational). Students can form abstract definitions, distinguish between necessary and sufficient sets of conditions for a concept, and understand and sometimes even provide logical arguments in the geometric domain. They can classify figures hierarchically (by ordering their properties) and give informal arguments to justify their classifications; a square, for example, is identified as a rhombus because it can be thought of as a "rhombus with some extra properties." At this level, the students can discover properties of classes of figures by informal deduction. For example, they might deduce that in any quadrilateral the sum of the angles must be 360° because any quadrilateral can be decomposed into two triangles, each of whose angles sum to 180°.

Definitions can be seen not merely as descriptions but as a method of logical organization. It becomes clear why, for example, a square is a rectangle. This logical organization of ideas is the first manifestation of true deduction. However, the students still do not understand that logical deduction is the method for establishing geometric truths.

At this level, the objects about which students reason are properties of classes of figures. Thus, for instance, the "properties are ordered, and the person will know that the figure is a rhombus if it satisfies the definition of quadrangle with four equal sides". The product of this reasoning is the reorganization of ideas achieved by interrelating properties of figures and classes of figures.

Level 4 (Formal Deduction). Students establish theorems within an axiomatic system when they reach this level. They recognize the difference among undefined terms, definitions, axioms, and theorems. They are capable of constructing original proofs; that is, they can produce a sequence of statements that logically justifies a conclusion as a consequence of the "givens."

At this level, students can reason formally by logically interpreting geometric statements such as axioms, definitions, and theorems. The objects of their reasoning are relationships between properties of classes of figures. The product of their reasoning is the establishment of second-order relationships—relationships between relationships expressed in terms of logical chains within a geometric system.

Level 5 (Rigor/Metamathemathical). At the fifth level students reason formally about mathematical systems. They can study geometry in the absence of reference models, and they can reason by formally manipulating geometric statements such as axioms, definitions, and theorems. The objects of this reasoning are relationships between formal constructs. The product of their reasoning is the establishment, elaboration, and comparison of axiomatic systems of geometry.

1.2 Characteristics of the theory

According to the theory of Pierre and Dina van Hiele, students progress through levels of thought in geometry, from a *Gestalt*-like visual level through increasing sophisticated levels. The theory has the following defining characteristics (Crowley. 1987 : 4):

1. *Sequential.* As with most development theories, a person must proceed through the levels in order. To function successfully at a particular level, a learner must have acquired the strategies of the preceding levels.

2. *Advancement.* Progress from level to level depends more on the content methods of instruction received than on age or biological maturation.

3. *Intrinsic and extrinsic.* Concepts implicitly understood at one level become explicitly understood at the next level.

4. *Linguistics.* Each level has its own language. A relation which is correct at one level can reveal itself to be incorrect at another level. Two people who reason at different levels cannot understand each other. In particular, if a teacher uses instructional materials, contents or vocabularies at a higher level than that of the learner, the students will not be able to follow the thought processes being used.

5. *Mismatch.* If the students is at one level and instruction is at a different level, the desired learning and progress may not occur. In particular, if the teacher, instructional

materials, content, vocabulary, and so on, are at a higher level than the learner, the student will not be able to follow the thought process being used.

1.3 Phases of Instruction

The van Hieles asserted that progress through the levels is more dependent on the instruction received than on age or maturation. The teacher plays a special role in facilitating this progress, especially in providing guidance about expectations. The van Hieles proposed five sequential phases of instruction (Clements; & Battista.1992 : 430-431). They assert that instruction developed according to this sequence promotes the acquisition of a level. The five phases of learning are as following:

Phase 1: Information. The students become acquainted with the content domain. The teacher discusses materials clarifying this content, placing materials at the child's disposal. Through this discussion, the teacher learns how students interpret the language and provides information to bring students to purposeful action and perception.

Phase 2: Guided Orientation. In this phase, students become acquainted with the objects from which geometric ideas are abstracted. The goal of instruction during this phase is for students to be actively engaged in exploring objects (for example, folding, measuring) so as to encounter the principal connections of the network of relations that is to be formalized. The teacher's role is to direct students' activity by guiding them in appropriate explorations—carefully structured, sequenced tasks (often one-step, eliciting specific responses) in which students manipulate objects so as to encounter specific concepts and procedures of geometry. Teachers should choose materials and tasks in which the targeted concepts and procedures are salient.

Phase 3: Explicitation. Students become conscious of the relations and begin to elaborate on their intuitive knowledge. Thus, in this phase, students become explicitly aware of their geometric conceptualizations, describe these conceptualizations in their own language, and learn some of the traditional mathematical language for the subject matter. The teacher's role is to bring the objects of study (geometric objects and ideas, relationships, patterns, and so on) to an explicit level of awareness by leading students' discussion of them in their own language. Once students have demonstrated their awareness of an object of study and have discussed it in their own words, the teacher introduces the relevant mathematical terminology.

Phase 4: Free Orientation. Students solve problems whose solution requires the synthesis and utilization of those concepts and relations previously elaborated. They learn to orient themselves within the "network of relations" and to apply the relationships to solving problems. The teacher's role is to select appropriate materials and geometric problems (with multiple solution paths), to give instructions to facilitate student thinking and to encourage students to reflect and elaborate on these problems and their solutions. In addition, teachers introduce terms, concepts, and relevant problem-solving processes as needed.

Phase 5: Integration. Students build a summary of all they have learned about the objects of study, integrating their knowledge into a coherent network that can easily be described and applied in the language and concepts of mathematics. The teacher's role is to encourage students to reflect on and consolidate their geometric knowledge by increasing emphasis on the use of mathematical structures as a framework for consolidation. Finally, the consolidated ideas are summarized by embedding them in the structural organization of formal mathematics. At the completion of Phase 5, a new level of thought is attained for the topic studied.

1.4 History of the van Hiele geometric thinking test

The van Hiele Geometric Thinking Test used in this study was developed in 1982 by Usiskin for the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) Project. Usiskin designed this test to determine the van Hiele level of the students. From quotes of the van Hiele themselves regarding students behaviors to be expected at each level. Questions were written for each level that would test whether a student was at that level. After that, the questions were given individually to students in oral interviews by three project personnel in three different states. Utilizing the responses of students, a 25-item multiple-choice test with 5 items at each level was created. This test was piloted with entire classes in four schools to ensure that it would not be too long for 35-minute time limit.

Usiskin did not use the questions on the standardized test; he constructed his own van Hiele level test for this project because not every geometry question was assignable to a van Hiele level. Some questions were identified "No level" if they involved parroting a definition or theorem (Usiskin. 1982 : 20 citing van Hiele. 1979). The questions on the van Hiele level test were generally more conceptual than those found on standard exams.

The test was administered to all the students enrolled in the tenth grade geometry course in 13 schools chosen to give a representation of schools in the United States. Based on the results of this test, Usiskin concluded that Level 5 either did not exist or was not testable. All other levels were testable. This test was criticized by Wilson (1990) and Crowley (1990) and defended by Usiskin (1990).

Therefore, in this study, the researcher used only the first twenty items of this test for assessing students' level of geometric thinking.

1.5 The framework used to analyze geometric thinking

Gutierrez & Jaime (1998) analyzed the list of descriptors of van Hiele levels from many publications (Burger; & Shaughnessy. 1986; Crowley. 1987; Fuys; Geddes; & Tischler. 1998; Jaime; & Gutierrez. 1990; Usiskin. 1982). They identified different key constructs as characteristics of the several (but not all) van Hiele levels as follows:

1. Recognition. Recognition of types and families of geometric figures, identification of components and properties of the figures.

2. Definition. Definition of a geometric concept. This construct can be viewed in two ways: as the students use a given definition read in a textbook, heard from the teacher or other students, and as the students formulate definition of the concept they are learning.

3. Classification. Classification of geometric figures or concepts into different families or classes.

4. Proof. Proof of properties or statements by explaining in some convincing way why such a property or statement is true.

In this study, the researcher has numbered the van Hiele levels from 1 to 5. Level 5 is not considered since its existence has not been clearly established in the middle school students in Thailand. Thus the researcher has numbered from Level 1 to 4.

Each construct is a component of two or more levels of geometric thinking. How students consider and use the constructs is an indicator of the their level of thinking. The key constructs across the levels are as follows:

Level 1 (Visual)

Recognition. The students recognize figures on the basis of physical, global attributes of figures, such as size of elements, position, etc. They sometimes use geometric vocabulary but such terms have a visual meaning more than a mathematical one. For instance, when describing a rectangle, some students use the term "vertical" for a side when they mean "perpendicular."

Use of definitions. Students take into consideration only an attribute which refers to physical objects in a global way, or non-mathematical properties like "round" for circle, so they are not able to use given mathematical definitions.

Formulation of definitions. The only definition they can formulate consists of descriptions of physical attributes of the figures they are looking at, such as "round" or "longer than" "wider" and perhaps some basic mathematical property. Sometimes the name of the concept is the definition itself; for example, students quite often say that "a square is a square" when they are asked to define a square.

Classification. Students at this level can understand only exclusive classifications, since they do not accept any kind of logical relationship between two different classes, or, many times, among two elements of the same class having a quite different physical appearance; for instance, two isosceles triangles having angles of 50°, 50°, 80° and 82°, 82°, and 16°, respectively.

Proof. Students at this level cannot understand the concept of proof.

Level 2 (Descriptive/Analytic)

Recognition. Students recognize geometric figures or concepts on the basis of their mathematical properties.

Use of definitions. When students are provided with a mathematical definition and they know every property contained in the definition, they can use it. But these students may experience difficulties when using some logical expressions, such as "and", "or" or "at least".

Formulation of definitions. Students at Level 2 do not understand the logical structure of definitions (i.e., sets of necessary and sufficient properties of the defined concepts), so when they are asked for a definition that has not been learned by rote, they often provide a long list of properties of the concepts, unaware of redundancies. In addition, the definitions may not include some necessary property which they are using implicitly. For example, some students define a rectangle as "a parallelogram having two pairs of equal sides, having two sides longer than the other two" (they omit the reference to the right angles). Other times, they provide a list with more properties than needed, for examples, students define a rectangle as "a parallelogram having two pairs of equal parallel and sides, having two sides longer than the other two, four right angles and two equal diagonals (they include extra properties).

Classification. The students at Level 2 experience difficulties in establishing logical connections between properties. They do not relate families based on the attributes provided in the definitions. Therefore, the classifications produced by students at Level 2 are usually exclusive. In the same way, when students at this level of reasoning are provided with a definition different from the one they have learned previously, the students usually do not accept the new definition, and they continue using their "own" definition. This happens very often with quadrilaterals, when students are habituated to use the exclusive definition and they are given the inclusion definitions.

Proof. For students at this level, a typical proof consists of some experimental verification of the truth of the property in one or a few cases. They may be convinced with just a special example, or they may need a more elaborated set of examples. Here is one student's answer when she was asked to prove that the angles of a quadrilateral add up to 360°

> "Let's suppose that we have a square. Each angle is 90°. They add
> 90° times 4 (sides)=360°
> Now, let's suppose any quadrilateral. Each angle is 80°, 92°, 66°, 122°.
> They add $80^{\circ}+92^{\circ}+66^{\circ}+122^{\circ}=360^{\circ}$.
> If the angles add to more than 360° the figure is no longer a quadrilateral."

Level 3 (Abstract/Relational)

Use of definitions. Students at Level 3 can establish logical relationships between mathematical properties. They are able to use mathematical definitions.

Formulation of definitions. When formulating a definition, students try to be non redundant, although redundancies may appear when the relationships among the properties do not consist of one-step implications.

Classification. Students at this level, have the ability to make inclusive classification of families, thus, those students who say that a square is not a rectangle would not be classified as Level 3 but as Level 2. The students are able to change their mind when a new definition of a concept is given, they also can accept and identify non-equivalent definitions of the same concept.

Proof. Students at this level are able to make deductions and logical proofs, and are able to give informal reasons for the truth of properties. The specific examples are used as only a help and no longer as the proof itself.

Level 4 (Formal Deduction)

Use of definitions and Formulation of definitions. The progression from Level 3 reasoning to Level 4 reasoning consists of a better understanding of definitions and the ability to prove the equivalence of different definitions of the same concept.

Proof. Students may understand and write standard formal proofs. Specific figures are used only to help select the adequate properties for the proof. The students are aware that a figure is only a specific case and that to prove a statement, it is necessary to develop a sequence of implications based on already established properties.

Table 1 summarized the main characteristics of each construct used to distinguish among students at the different van Hiele levels.

TABLE 1 SUMMARY OF THE MAIN CHARACTERISTICS OF EACH CONSTRUCT USED TO DISTINGUISH AMONG STUDENTS AT DIFFERENT VAN HIELE LEVELS

	Level 1	Level 2	Level 3	Level 4
Recognition	Physical attributes	Mathematical properties	-------------------	-------------------
Use of definitions	-------------------	Only definitions with simple structure	Any definition	Accept several equivalent definitions
Formulation of definitions	List of physical properties	List of mathematical properties	Set of necessary and sufficient properties	Can prove the equivalence of definition
Classification	Exclusive, based on physical attributes	Exclusive, based on mathematical attributes	Can move among inclusive and exclusive	-------------------
Proof	-------------------	Verification with examples	Informal logical proofs	Formal mathematical proofs

(Gutierrez & Jaime. 1998 : 32)

This framework was used to examine changes in levels of geometric thinking of target students in the experimental group while analyzing the protocols.

1.6 Implications for instruction to develop geometric thinking

From the review of research on geometric thinking, the researcher drew a number of implications that could be used to develop student geometric thinking in this study. These implications relate to:

1. *Manipulatives and "real world" objects*. Students should manipulate concrete geometric shapes and materials so that they can deal with geometric shapes on their own.

Further research has concurred that students respond favorably to initial introduction of concepts in real world settings and that manipulatives are important and helpful, especially at levels 1 and 2. The visual approach based on concrete materials seemed not only to maintain students' interest but also to assist students in creating definitions and conjectures and in gaining insight into relationships (Fuys; et al. 1988 : 14). Considering this information along with the deficiencies in textbooks, it is imperative that teachers not rely only on the text.

2. *Classroom environment and social interaction.* Baroody; & Caslick. (1998 : 33) suggested that instruction should involve students participating in a mathematical community. Although teacher-to-student communication is important, student-to-student interactions are crucial for a number of reasons. Student-student interactions give students the opportunity to propose ideas and to test their own and other's ideas. This is essential for the development of mathematical thinking and understanding. With small groups, students can speak simultaneously, and practically everyone can be involved in providing feedback. Geddes; & Fortunato (1993 : 211) suggested that a mathematics classroom should offer students opportunities to explore new ideas, to develop thinking skills, to experience the fun of solving real-world problems and asking "what if" questions, and to reflect on their own mathematical thinking.

In this research, the researcher engaged students to work with manipulatives and also involved them to work in small groups.

For this part, the researcher provided an overview of theory about geometric thinking, van Hiele level of geometric thinking theory and its characteristics. To assess the geometric thinking level, the researcher provided the history of the van Hiele geometric thinking test which is the paper-pencil test. The researcher also provided Gutierrez and Jaime (1998) framework for assessing the processes which were integrated in each level of van Hiele geometric thinking. As the van Hiele asserted that progress through the levels is more dependent on the instruction rather than on age or maturation, they provided the phases of instruction which help to promote students' level of geometric thinking. In addition to van Hiele's phases of instruction, other implications were provided for developing instruction which help students progress to the next level. An alternative way which might help to promote students' level of thinking is teaching by the open approach. The following part provides the model for developing the instruction and the key idea of the open approach.

2. Model for Developing Instruction

In this study, the researcher adapted Taba Model : Grass-roots Rationale (Taba. 1962 : 347-379) to develop the instruction. Details of Taba Model are as follow:

Hilda Taba believed that the curriculum should be designed by the uses of the program. Teacher should begin the process by creating specific teaching – learning units for their students. She advocated that teachers take an inductive approach to curriculum development – starting with specific and building to a general design. Taba noted seven major steps to her grass roots model in which teachers would have major input:

1. *Diagnosis of needs.* The teacher starts the process by identifying the needs of the students for whom the curriculum is to be planned.

2. *Formulation of objectives.* After the teacher has identified needs that require attention, he or she specifies objectives to be accomplished.

3. *Selection of content.* The objectives selected or created suggest the subject matter or content of the curriculum. Not only should objectives and content match, but also the validity and significance of the content chosen needs to be determined.

4. *Organization of content.* A teacher cannot just select content, but must organize it in some type of sequence, taking into consideration the maturity of the learners, their academic achievement, and their interest.

5. *Selection of learning experiences.* Content must be presented to pupils and pupils must engage the content. At this point, the teacher's instructional methods involve the students with the content.

6. *Organization of learning activities.* Just as content must be sequenced and organized, so must the learning activities. Often the sequence of the learning activities is determined by the content. But the teacher needs to keep in mind the particular students whom he or she will be teaching.

7. *Evaluation and means of evaluation.* The curriculum planner must determine just what objectives have been accomplished. Evaluation procedures need to be considered by the students and teachers.

The research applied Step 3-6 of Taba Model to develop the instruction. For the instructional methods, the researcher used the open approach which provides details of the key ideas in the next part.

3. The Open Approach

Most of the recent research studies focus on students' individual mathematical thinking. Development of teaching methods that are tuned to a variety of students' ways of thinking is also a major issue. In other words, students' mathematical thinking, mathematical perspectives and development of teaching methods have been integrated, which constitutes a remarkable feature in recent Japanese mathematics education and mathematics educational research.

The origin of such a trend was the research on evaluation conducted at the early of 1970s. One of leading investigations was the one by Shimada and others, Shigeru Shimada, Toshio Sawada, Yoshihiko Hashimoto and Kenichi Shibuya, concerning the method of evaluating higher-order thinking in mathematics education using open-ended problems as a theme. Open-ended problem refers to a problem that is formulated so as to have multiple correct answers. Later years, Shimada and others conducted research in collaboration with teachers in elementary and secondary schools. As a result of this collaborative work, they published the book which translated into English under the name "The open-ended approach: A new proposal for teaching mathematics", by National Councils of Teachers of Mathematics in 1997. The open-ended approach proposed by Shimada, emphasis was placed on the problem whose end was not close to one answer (open – ended problem). The lesson then proceeds by using many correct answers to the given problems to provided experience in finding something new in the process.

Nodha, one of the project team, has extended the concept of an open-ended approach to the open-approach method. The method of using the open approach depends on the problems used in classroom activities. These problems include problem situations, process problems and open-ended problems, and procedures for using these problems including classroom conditions and teaching objectives (Inprasitha. 1997: 239).

3.1 Teaching objectives of the open approach

Teaching by the open approach assumes the following (Nodha. 2000 : 41):

1. All students can learn mathematics in response to their own mathematical power, accompanied by a certain degree of self-determination of their learning.

2. All students can elaborate on the quality of their processes and products toward mathematics.

Therefore, individualized instruction, i.e. the open approach, need to employ the following processes:

(1) understand students' ideas as much as possible.

(2) enrich the ideas during mathematics activities by means of students negotiations with others and/or teachers' advice.

(3) encourage their self- determination in elaborating the activity mathematically.

3.2 The principles of teaching by the open approach

Teaching by the open approach also assumes three principles (Nodha. 2000 : 42)

1. *The autonomy of students' activities.* The teachers should appreciate the value of students' activity by being as non-interfering as possible.

2. *The evolutionary and integral nature of mathematical knowledge.* Content of mathematics is theoretical and systematic. Therefore, the more essential certain knowledge is, the more comprehensively it derives analogical, special and general knowledge. That is, the essential original knowledge can be reflected on many times later in the course of the evolution of mathematical knowledge. This repeated reflection on the original knowledge is a driving force to continue to, metaphorically, open the door ahead more widely.

3. *Teachers' expedient decision-making in class.* In mathematics classes, teachers often encounter students' unexpected ideas. Hence, teachers have an important role to give the ideas full play, and to take into account other students' ideas.

3.3 Open-ended problems

The problems usually used in teaching mathematics in both elementary and secondary school classrooms have one and only one correct answer. These problems are called "complete" or "closed" problems. Another type of problem that is formulated to have multiple correct answers is "incomplete" or "open-ended" problems. In the open-ended approach proposed by Shimada and his colleagues (e.g. Shimada, 1977), the problems they used were only one with multiple correct answers. Here, in the open approach (Nodha.

2000 : 43), the meaning of openness is considered more broadly than that of the open-ended approach. In addition to the problem whose end was open, the problem that produces multiple correct solutions and the problem that produces multiple problems are included. By this extension, the difficulty of constructing the open problems is overcome. Moreover, it becomes possible to provide more opportunities for students with different abilities and needs to participate in the class.

3.3.1 Types of problem used in the open approach

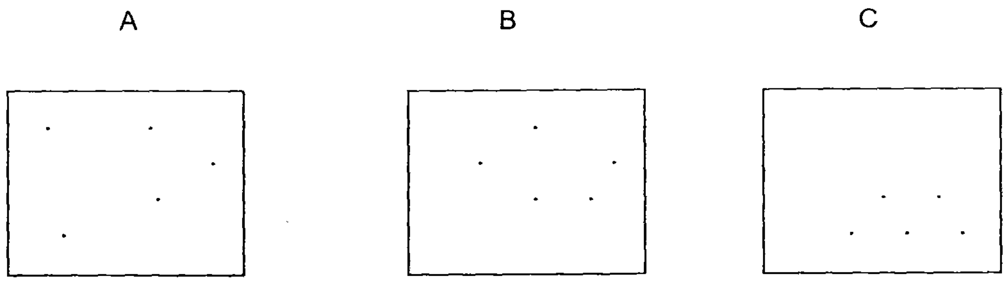
Based on the openness described above, problems used in the open approach are non-routine problems that are classified into three types.

1. *Process is open.* This type of problem has multiple correct ways of solving the original problem. All mathematical problems are inherently open in this sense. For example, suppose 37 pupils will make birthday cards for the teacher in the classroom meeting, and it has been decided that everyone will make cards. Then, they have to make some small cards (in the shape of a rectangle 15 cm. long and 10 cm. wide) from a bigger rectangular sheet (45 cm. long and 35 cm. wide). The problem will be, "How many small cards can you make from the bigger one?"

Students may cut or divide the rectangular sheet into the size of card and get the arrangement. Students also may calculate $(35\times45)\div(15\times10)$ and get the answer 10.5 numerically.

Multiple solutions enable students to carry out of the activity according to their abilities and interests, and then through group discussion, to seek a better process of problem solving.

2. *End products are open.* This type of problem has multiple correct answers. Shimada and his colleagues (e.g. Shimada. 1977) have developed this type of problem. A well known representative problem in the open-ended approach is the "marble problem".

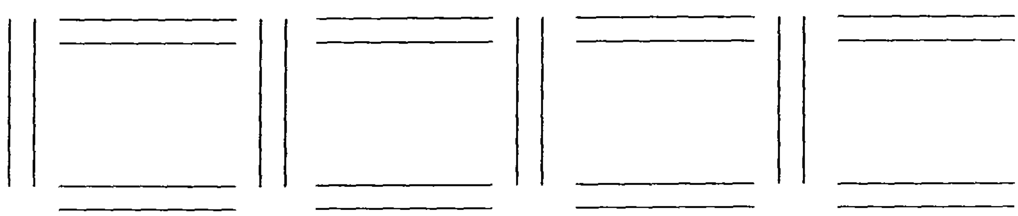


The figure shows scattering patterns of marbles thrown by the three students A, B and C. In this game, the student who has the smallest scatter is the winner. In this example, the degree of scattering ranges from A to C. In such a case, it is convenient if we have some numerical measure to indicate the degree of scattering. Then, think about it from various points of view, and show ways of indicating the degree of scattering by itemized statements. After that, think of the best answer for this problem.

Students may discover "measure the area of a polygonal figure" as a measure of degree of scattering. Another student may think of "measure of the length of all segments connecting two points", and still another may do "measure the radius of the smallest circle including all points." These methods of measure have advantages and disadvantages. The teacher can help students see both the advantages and disadvantages in generalizing the purposed methods. After students have solved the problem, they can develop new problems by changing the conditions or attributes of the original problem.

3. *Ways to develop the problem are open.* After students solve the problem, they can develop new problems by changing the conditions or attributes of the original problem. An example of this type of problem is the "matchstick problem"

"Squares are made using matchsticks as shown in the picture below. When the number of squares is eight, how many matchstick are used?



(1) Write a solution and the answer to the problem above.

(2) Make up your own problems like the one above and write them down.

(3) Make as many problems as you can. You don't need to find the answers to your problems.

(4) Choose the one problem you think is best from those you wrote down above, and write the number of the problem in the space. Write down the reason(s) you think it is the best.

Here, students may develop problems by changing the number of squares. They may further change the condition "square" to "triangle" or "rhombus," for example. They may also develop problems to ask for the number of squares when the number of matchsticks used is given (inverse problem). Students can enjoy developing their own problems. Furthermore, by comparing with their peers they can discuss mathematical structures of the problem and generalizability of their solutions in the lesson.

In this study, the researcher referred "open-ended problems" to the first and second types of problems mentioned above, that were, the process is open and end products are open.

As the second type of problem, end products are open which stated by Shimada and his colleagues, stated earlier, Becker and Shimada (1997 : 27) classified different kinds of problems into another three kinds as follows.

1. Finding relations. Students are asked to find some mathematical rules or relations.

2. Classifying. Students are asked to classify according to different characteristics which may lead them to formulate some mathematical concepts.

3. Measuring. Students are asked to assign a numerical measure to a certain phenomenon. Problems of this kind involve several facets of mathematical thinking. Students are expected to apply mathematical knowledge and skills they have previously learned in order to solve the problems.

3.3.2 How to construct a problem

Becker and Shimada (1997: 28-31) provide guidelines for creating open-ended problems as follows:

1. Prepare a physical situation involving some variable quantities in which mathematical relations can be observed.

2. Instead of asking students to prove a geometry theorem like "if P, then Q," change this problem to "if P, then what kind of relationship among the elements in the figure can you find?"

3. Show students some geometric figures that concern a geometry theorem, then have them draw other figures like the given ones and ask them to conjecture a theorem that is suggested by the figures.

4. Show students a number sequence or number table and then ask them to discover some mathematical rules.

5. Show students several concrete instances in several categories. Point to one of the instances as an example and ask student to enumerate others that have the same characteristics as the example. Multiple methods of enumeration can be based on a multitude of ways to characterize the examples.

6. Show the students a group of several similar exercises or problems. Ask the students to solve them and then to find as many common properties as possible among at least two of them.

In this study, as mentioned that the researcher used the words "open-ended problems" to refer to the first two types : process is open and the end products are open, of the problems used in the open approach. The open-ended problems were constructed from some of the guidelines proposed by Becker and Shimada (1997).

3.3.3 Advantages from using open-ended problems

1. The open-ended problems cater for individual differences among students both in their abilities and interests and in the development of mathematical ways of thinking.

2. The open-ended problems support the investigative process of solving and generating problems.

Through this kind of activities, students are expected to learn not only mathematical knowledge but also process associated with mathematical problem solving such as mathematical ways of thinking, beliefs, and meta-knowledge of "how to learn."

3.4 How to develop a teaching plan

Becker and Shimada (1997 : 31-33) provide some considerations before developing teaching plan as follows:

3.4.1 Determine of the problem is appropriate. There are three points of concern before using the problem in classroom teaching.

3.4.1.1 The problem should encourage students to think from different viewpoints and should allow the high-achieving and low-achieving students to solve it using several different approaches.

3.4.1.2 The problem should be commensurate with the students' ability.

3.4.1.3 The problem should connect or relate to some higher mathematical concepts or be capable of further development to a higher level of mathematical thinking.

3.4.2 Develop the lesson plan. The teacher should consider the following points in developing a good lesson plan.

3.4.2.1 List the students' expected responses to the problem. Students are expected to respond to the problem in different ways. The teacher should write a list of their anticipated responses to the problem. The list should include more higher - level responses than may be expected for the level of the students. Afterward, these responses should be rearranged and grouped according to the view points involved. Finally they should be summarized into a general proposition for each viewpoint.

3.4.2.2 Make the purpose of using the problem clear. The problem can be treated as an independent topic, as an introduction to a new concept, or as a summary of students' learning. The teacher should understand the role of the problem.

3.4.3 Devise a method of posing the problem. The problem has to be expressed in a way that students can easily understand it and find an approach to solve it.

3.4.4 Make the problem as attractive as possible. The problem should be concrete and familiar to students. It should also include aspects that arouse their intellectual curiosity.

3.4.5 Allow enough time to explore the problem fully. Sometimes more time than expected is required to pose a problem, have students solve it, discuss the approaches and solutions, and summarize what has been learned. Accordingly, teachers need to allow enough time to explore the problem. Active discussion among the students and between students and the teacher is one of the crucial aspects of using open-ended problems. The teacher may use two class periods for one problem. In the first period, the students may work individually or in groups to solve the problem and to summarize their findings. Then the second period, the whole class discusses the approaches and solutions and the teacher gives concluding remarks.

3.5 Ideas on teaching the problem

When actually teaching the problem, the teacher must consider the following points.

3.5.1 Posing the problem. In helping students understand the meaning of the problem, the following approaches are effective:

3.5.1.1 Encourage students to focus on the same issue by projecting the problem on a screen using an overhead projector.

3.5.1.2 Add more data for generalization, for example, by introducing variety in the problem situation or by showing more concrete data.

3.5.1.3 Give examples that do not restrict students' way of thinking about the problem.

3.5.1.4 Make good use of concrete materials such as models.

3.5.2 Organizing the teaching. The teacher must be careful not to impose a particular orientation on all students by adopting the opinion of particular students. The teaching should consist of a combination of two things: (a) individual work and (b) discussion by the whole class.

3.5.3 Recording students' responses. It is important to have a written record of the responses, approaches, or solutions to the problem that are taken by each individual and group. Also, by collecting work sheets after the lesson, the teacher can use them in evaluating individual and group learning. The teacher may walk around to provide suggestions to stimulate students to think in a relevant way about the problem.

3.5.4 Summarizing what students have learned. The teacher or students should write their individual or group work on the chalkboard for all to see. The teacher should include all student propositions even when some propositions are erroneous or incomplete (in expression). The teacher should regard students in a positive manner and modify them by comments from the other students. When the students contribute too many propositions, the teacher should concentrate on one point of view and guide students to a conclusion.

3.6 Teaching situation for an open-approach

Teaching by an open-approach method consists of three situations in general:

3.6.1 *Situation A: Formulating a problem mathematically*

In this situation, teachers show students the original situations or problems, and students try to formulate them as mathematical problems in response to their own learning experience.

3.6.2 *Situation B: Investigating various approaches to the formulated problems*

Students are expected to find their own solutions on the basis of their experience. Teachers direct students to discuss the relations among the wide variety of solutions proposed, and lead the students to integrate seemingly unrelated solutions into more sophisticated one.

3.6.3 *Situation C: Posing advanced problems*

Students try to pose more general problems on the basis of their activities in Situation B. Through solving these problems, they are expected to find more general solutions.

3.7 Patterns of Teaching

Stigler and Hiebert (1999 : 76-81) analyzed the TIMSS videotape studies and characterized the lessons in the US, Germany and Japan. The pattern of teaching mathematics in Japan is consistent with the style of teaching in the open approach. The Japanese lessons often follow a sequence of five activities.

- *Reviewing the previous lesson.* The review is conducted through a brief teacher lecture, or by the teacher's leading a discussion, or by the students' reciting the main point. The day's lesson builds frequently and directly on the previous day's lesson, perhaps by using the methods that were developed on the previous day.

- *Presenting the problem for the day.* Usually, there is one key problem that sets the stage for most of the work during the lesson.

- *Students working individually or in groups.* This almost always follows the presentation of the problem and lasts anywhere from one to twenty minutes, often five to ten minutes. Students rarely work in small groups to solve problems until they have worked first by themselves.

- *Discussing solution methods.* After the students have worked on the problem, one or more solution methods are presented and discussed. Often, the teachers ask one or more students to share what they have found. Teachers often select students to share (rather than taking volunteers) based on the methods they have seen students developed as they circulated around the room. Sometimes, teachers themselves present methods they have seen students using or new methods, they wish to summarize and elaborate.

- *Highlighting and summarizing the major points.* Usually at the end of the lesson, and sometimes during the lesson, the teachers present a brief lecture on the main point(s) of the lesson.

Hashimoto & Becker (Conway. 1996 : 8; citing Hashimoto; & Becker. 1999.) suggested that teachers should prepare detailed lesson plans organized as follows:

- presenting the problems or topics and directions,
- understanding the problems,
- problem solving by students using their own natural mathematical thinking ability,
- comparing and discussing students' solutions and,
- presenting a summary of the lesson.

Becker. et al. (Nagasaki; & Becker. 1993; citing Becker; et al. 1990. *Arithmetic Teacher.*) suggested how to organize the lesson according to the following scheme.

1. Introducing the problem

The teacher presents or poses the problem on blackboard or poster.

2. Understanding the problem

The teacher ensures that students know what is expected before they begin work.

3. Problem solving by students

Students are given a worksheet with the problem written on it and work individually and/or in small groups. Emphasis is placed on appealing to students' natural ways of thinking "drawing out" a variety of responses. The teacher moves among the students, purposefully scanning their work and selecting the approaches or answers that will be discussed with the whole class.

4. Comparing and discussing approaches or answers

Individual student (or groups) writes their approaches or answers on the blackboard for all students to see. Them the teacher guides a comparison and discussion of the responses and groups or classifies them according to the same mathematical ideas for discussion of their mathematical quality.

5. Summing up the lesson

The teacher plays an important, even crucial, role in "pulling together" the outcomes of the discussion as it relates to the lesson's objective.

In Thailand, students are not familiar with the open approach, and the use of problem solving oriented method. Therefore, the researcher modified the teaching style of Japanese lessons by adding 'working individually' to the instructional sequence and the role of teacher so that it is appropriate for the Thai context:

1. Reviewing the previous lesson

The teacher reviews the previous lesson or introduces new concepts.

2. Presenting and understanding the problems

The teacher poses problems to the class and help students clarify the problems by asking students to discuss with each other.

3. Solving the problems by students

Students solve the problems using their own natural mathematical thinking ability with their group. During this time, the teacher may provide some hints as needed.

4. Presenting and discussing the solutions

Students present their solutions to the class. The teacher guides a comparison and discussion of the solutions of each group and helps students to extend their ideas.

5. Summarizing the lessons

The teacher pools the outcomes of the discussion and summarized the lessons.

6. Working individually

Students answer questions in the worksheets individually.

3.8 Research on the open approach and open-ended problems

Conway (1996) studied the effect of instruction using the open approach on elementary pre-service teachers' problem solving performance, attitudes toward mathematics, and beliefs about mathematics. The subjects were taught and assessed with respect to three different components of "open-ended" problem solving, namely, a problem with unique answer, but which could be approached in many ways; a problem with many correct answers and a problem from which subjects formulated problems of their own like the solved one.

As a result of the treatment the problem solving performance of the treatment group, that is, flexibility, originality, and elegance, increased for the first of these types of open-ended problems, a problem with a unique answer, in comparison to a control group that was not taught using this instructional approach. No increased performance was shown in the treatment group with respect to the originality and elegance. There were no significant difference between the treatment group and control group on attitudes toward mathematics, attitudes toward problem-solving, and beliefs about the nature of mathematics.

Walton (2002) evaluated how successful an elementary and middle school mathematics teacher enhancement program was in effecting changes in curriculum and instruction in the mathematics classroom. The subjects were 299 K-8 teachers from schools in and around Belleville, Illinois. Data are from (a) pre and posttests which measured teachers' beliefs and attitudes with regard to mathematics, mathematical problem solving, and teaching mathematical problem solving, (b) reflection papers written by teachers after one year of program implementation, (c) closed-ended questionnaires which elicited teachers' responses regarding various aspects on the program and (d) external program evaluations which were written at the end of each of the three summers of the program. Sixteen teachers were observed while teaching a mathematics class and were interviewed individually.

The results from all data indicated changes in the mathematics curricula and instruction in the teachers' classroom. The changes in curricula include more attention on (a) problem solving, in particular open-ended problem solving, (b) number sense and patterning, (c) geometry, and (d) graphing. The changes in instruction include the use of (a) the open approach to teaching mathematics in general and problem solving in particular, (b) more instructional aids such as overhead projectors, manipulatives, and games, (c) small cooperative groups, and (d) the mathematics text book as a resource only.

Residual effects of the program included changing teachers' attitudes and beliefs with regard to mathematics, mathematics teaching, and mathematical problem solving and increasing teachers' level of confidence in teaching mathematics.

Nowyenphon (2001) developed mathematics teaching and learning activities through open-ended problem solving for Mathayom suksa 1 students. He studied the effect of the activities on students' ability to solve problems, behaviors reflecting problem solving thoughts, attitudes towards mathematics and mathematical learning achievement. The samples were Mathayom Suksa 1 students who were studying in the first semester of 2000 academic year at Pak Kred school, Nonthaburi province. The researcher developed a package of 15 open-ended problem solving activities, each of which required 100 minutes of instruction time. The result showed that the mathematics instructional activities with the open-ended solving technique met the 75/75 efficiency criteria. Students' problem solving abilities gradually improved from the problem solving level that required the constant use of questions and problem solving guidelines to levels that required fewer and fewer motivating questions. Most students could solve problems with structure similar to the one. They had experienced before better than those that were not familiar to the one. For the problem solving behaviors, namely, problem investigation, problem solving strategy, employment of fundamental mathematics knowledge, flexibility, originality and communication of thought in problem solving, students' problem solving behavior was the "good" level. The result showed that students in the experimental group possessed positive attitudes towards mathematics and the mathematics learning achievement of the experimental group was higher than the achievement norm for the school.

Srilumduan (2003) assessed 8 sixth grade students in mathematical reasoning and proof, and modes of representation and communication. She used open-ended problems in the geometric content area. The eight case study students were divided into small groups of different mathematical achievement. They were assigned to cooperatively solve the open-ended problems in 100 minutes problems. Audio tapes and video tapes were used to record the students' speech (think aloud) as well as the behaviors of body movement. She analyzed her data which gathered from transcription of audio tape and videotape by using protocol analysis technique.

The results showed that 1) open-ended problems are suitable for assessing students' mathematical processes 2) protocol analysis is an alternative research methodology for analyzing students' mathematical processes 3) all three of their mathematical processes, reasoning and proof, representation and communication, are closed, related and influences one another during solving of open-ended problems.

In summary from the research about the open approach, the open approach helped to increase students' problem solving performances, e.g. flexibility, originality and elegance but not on attitudes or beliefs in mathematics. However, it helped to increase the teachers'' level of confidence in teaching mathematics. For the use of open-ended problems, they helped to improve in problem solving behaviors attitudes towards mathematics and mathematics achievement. Open-ended problems were suitable for assessing mathematical processes such as reasoning and proof, representation and communication.

The researcher uses qualitative research to address the third and the forth purposes, that are, to trace changes in levels of geometric thinking of target students and to examine sociomathematical norms and classroom mathematical practices emerged during the instruction. Therefore, the researcher provides an overview of qualitative research in the next part.

4. An Overview of Qualitative Research

According to Miles and Huberman (1994 : 1), qualitative research has been described as a field of inquiry that cuts across disciplines and subject matter, finding application in such areas as anthropology, sociology, psychology, sociolinguistics, political science, and education. Qualitative research focuses on processes, meanings, and the socially constituted nature of reality and provides insights into the phenomena being studied that cannot be obtained by other means.

Denzin & Lincoln (1994) offer their following generic definition that recognizes the cross-disciplinary nature of the field and the fact that quantitative research means many things to many people:

> " Quantitative research is multimethod in focus, involving an interpretative, naturalistic approach to its subject matter. This means that qualitative researchers study things in their natural settings, attempting to make sense of, or interpret, phenomena in terms of meaning people bring to them. Qualitative research involves

> the studied use and collection of a variety of empirical materials—case study, personal experience, introspective, life story, interview, observational, historical, interactional, and visual texts—that describe routine and problematic moments and meanings in individuals' lives". (p.2)

The data from quantitative research methods can tell what is happening, but it cannot so readily tell why or how to address the kinds of questions we need to look at in-depth. In other words, qualitative research methods best address a different set of questions from those best addressed by quantitative methods.

4.1 Data collection of qualitative research

The term "qualitative methods" implies that data consist of words rather than numbers (Miles & Huberman. 1994 :1); thus these methods are focused on ways to collect and analyze words. The common data collection methods are as follow:

1. Questionnaires
2. Interview and focus groups
3. Observations
4. Audio- and Video taping
5. Written artifacts
6. Case studies

Details of these methods are provided as follow:

1. *Questionnaires*. The term of questionnaires means a set of written questions to which a sample of respondents provide written answers.

2. *Interviews and focus groups.* Interviews and focus groups are based on a set of questions asked of a sample of respondents. In the case of interviews and focus groups, however, administration and responses are oral rather than written. Interviews typically occur with one subject; focus groups occur with a small group of subjects participating in an interview simultaneously (3-5 per group). In both cases, and interviewer asks a series of questions to which a subject responds in his her own words. Many interviews occur in person, but sometimes interviews can be conducted by telephone to save on travel costs.

Like questionnaires, interviews are effective when there is a clear statement of the research problem.

3. *Observations.* At any given moment in an observation situation, many phenomena occur simultaneously. If an observer is watching students do their homework, it may be that one student is talking, another is listening, another is writing and another is walking to the refrigerator. It would be easier, and less thorough, for the observer to record only a subset of the events: what the speaker is saying, what tone the speaker is using (e.g., supportive, tentative, condescending), how the listener is reacting (e.g., nodding his head, looking confused, verbalizing "Mm hm"), or what the writer is writing (or what the other student retrieves from the refrigerator).

Thus, the researcher needs to determine ahead of time which phenomena are of interest: verbal behaviors, non-verbal behaviors, misbehaviors, student-teacher interactions, student-student interactions, emphasis in content.

Once the researcher has determined the phenomena of interest, sometimes an observation guide is designed. Guides vary from a checklist of behaviors that did not occur to a very open-ended list of the questions to consider during the observation. The observation guide should also include some indication of how to use time.

An addition method for recording observations is field notes: notes that an observer takes as he or she watches events unfold. An observer records not only the event occurring, but also his or her reactions to, thoughts about, and interpretations of those events.

4. *Audio- and videotaping.* Because it is difficult for any one person to focus on multiple events at the same time, sometimes it is desirable to record the events so that they can be analyzed at a later time. Of course, the presence of a recording device may make the subject(s) feel more uncomfortable and thus less open in answering questions or in exhibiting behaviors. However it is usually possible to position the recording device so that the subject(s) is less aware of it and is thus less inhibited by its presence. The advantages of having a taped archive of it and is thus less inhibited by its presence. The advantages of having a taped archive of an interview or observation make it worth the extra work needed to establish a less threatening environment for the subject(s).

Taping interviews can also help to reduce interviewer bias because the recordings document the complete verbal interactions, not just those that the interviewer considers important at the time.

To save time a researcher might only transcribe the parts of the tape that do yield interesting or useful data. On the other hand, particularly in exploratory studies, it is not always clear which parts of an interview or observation will turn out to be interesting or useful as the study progress. These circumstances the researcher might choose to transcribe the entire tape. Transcripts provide data in a form convenient for analysis.

5. *Written artifacts*. Another type of data, involving written words, can be found in documents and records such as textbooks, course syllabi, student work, student journal entries, instructors' statements of teaching philosophy, transcript records, department mission statements, and publications of professional organizations. This kind of data can be collected in order to explore such questions as "What type of questions do instructors put on tests?", " What kind of mistakes do students make when using the chain rule to take a derivative?"

6. *Case studies*. A case study is a particular instance (case) of a phenomenon. The unit that constitutes a case depends on the purpose and design of the study. Whereas statistical research might seek generalities about a population, case study research probes deeply into a single instance. Because of its flexibility and capacity for detail, case study designs are popular for qualitative research in education.

Case studies tend to be used to describe a phenomenon, to explain a phenomenon, or to evaluate phenomenon. Any of the data collection methods described above can be used with a case study. Triangulation (the use of multiple sources or instruments) is often the most effective strategy to ensure depth and richness of data as well as reliability and validity of results.

4.2 Data analysis of qualitative research

Miles & Huberman (1994 : 10-12) outlined three current flows of activity called three part analysis to analyze data from qualitative research.

1. *Data reduction*. Data reduction refers to the process of selecting, focusing, simplifying, abstracting, and transforming the data that appear in written-up field notes or transcriptions. The researcher desires to sort, chunk, and categorize the words, perspectives, and behaviors of the research participants. As data being collected, the researcher reads through transcripts or note (or listens to the audiotapes or watches the videotapes) enough times that the researcher is intimately familiar with the contents.

Then researcher begins to see patterns and themes, and can begin to construct labels or codes to mark particular features or events. If the researcher has a transcription of a taped interview, he or she might mark passages in the file (or in the margins on paper) as exemplifying one of the patterns that is emerging (e.g., student is using short term memory or student makes a notation-related error). As pieces of the data get coded they can be sorted (e.g., by copying and pasting into another computer file, or by cutting the paper and sorting the strips into piles), to help refine (i.e., make more specific, make more general, re-organize, redefine) the categories. To enhance validity, additional researchers can go through the data using the same procedure and the categories created can be compared for yet further rounds of revision.

2. *Data display.* The most frequent form of display for qualitative data in the past has been extended text. It is terribly cumbersome and extremely bulky. So the better display should be the matrices, graph, charts, and networks. All are designed to assemble organized information into an immediately accessible, compact form so that the analyst can see what is happening and either draw justified conclusions or move on the next step of analysis the display suggests may be useful.

3. *Conclusion Drawing and Verification.* Conclusion are verified as the analyst proceeds. Verification may be as brief as a fleeting second thought crossing the analyst's mind during writing, with a short excursion back to the field notes, or it may be through and elaborate, with length argumentation and review among colleagues to develop "inter subjective consensus" or with extensive efforts to replicate a finding in another data set. The meanings emerging from the data have to be tested for their plausibility, their sturdiness, their conformability—that is, their validity.

To address the third purpose, the researcher selected case-study students for tracing changes in their levels of geometric thinking according to the Gutierrez & Jaime Framework (1998). The researcher collected data by observations, video- and audio taping and written artifacts methods. The researcher analyzed protocols which transcribed from the video and audio tapes and then triangulated by students' written work and witness reflections. For the forth purpose, the researcher used video taping method to record students' discussions during the whole class. Then, the researcher examined sociomathematical norms and classroom mathematical practices by analyzing from the video tapes and witness's observation and field notes.

5. Meaning and Examples of Sociomathematical Norms and Classroom Mathematical Practices

The context and culture of classroom are considered regarded to be the important factors of students' mathematical thinking. Simon (1995 : 115) who takes strong psychological perspective acknowledges the influence of social interaction by showing aspects of relationship between individual psychological processes and classroom social process. Vygotsky also tended to elevate social processes above psychological processes. For example, it is argued in some formulations that the quality of students' mathematical thinking is generated by or derived from organizational features of the social activities in which they participated. From these studies, Cobb (2000b : 307) mentioned that individual students' activity is seen to be located both within the classroom micro-culture and within broader systems of activity that constitute the sociopolitical setting of reform in mathematics education. For analyzing individual and collective mathematical activity in the classroom, Cobb (2000b : 321) developed an interpretative framework (see Table 2). This framework has two columns, social perspective and psychological perspective that refer to its two reflexively related perspectives. The psychological perspective comprises the emergent viewpoints that correlate social perspective. The emergent viewpoint is the alternative stance which assumes that learning can be characterized as both a process of active individual construction and a process of mathematical enculturation (Cobb. 2000a : 309 citing Dorfler. 1995). Therefore students' mathematical development – increasingly sophisticated ways of reasoning – are seen to be related to their participation in particular communities of practice. The details of this correlation are explained in the following paragraphs.

TABLE 2 AN INTERPRETIVE FRAMEWORK FOR ANALYZING INDIVIDUAL AND COLLECTIVE ACTIVITY

Social Perspective	Psychological Perspective
Classroom social norms	Beliefs about our own role, others' role, and the general nature of mathematical activity
Sociomathematical norms	Specific mathematical beliefs and values
Classroom mathematical practices	Mathematical conceptions and activity

Classroom Social Norms. Although Cobb & Yackle (1996 : 175-190) observed that classroom social norms pertain to classroom participation structure and include making sense of explanations and questioning alternative viewpoints ,Cobb (2000b : 322) only focused on the norm of classroom discussion which he called the negotiation process. Example of norms include explaining and justifying solutions, attempting to make sense of explanations given by others, indicating agreement and disagreement, and questioning alternatives in situations where a conflict in interpretations or solutions has become apparent. From the emergent perspective, in a classroom where disagreements exist, the role of the teacher is to initiate and guide the renegotiation process. The students have to play their part in contributing to the establishment of social norms. In making these contributions, students reorganize their individual beliefs about their own role, others' roles, and the general nature of mathematical activity. As a consequence, these beliefs are taken to be the psychological correlates of the classroom social norm.

Sociomathematical norms. The sociomathematical norms are specific social norms. Only the renegotiations of mathematical activities are considered to be sociomathematical norms. Examples of such sociomathematical norms include what counts as a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution, and an acceptable mathematical explanation. The negotiation process associated with different mathematical solutions, for example, emerges because students do not know what would constitute a mathematical difference until the teacher and other students accept some of their contributions but not others.

When students establish sociomathematical norms, they are constructing specific mathematical beliefs and values that enable them to act as increasingly autonomous members of classroom mathematical communities (Cobb & Yackel. 1996 : 176). These beliefs and values are psychological constructs and constitute a mathematical disposition. As shown in Table 2, they are taken to be the psychological correlates of the sociomathematical norms.

Classroom Mathematical Practices. Cobb (1999) defined *mathematical practice* as " the taken-as-shared way of reasoning, arguing, and symbolizing established while discussing particular mathematical ideas" (p.9). Mathematical practices include individual students' mathematical interpretations and solution strategies that become generally understood, accepted and used by the entire classroom community. As dictated by socio-mathematical norms, students are often obliged to provide a reasonable explanation of how and why strategy works. In the course of discussion, the strategy becomes taken-as-shared by the classroom community. Thus the strategy is translated into a classroom mathematical practice because it no longer needs detailed explanation.

Although classroom mathematical practices are established in the classroom community, they can be seen to constitute the immediate local situations of the students' mathematical development. In identifying sequences of such practices, the analysis documents the evolving social situations in which students participate and learn. Individual students' mathematical conceptions and activities are taken as psychological correlates of these practices, and the relation between them is considered to be reflexive.

The social norms, sociomathematical norms and classroom mathematical practices are the aspects of classroom social interaction that became normative. Even of the teacher does not to cultivate them; every class has these norms that are operative for that particular class. In this study, the researcher focused on sociomathematical norms and classroom mathematical practices. Here are some examples of the sociomathematical norms and classroom mathematical practices that operated in classroom associated with recent research.

Wares (2001) used a teaching experiment to examine how middle school students constructed statistical mathematical models. Wares designed and modified activities to help students extend their reasoning to construct mathematical models. Two sociomathematical norms appeared. One was that students should be able to defend and justify their mathematical models. The other related to the idea of a valid comparison of mathematical quantities. Classroom mathematical practices identified during the teaching experiment included student's use of proportional reasoning to defend their mathematical model, their use of graphs for comparison, and the comparison of mathematical quantities using the concept of rate.

Polaki (2000) used two versions of a teaching experiment to compare elementary students' growth in probabilistic reasoning. He found that both versions of the teaching experiment had a profound impact on student's reasoning. Further, the study generated a number of mathematical practices that were observes during the teachers experiments. The mathematical practices that emerged from the teaching experiment included use of invented formal mathematical language, a systematic method for generating two-dimensional outcomes, and using the sample space to reason about probability problems. These mathematical practices were considered instrumental in building students' conceptual understanding of probability.

Zimmermann (2000) studied students' reasoning about probability simulation. During the teaching experiment four sociomathematical norms were identified as well as number of classroom mathematical practices. The first sociomathematical norm that appeared was "what counts as a valid simulation." Connected to this norm were three classroom mathematical practices: (a) a valid probability generator was required, (b) a valid trials is needed, and (c) at least 30 trials per group should be done and the trails aggregaled. The second sociomathematical norm that emerged was related to the justification of a valid probability generator. The practices that emerged within this norm included the following: (a) the probability generator must have a one-to-one correspondence to the probabilities in the problem, (b) equivalent probability generators are acceptable, and (c) the calculator can serve as the mechanism for generating a trail. The concept of a valid trial encapsulated the third sociomathematical norm. From this norm developed the classroom mathematical practice that the number of outcomes in a trial must correspond outcomes in the. Finally, the fourth sociomathematical norm required the justification of what was a sufficient number of trials. Two practices were identified related to this norm: (a) a group's simulation results would be the combined outcomes of 10 trials per person, and (b) the number of trials depended on whether the students were dealing with an ideal or a practical situation. Some of the classroom mathematical practices became taken-as-shared by the students, such as the need to ensure a valid trial and the requirement of having a one-to-one correspondence to the probabilities given in a problem. Other practices were observed but not necessarily shared by all students. For example, one such mathematical practice was that one must conduct at least 30 trials for a simulation problem.

Yackel; Rasmussen; & King (2000) documented social and sociomathematical norms designed in an advance undergraduate mathematical course. The primary data were from a semester-long classroom teaching experiment conducted in a university-level introductory differential equations class in an American university. The typical class session began with a short problem introduction by the instructor and subsequently students worked in small groups. Small group work was followed by whole class discussion of students' problem solving approaches, interpretations, and thinking. The researcher compared two different classrooms in differential equations that had sharply contrasting social norms regarding students' explanations. In the first class, students were neither obliged to explain their thinking nor were they expected to make sense of other students' explanations. In the second class (project class), they were so obliged. The two social norms, explaining their reasoning and making sense of other students' thinking, helped students form the basis for furthering their own mathematical development. For sociomathematical norms, the researcher offered two contrasting images of what constitutes an acceptable mathematical explanation in two different classrooms in differential equations. In the first class, the instructor was discussing the linearization of the system of differential equations that model the behavior of the pendulum. In the course of discussion, the instructor drew a picture of situation, calculated the Jacobian, and concluded that the linearization is $x' = y$ and $y' = x$. One of the students asked how the conclusion came from, the instructor responded "because that's the rules." This response was implicitly accepted as sufficient mathematical justification. In contrast, for the project class, procedural explanation were insufficient, explanation had to be grounded in an interpretation of the rate of change as expressed by the differential equation. The instructor asked students to discuss with whole class about the differential equation which model the rate of change in the number of fox squirrels in the Rocky Mountains. One student explained in terms of procedural instruction but the instructor probed further into the meaning of what students said in terms of the rate of change and the context of the problem. As the discussion progressed, the instructor asked another student how his group thought about the analysis. His response reflected a shift from explanation as procedure to explanation in terms of rate of change and the significance of the rate of change for the population. This indicated that students understand the significance of this relationship because they engaged in discussions. Therefore, the sociomathematical norm for this class was the explanations which grounded in an interpretation of the rates of change as expressed by first-order different equations. Sociomathematical norms are constituted in interaction. As students acted in accordance with a norm, they contributed to its ongoing constitution. In this study also found that explanation involving technology were also grounded in rates of change. Thus, technology functioned to enlarge students' means of reasoning rather than to generate answers.

In summary, the purposes of this study are to develop the instruction based on the open approach which assumed that it might help students progress to the higher level of geometric thinking. Therefore, this review has looked at geometric thinking, particularly, the van Hiele thinking model which provides the basis of what geometric thinking means in this study. From the review, researcher also noted certain critical things to design the instruction based on the open approach. Finally, to look at changes in levels of geometric thinking according to the constructs and examination of the emergent perspective by Cobb that the researcher uses qualitative research, the literature review in qualitative research and the emergent perspective has provided important ideas in carrying out that examination.

CHAPTER 3

METHODOLOGY

This study adopted a sequential mixed method design: sequential QUAN-QUAL analysis which started with a quantitative data analysis and follows by qualitative data collection and analysis (Tashakkori; & Teddlie. 1998 : 127). The purposes of this study were:

(1) to develop instruction based on the open approach and to study its impact on levels of geometrical thinking of eighth grade students.

(2) to evaluate the effect of the instruction based on the open approach on geometric achievement of eighth grade students.

(3) to trace changes in target students' levels of geometric thinking.

(4) to examine sociomathematical norms and classroom mathematical practices emerged during the instruction.

Two types of analyses, quantitative analysis and qualitative analysis were used. Quantitative analysis was used to address purposes (1) and (2). Qualitative analysis was used to address purposes (3) and (4). This chapter describes the samples of the study, instructional design, assessment instruments, procedures and data collection, data analysis and statistics used in this study.

Samples of the Study

The participants were two classes which were randomized from five classes of eighth grade with mixed ability students of the Demonstration School of Chiang Mai University, Thailand. Before random these two classes, students' abilities among five classes were examined by using Analysis of Variance (ANOVA) to compare the mean scores from final scores of the fundamental mathematics achievement test (MA 102) (see details in APPENDIX D). The result showed that there was no significant difference on mean scores among five classes (F (.05 : df 4,202) = 0.562). Therefore, the mathematics abilities of students among five classes were not different. After that, the researcher randomized two classes and then assigned one class as the experimental group with 40 students. Another class was assigned as the control group with 42 students. The experimental group was taught by the researcher with the instruction developed by the researcher while the control group was taught by another teacher with the conventional method. Both classes were taught the same contents and used the same exercises.

Two weeks before the instruction, each student in the experimental group was administered the van Hiele Geometric Thinking test and assigned a level of geometric thinking according to his/her ability. Then, the researcher selected purposively one group which all students were at Level 2 and another group which all students were at Level 3 to be case-study students. The purpose for selecting the case-study students was to trace changes in their levels of geometric thinking according to the constructs. The criteria for selecting case-study students were the abilities to work collaboratively and to share their thoughts with their friends in group. The six case-study students were called target students.

Instructional Design

The researcher adapted Taba Model : Grass-roots Rationale (Taba. 1962 : 347-379) to be the framework for developing the instruction. The researcher applied Steps 3-6 from this model. Details of designing the instruction were preceded as follow:

1. Determining contents : Taba Step 3-4

The researcher studied curriculum and supplementary documents about geometry in the lower secondary school and then selected topics in geometry: congruence of triangles, parallel lines and parallelograms. The researcher determined learning objectives and listed the important concepts in those topics.

2. Selecting the learning experience : Taba Step 5

The researcher selected the open approach to be the instructional method.

3. Organizing the learning activities : Taba Step 6

The researcher organized the learning activities according to the open approach as following steps:

3.1 Constructing open-ended problems.

The open-ended problems were constructed according to concepts in each topic. There were twelve problems. The open-ended problems used in this study were problems that have multiple correct answers or multiple correct ways to solve. They were used as introducing the new concepts or used as the application of concepts which students had previously learned. The problems must meet certain requirements: (a) measuring four constructs: recognition, definition, classification and proof (b) measuring four constructs across the levels of van Hiele geometric thinking (c) providing students' opportunity to explain the reasons for their answers.

3.2 Listing the expected responses to the problems.

The researcher wrote samples of expected students' responses in each problem.

3.3 Determining the instructional activities.

Instructional activities in each session were in accord with the following key steps:

1. Reviewing the previous lesson. The teacher reviewed the previous lesson or introduced new concepts.

2. Presenting and understanding the problems. The teacher presented or posed problems to the class and helped students to clarify the problems by asking them to engage in group discussion.

3. Solving the problems by students. Students solved the problems with their group using their own natural mathematical thinking ability. During this time, the teacher provided some hints as needed.

4. Presenting and discussing the solutions. Students presented their solutions to the class. The teacher guided a comparison and discussion of the solutions of each group and helped students to extend their ideas.

5. Summarizing the lessons. The teacher pooled the outcomes of the discussion and summarized the lessons.

6. Working individually. Students answered questions in the worksheets individually.

4. Constructing the lesson plans

After preparing the open-ended problems and the instructional activities, the researcher constructed the lesson plans with the twelve 100-minute sessions (see the example of lesson plan in APPENDIX B). The outline of the session was presented in Table 3. Each lesson plan consisted of learning objectives, instructional activities, students' expected responses, ways of discussion and ways of evaluation. The researcher constructed the lesson plans and gave them to three educational experts who were the mathematics educators to comment on sequences of teaching, open-ended problems, activities, exercises, and time allocation. Each expert rated lesson plans with respect to the correspondence between the objectives and activities in each lesson plan. All lesson plans got the index of item objective congruence (IOC) 1.00. The experts gave comments on sequences of teaching and the appropriateness of the open-ended problems, learning objectives, exercises, and time allocation. After that, the researcher revised the lesson plans and then tried out the lessons with six non-participants. The purpose of trialing was to examine the language used in each problem, teaching approach, materials used in each problem, responses from the students. After trialing, the lesson plans were refined and applied in this study.

TABLE 3 THE OUTLINE OF TWELVE SESSIONS

Session	Contents
1 and 2	The congruence of figures and congruence of triangles
3- 5	SAS, ASA, AAS and SSS Theorems
6 and 7	Isosceles triangles, their properties and applications
8	The parallel lines theorems
9	The converses of parallel line theorems
10	Theorems of interior and exterior angles of a triangle
11	Parallelograms and their properties
12	Applications of parallel lines and parallelograms

Assessment Instruments

In this study, here were three assessment instruments: the van Hiele Geometric Thinking Test, the Geometric Achievement Test and Gutierrez & Jaime Framework.

van Hiele Geometric Thinking Test

The van Hiele Geometric Thinking Test adapted from that of Usiskin (1982) was used to determine students' level of geometric thinking in the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) Project. It comprised 20 multiple-choice items with five alternatives in each item. It was discriminated into four levels with five items in each level (see the test in APPENDIX A). The test was graded according to the following rule: If a student meets the criterion on passing each level up to and including level n and failed to meet the criterion of level n+1, then the student are assigned to level n. To pass a level, students must get 3 out of 5 items in that level. This test was administered to all students in the experimental group before and after the instruction.

Geometric Achievement Test

The Geometry Achievement Test was constructed by the researcher. It was used to measure the geometric achievement of the students about congruence of triangles, parallel lines and parallelograms. The processes for construction were the following:

1. The researcher constructed the item specification table and then constructed the item consistent with the learning objectives and the measure of cognitive domains: computation, comprehension, application and analysis.
2. The researcher constructed 32 items: 26 multiple-choices with four alternatives in each item and 3 written items.
3. Four experts, 2 university mathematics lecturors and 2 mathematics Teachers, commented the corresponding of the items to the learning objectives and the measure of cognitive domain.
4. The expert's comments were about errors of typing, unclear statements and the measure of cognitive domains. The researcher adjust the test according to the experts' comments.
5. The researcher tried out the 32 items with 50 ninth-grade students.
6. After revising and modifying the items, 20 multiple choices and those 3 written items were tried out again with 55 ninth-grade students.

After the second trial, the test comprised 20 multiple choices and 3 written items (see the test in APPENDIX A). The reliability of this test was 0.72. The difficulty indices were between 0.47 – 0.78 and the discrimination indices were between 0.20-0.36 (see details in APPENDIX C). This test was administered to both the experimental group and the control group.

Gutierrez and Jaime Framework

The researcher used Gutierrez & Jaime (1998 : 32) Framework to trace changes in target students' level of geometric thinking according to the following constructs : recognition, definition, classification and proof (see details in Chapter 2, p. 12-16)

Procedure and Data Collection

Two weeks before instruction, the researcher administered the van Hiele Geometric Thinking Test to all students in the experimental group. Then, each student was assigned a level of geometric thinking according to criteria of the test. The researcher selected two groups of students whom three of them were at Level 2 of geometric thinking and the other three were at Level 3 to be the target students. The target students could work collaboratively in their group and permit to share their ideas with their friends. Both groups of target students were focused on their problem solving behaviors during teaching.

The students in the control group were taught by another teacher with the conventional method, taught by followed the teacher's instruction manual, with the same contents and exercises as the experimental group.

During each instructional session, the researcher had a dual role, as a researcher and as a teacher in the experimental group. The researcher taught as set in lesson plans and, at the same time, observed the behaviors of both target groups. Two video cameras recorded the behaviors of the target groups while they were solving the problems. At the same time, audio recorders recorded both the students "local talk" with friends and their thinking aloud. In this study, one witness, an experienced mathematics teacher, helped the researcher to observe thinking behaviors of both target groups as they solved the problems. The witness also observed all students during the whole class discussion to examine the sociomathematical norms and classroom mathematical practices. After the instruction, the researcher constructed the protocol by transcribing students' talk and behaviors from the tape recorder and video recorders and then discussed with witness until consensus was reached.

A week after the last session, the students in the experimental group and the control group were administered the Geometric Achievement Test. The van Hiele Geometric Thinking Test was used again for assessing the levels of geometric thinking of students in the experimental group.

Data Analysis

To address all purposes of the study, two kinds of analysis: quantitative analysis and qualitative analysis were used.

To address purposes (1) and (2), the researcher used quantitative analysis. For purpose (1), the researcher studied the impact of the open approach on students' level of geometric thinking in the experimental group by using chi-square test. To address purpose (2), the researcher evaluated the effect of instruction by comparing means of scores on the Geometric Achievement Test for students in both the experimental group and the control group by using a t-test.

To address purpose (3) and (4), qualitative analysis was used to explain how the levels of geometric thinking according to the constructs of target groups changed during instruction. The qualitative data were obtained from protocols of each target group, and their written works while they were solving the problems. The qualitative analysis began by viewing the protocols in each session. Then, the researcher and the witness analyzed the protocol according to the constructs (recognition, definition, classification and proof) in Gutierrez and Jaime Framework. The aim in the analyzing protocol was to search for indicators of each construct in the levels. This analyzing process was carried out with all twenty four protocols. The protocols were triangulated by students' written work.. For examination the collective activities, the researcher and the witness captured the students' interaction and discourse and then determined the sociomathematical norms and classroom mathematical practices from the videotapes.

Statistics used in the Study

Statistics used in this study were divided into two groups: the statistics used for indicating the efficiency of instruments and the statistics used for testing the hypotheses. These statistics were calculated by SPSS for Windows.

The statistics used for indicating the efficiency of instruments

The statistics used for indicating the efficiency of instruments were the index of item objective congruence (IOC), Kuder-Richardson method (KR-20), the difficulty index (p) and the discrimination index (D).

The index of item objective congruence (IOC)

The index of item objective congruence is used for indicating the correspondence between the objectives and activities in each lesson plan. The method consists of having specialists judge each activity on each of the objectives by assigning a value of +1, 0 or -1. A rating of +1 indicates a definite feeling that an activity is a measure of the objectives; 0

shows that the judge is undecided about whether the activity is a measure of the objective; -1 shows a definite feeling that an item is not a measure of the objective. The formula for calculating the index is:

$$IOC = \frac{\sum R}{N}$$

where IOC is the index of item objective congruence

$\sum R$ is sum of the rating (-1,0,+1) of items as a measure of objectives by specialists

N is the number of specialists

If the value of IOC is less than 0.5, the lesson plan or item should be modified.

(Berk. 1980 : 89)

Kuder-Richardson Method (KR-20)

The Kuder-Richardson method is used for indicating the reliability of the Geometric Achievement Test. The formula is:

$$r_{tt} = \frac{k}{k-1}\left[1 - \frac{\sum p_i q_i}{s_t^{2}}\right]$$

where r_{tt} is the reliability of the test

k is the number of items in the test

p is the proportion of students who answered right

q is the proportion of students who answered wrong

s_{ti}^{2} is the variance of a total item

(Ebel & Frisbie. 1986: 78)

The difficulty index (p)

The difficulty index is used to identify the difficulty or simplicity of each item in the Geometric Achievement Test. The formula for calculating the difficulty index is

$$p = \frac{\text{Number of correct responses to an item}}{\text{Total number of persons responding}} .$$

The difficulty index p is an inverse index, the lower the value the more difficult the item. For example, if p = 0.2, the item is difficult but if p = 0.9, the item is very easy.

(Wiersma & Jurs. 1990 : 143)

The *discrimination index (D)*

A discrimination index is the quality of the item to discriminate between the high-ability students and the low -ability students in the Geometric Achievement Test. The formula is:

$$D = P_H - P_L .$$

where D is discrimination index

P_H is the proportion getting the item correct of the highest group

P_L is the proportion getting the item correct of the lowest group.

If D is 0.40 and greater, the item is considered to be good discrimination.

D is 0.30 to 0.39, reasonably good but possibly subject to improvement.

D is 0.20 to 0.29, marginal items, need some revision.

D is below 0.19, poor items, need major revision or should be eliminated.

(Wiersma & Jurs. 1990 : 146-147)

The statistics used for testing the hypotheses

The statistics used for testing the hypotheses are the Analysis of Variance (ANOVA), the chi-square test, the t-test.

The Analysis of Variance (ANOVA)

The ANOVA test is used for comparing means score from final scores of the fundamental mathematics achievement test (MA 102) of five classes of eighth grade students.

It is often convenient to arrange the statistics needed for calculation into a table such as the one below:

Source	df	SS	MS	F
Between group	k-1	SS_b	MS_b	$\frac{MS_b}{MS_w}$
Within group	n-k	SS_w	MS_w	
Total	n-1	SS_t		

where SS_b = sum of squares between groups

SS_w = sum of squares within groups

SS_t = total sum of squares

MS_b = means square between groups

MS_w = means square within groups

k = number of groups

n = number of students

N_j = number of subjects within each group.

We can find the value of each item as follow:

$$SS_b = \sum_{j=1}^{k} N_j(\bar{x}_j - \bar{x})^2$$

$$SS_t = \sum_{j=1}^{k}\sum_{i=1}^{n}(x_{ij} - \bar{x})^2$$

$$SS_w = SS_t - SS_b$$

$$MS_b = \frac{SS_b}{k-1}$$

$$MS_w = \frac{SS_w}{n-k}$$

$$F = \frac{MS_b}{MS_w}$$

(Sprinthall. 2003 :326)

The Chi-Square Test

The chi-square test is used for addressing the impact on the instruction based on the open approach on the levels of geometric thinking of students in the experimental group. The researcher used the chi-square test to test the independence between the instruction based on the open approach and the levels of geometric thinking.

The formula of the chi-square test is as follows:

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

where O_i = observed values

E_i = expected values.

The degree of freedom (df.) can be expressed by the following formula:

$$df = (r-1)(c-1)$$

where r = number of rows in the table

and c = number of columns in the table.

(Sprinthall. 2003 :243)

The t-test

The t-test is used for comparing mean scores on geometric achievement for students in the experimental group and students in the control group. Before testing by t-test, the researcher tested the sample means $\overline{X}_1, \overline{X}_2$ and found that they were normally distributed. The equality of population variances σ_1^2, σ_2^2 was tested by F-test.

$$F = \frac{s_1^2}{s_2^2}$$

where degree of freedom, $df_1 = n_1 - 1$ and $df_2 = n_2 - 1$.

After testing and found that the population variances were equal, $\sigma_1^2 = \sigma_2^2$, then the researcher used the following test statistic:

$$t = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

with degree of freedom, df = $n_1 + n_2 - 2$

where $\bar{X}_1$ is a mean score of the experimental group

$\bar{X}_2$ is a mean score of the control group

s_1^2 is variance of the experimental group

s_2^2 is variance of the control group

n_1 is number of students in the experimental group

n_2 is number of students in the control group.

(Glass & Stanley. 1970: 295)

CHAPTER 4

FINDINGS

The purposes of this study were:

(1) to develop instruction based on the open approach and to study its impact on levels of geometric thinking of eighth grade students.

(2) to evaluate the effect of the instruction based on the open approach on geometric achievement of eighth grade students.

(3) to trace changes in target students' levels of geometric thinking.

(4) to examine sociomathematical norms and classroom mathematical practices emerged during the instruction.

The researcher used both quantitative analyses and qualitative analysis. Quantitative analyses were used to describe the first and second purposes: the impact of the instruction on levels of geometric thinking of the experimental group before and after the instruction ; the effect of the instruction on geometric achievement between the experimental group and the control group. Qualitative analysis was used to describe the third purpose: changes in target students' levels of geometric thinking. Analysis of this purpose was divided into four sections. The first section described changes in geometric thinking levels of target students. The second section described changes in each constructs of each target student before and after the instruction. The third section described changes in each construct of each target student during the instruction. The forth section described summaries of changes in each construct. The researcher also uses qualitative analyses to describe the forth purpose: the emergence of sociomathematical norms and classroom mathematical practices during the instruction.

The researcher presented the results of this study according to the purposes: the result of instruction developed according to Taba Model and based on the open approach and its impact on levels of geometric thinking, the effect of the instruction on geometric achievement, changes in target students' levels of geometric thinking and sociomathematical norms and classroom mathematical practices during the instruction.

The Instruction and Its Impact on Levels of Geometric Thinking

To address the first purpose, the result showed that the instruction which developed according to Taba Model and based on the open approach could be applicable. For the impact of the instruction on levels of geometric thinking, the researcher described changes of geometric thinking levels of students in the experimental group before and after the instruction.

All students were administered van Hiele Geometric Thinking Test before and after instruction. The results are shown in Table 4.

TABLE 4 FREQUENCIES OF GEOMETRIC THINKING LEVELS (n=40)

	Levels			Total
	1	2	3	
Before the instruction	1	16	23	40
After the instruction	1	7	32	40

Table 4 presented the frequencies of geometric thinking levels for all students (n=40) prior to and following the implementation of the instruction based on the open approach. To test that the levels of geometric thinking and the instruction based on the open approach was independent, the researcher used the Chi-Square Test (Sprinthall. 2003 : 243). The result from the chi-square test showed that there was a significant difference (χ^2 (df =2) = 14.10, p< .05) on levels of geometric thinking and the instruction based on the open approach. This showed that after the instruction, there were the increase in the number of students exhibiting Level 3 of geometric thinking and the corresponding decrease in the number of students exhibiting Level 2 of geometric thinking.

A classification of levels of geometric thinking before and after the instruction is shown in Table 5.

TABLE 5 CHANGES OF GEOMETRIC THINKING LEVELS

After the instruction / Before the instruction	Level 1	Level 2	Level 3	Total
Level 1	1	-	-	1
Level 2	-	4	12	7
Level 3	-	3	20	32
Total	1	16	23	40

Table 5 showed that after the instruction, 12 students moved to higher level of geometric thinking (the upper triangle with thick shade area) and 25 students (the diagonal of the rectangle) were still at the same level as that before the instruction but 3 students (the lower triangle with light shade area) whose level decreased to a lower level. However, a lot of students develop to higher level.

The Effect of the Instruction on Geometric Achievement

To address the second purpose, the researcher considered the effect of the instruction by comparing means on the Geometric Achievement Test of students in the experimental group and the control group by t-test.

Before the instruction, to examine that the abilities of eighth grade students among five classes were not different, the researcher used Analysis of Variance (ANOVA) to compare the mean scores which were from final scores of the fundamental mathematics achievement test (MA 102) among five classes of eighth grade students (see details in APPENDIX D). The result showed that there was no significant difference on mean scores among five classes (F(.05 : df 4, 202) = .562), that was, students' mathematics abilities among five classes were not different. Then the researcher randomized two classes and assigned one class to be the experimental group and another class to be the control group.

After the instruction, all students in the experimental group and control group were administered the Geometric Achievement Test constructed by the researcher. The results are shown in Table 6.

TABLE 6 THE EFFECT OF INSTRUCTION BASED ON THE OPEN APPROACH ON GEOMETRIC ACHIEVEMENT BETWEEN THE EXPERIMENTAL AND CONTROL GROUP

GROUP	$\bar{X}$	s	n	t
Experimental group	26.26	6.64	40	
				0.79
Control group	25.09	8.00	42	

There was no significant difference (p = .432) on geometric achievement between the experimental and control group. This showed that the instruction based on the open approach had not affected on geometric achievement.

Changes in Target Students' Levels of Geometric Thinking

To address the third purpose, the researcher subdivided into 4 sections as follows:

The first section described changes in geometric thinking levels of target students before and after the instruction and also focused only the target group who made progress from Level 2 to Level 3 of geometric thinking.

The second section described changes in levels of geometric thinking which was classified by the constructs according to the Gutierrez and Jaime Framework (1998) of each target student in Level 2 of geometric thinking who made progress to the higher geometric thinking level. For describing changes, the researcher classified the individual items according to the constructs on the van Hiele Geometric Thinking Test and also the Geometric Achievement Test.

The third section described changes in the constructs of each target student in Level 2 of geometric thinking group during the instruction. The researcher analyzed data from audio and video tapes, students' written work and also witness's reflections.

The final section described summaries of changes in each construct.

The first section: changes in geometric thinking levels of target students

Target students were administered the van Hiele Geometric Thinking Test before and after the instruction. The results are shown in Table 7.

TABLE 7 GEOMETRIC THINKING LEVELS OF TARGET STUDENTS

Student's name	Level	
	Before instruction	After instruction
Pam	2	3
Sand	2	3
Heart	2	3
Eak	3	3
Ball	3	3
Chai	3	3

Table 7 showed that Eak, Ball and Chai were at Level 3 of geometric thinking both before and after the instruction. However, before the instruction, Pam, Sand and Heart were at Level 2 of geometric thinking but after instruction, all students had progressed to Level 3 of geometric thinking.

Since the Level 2 group made progress to higher level of geometric thinking after the instruction and was no progress in Level 3 group, hence, the researcher studied in depth only the Level 2 group by analyzing changes in geometric thinking individually.

The second section: changes in the constructs of each target student before and after the instruction

Changes in each target students' geometric thinking were traced prior to and following the instruction. According to Gutierrez and Jaime Framework (1998), geometric thinking comprises the following constructs: **recognition, definition, classification and proof.** To analyze changes, the researcher categorized individual items on the van Hiele Geometric Thinking Test (see the test in APPENDIX A). Each item reflected the indicators of some constructs across some levels that presented in Table 8. To categorize each item, the researcher and two mathematics educators evaluated those items independently and then compared the evaluations and discussed the discrepancies, looking for an agreement. Each student's responses to all items were presented and then discussed their performance in each construct exhibited levels.

TABLE 8 CATEGORIZATION OF EACH CONSTRUCT ACROSS LEVELS IN INDIVIDUAL ITEMS ON THE VAN HIELE GEOMETRIC THINKING TEST

Level / Constructs	Level 1	Level 2	Level 3
Recognition	Recognize figures by physical attributes: Items 2-4	Recognize figures by mathematical properties: Item 10	-
Definition	-	Know and use sets of properties associated the definition: Items 6-9	Understand sets of necessary and sufficient properties of the defined concepts: Item 14
Classification	Classify figures by their different physical appearance: Items 1, 5	-	Classify figures by exclusive or inclusive relationships: Items 13,15
Proof	-	-	Give informal logical proof: - Apply properties to solve problems: Item 11 - Understand if-then statement : Item 12

The researcher also triangulated some items which indicated the indicators of some constructs across level from the Geometric Achievement Test (see the test in APPENDIX A) and used them to be the evidences for after the instruction. The categorization of items in the Geometric Achievement Test is presented in Table 9. The researcher chose some items to show that students exhibited the indicators of that constructs, for example Item 2, 5-7, 11-12, 18-20 and the written items.

TABLE 9 CATEGORIZATION OF EACH CONSTRUCT ACROSS LEVELS IN INDIVIDUAL ITEMS ON THE GEOMETRIC ACHIEVEMENT TEST

Level / Constructs	Level 1	Level 2	Level 3
Definition	-	Know and use sets of properties associated the definition: Items 1, 15	Understand sets of necessary and sufficient properties of the defined concepts: Items 2-5, 7, 14
Classification	-	-	Classify figures by exclusive or inclusive relationships: Written items 1-4
Proof	-	-	Give informal logical proof: - Understand if-then statement : Items 6,9 - Apply property to solve problems : Items10-13, - Follow the given proof : Items 8, 15-20

In tracing changes of target students' geometric thinking, the researcher traced changes in each construct across the levels before and after the instruction individually from the individual items on the van Hiele Geometric Thinking Test and the Geometric

Achievement Test (see both tests in APPENDIX A). The researcher described changes in each target student, Pam, Sand and Heart, respectively.

Pam was normally the leader of the discussion in her group. She was always the first who responded to a problem. The scores on each construct according to the Gutierrez & Jaime Framework and individual items on the van Hiele Geometric Thinking Test for Pam before and after the instruction are shown in Table 10. The table shows the points obtained by Pam for each category and level of items. The total number of points for each category and level is shown in parentheses.

TABLE 10 PAM'S SCORES ON EACH CONSTRUCT FROM THE VAN HIELE GEOMETRIC THINKING TEST BEFORE AND AFTER THE INSTRUCTION

	Recognition		Definition		Classification		Proof
	Level 1 (3 pt.)	Level 2 (1 pt.)	Level 2 (4 pt.)	Level 3 (1 pt.)	Level 1 (2 pt.)	Level 3 (2 pt.)	Level 3 (2 pt.)
Before	3	1	3	0	1	1	1
After	3	1	2	1	1	1	1

The following table (Table 11) is presented the items which Pam was correct and incorrect for each construct; recognition, definition, classification and proof across levels respectively, before and after the instruction. Remarks: / showed students are correct and x showed students are incorrect.

TABLE 11 PAM'S RESPONSES ON EACH ITEM ACCORDING TO EACH CONSTRUCT

	Recognition				Definition					Classification				Proof	
	L1			L2	L2				L3	L1		L3		L3	
Item	2	3	4	10	6	7	8	9	14	1	5	13	15	11	12
Before	/	/	/	/	/	/	/	x	x	/	x	/	x	/	x
After	/	/	/	/	x	/	/	x	/	/	x	/	x	x	/

The following paragraph described the responses of Pam in each item according to the constructs. Each item showed the indicators referred in Table 8. The researcher also presented the items which were categorized in Table 9 in the Geometric Achievement Test to support that Pam exhibited that constructs after the instruction. This pattern was also used with Sand and Heart.

--**Recognition. Before the instruction.** Pam was able to do correctly in all the Level 1 items that asked about the physical attributes of shapes. She was able to select the correct shape by its physical appearance. For example, Item 2 asked her to identify which figure was a rectangle; she showed the correct responses (c) purely by recognizing the shape. Another example was Item 3 which Pam was able to select correctly (b). She was able to differentiate the square from the other to figures.

Pam was also correct in Level 2 item (Item 10) which involved the properties of shapes and line symmetry.

Pam was able to select almost correctly in all Level 1 items and also in Level 2 item for recognition, this implies she might be able to recognize shapes not only by their physical attributes but also their properties which indicated Level 2 thinking.

As referred in the Gutierrez & Jaime Framework, the ability to do recognition does not discriminate among students at the van Hiele Levels 2, 3 and 4; that is, Level 2 is the highest level for recognition. This implies that Pam was able to perform at Level 2 even for recognition prior to instruction.

After the instruction. Pam still was able to do correctly in Level 1 items and also Level 2 item for recognition as she did before the instruction.

--**Definitions. Before the instruction**. Pam was able to do 3 out of 4 on Level 2 items. For example, in Item 6 she responded correctly (c), indicating that she was able to correctly state the property of isosceles triangles: at least two angles had the same measure. She was also correct in Item 8 (c) concerning the properties of a rectangle. She was able to determine that the four sides of a rectangle were not necessarily all the same length. These two items implies that she was able to know and use properties associated in either an isosceles triangle or a rectangle.

However, Pam was unable to correct in Item 9. She chose the answer (d) that the two diagonals of rhombus are perpendicular which is, in fact, true for rhombus. The property which is not true for rhombus is the two diagonals have the same length.

For Level 3 item, Pam was unable to correct in Item 14 which she chose answer (c). This implies that she might not be able to understand the necessary and sufficient conditions for parallelograms, the opposite sides parallel. She recognized only the familiar figure of parallelogram. She did not concern that S and R also had the opposite sides parallel.

After the instruction. Pam was able to do correctly to 2 out of 4 on the Level 2 items. She was incorrect in Item 6 and still incorrect Item 9. For Level 3 item, Item 14, she was able to respond correctly (a). This indicates that she was aware of the necessary and sufficient conditions for a parallelogram. She did the same thing to show that she understood the necessary and sufficient conditions of congruent triangles in the Geometric Achievement Test, for example, the question of Item 2 (see figure 1) is presented as follows:

Item 2. Which one is not true?

a. Two figures are congruent when one figure is fitting on top of the other.
b. Two segments are congruent when they have the same length.
c. If two lines are intersecting, the opposite angles are congruent.
d. Two triangles which have 2 angles the same measure and 1 side the same length are congruent.

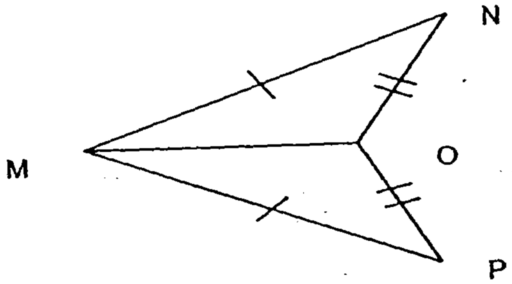
Figure 1 Item 2 in the Geometric Achievement Test

Pam was able to select (d) that is not true. This indicated that she understood the necessary and sufficient conditions for congruent triangles (ASA) that the side must be included between the two angles or AAS which is the two angles and a non-included side of one triangle must be congruent to the corresponding angles and side of another triangle.

She also responded correctly in Item 5 question (Figure 2) that asked her to find the other parts which were needed for the triangles to be congruent triangle by SSS, she was able to choose the correct answer (c). She knew the two pairs of corresponding sides

NO = OP and knew that MO = MO because they are the same segments. Then she was able to find the other pair of corresponding sides, MN = NP, of the two triangles.

Item 5. Suppose NO = OP and $N\hat{O}M = P\hat{O}M$. If ΔMNO and ΔMPO are congruent by SSS Theorem, which one should be determined?



a. $M\hat{N}O = M\hat{P}O$

b. $N\hat{M}O = P\hat{M}O$

c. MN = MP

d. none of these is true.

Figure 2 Item 5 in the Geometric Achievement Test

–Classification. Before the instruction. For Level 1 items, she was able to respond correctly in Item 1 even though the two triangles did not have the same appearance. However, she was not able to choose correctly in Item 5 which asked her to choose parallelograms that had different appearance. Pam indicated that only figure P was a parallelogram. This implied that she chose only the parallelogram with which she was familiar.

For Level 3 items, in Item 13 she was able to select the correct answer (e). She understood that a square is not a rectangle, noted that, in Thai curriculum, a rectangle is defined as a parallelogram which two adjacent sides are not equal and has four right angles, so the square is not a rectangle because it has four sides equal. In Item 15, she chose the answer (e) which is incorrect. She might think that the property "diagonal equal" is the property which is true for rectangles and also parallelogram. In fact, this property is true only in rectangle for this item.

After the instruction. Pam did correctly and incorrect in Level 1 and Level 3 items as before the instruction. However, Pam indicated that she was able to understand the inclusive relationships in the Geometric Achievement Test, for example, she was able to give the reason why the rectangle was a parallelogram. She understood that because a rectangle had two pairs of sides parallel as the parallelogram. She also understood that a rhombus was not a square because its angles were not right angles (see Figure 3).

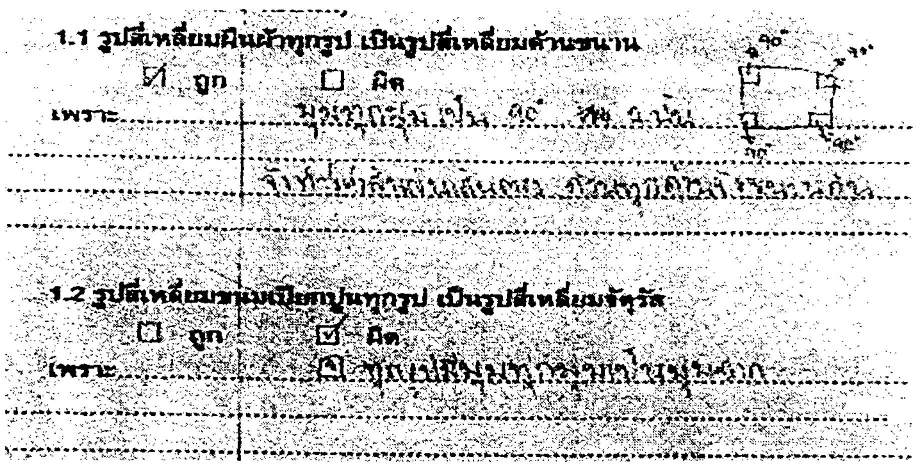
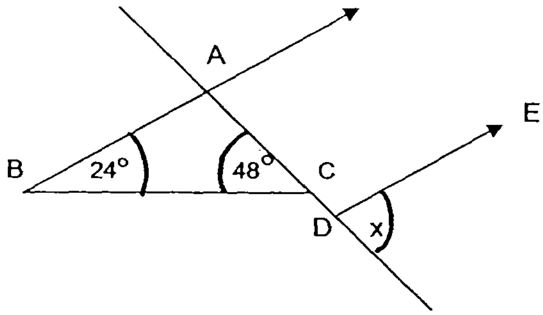


Figure 3 Pam's responses for Classification items on the Geometric Achievement Test

--**Proof. Before the instruction.** Pam was able to do correctly only 1 out of 2 for Level 3 items. She was able to find the measurement of angle x in Item 11 which required an understanding of relationships among properties such as the angles of square, the angles of straight lines and sum of the interior angles of triangles and applied them to find the measure of x. However, she was not able to understand if-then statements in Item 12.

After the instruction. Pam still was able to do correctly only 1 out of 2 for Level 3 items. Interestingly, she was not able to find the measure of angle x in Item 11. However, Pam showed that she was able to correct 3 out of 4 for the items which requires an ability to apply properties to solve the problem in the Geometric Achievement Test. For example; she was correct in Item 11 (Figure 4) and Item 12 (Figure 5). In item 11 requires the properties about the corresponding angles of the parallel lines, and the exterior angle of a triangle equal sum of the opposite interior angles to find the measure of x. In items 12, it needs to know the properties about the alternate interior angles of parallel lines and the necessary and sufficient conditions of triangle to be congruent.

Item 11. Suppose $\overrightarrow{BA} \parallel \overrightarrow{DE}$. Find the measure of x.



a. 24°

b. 48°

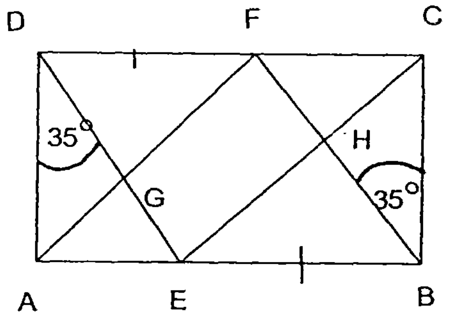
c. 72°

d. 108°

Figure 4 Item 11 in the Geometric Achievement Test

Item 12. Suppose ABCD is a rectangle, DF = BE, $A\hat{D}E = 35^{\circ}$ and $C\hat{B}H = 35^{\circ}$.

Which one is not true?



a. $\overline{DE} \parallel \overline{FB}$

b. $B\hat{F}A = 35^{\circ}$

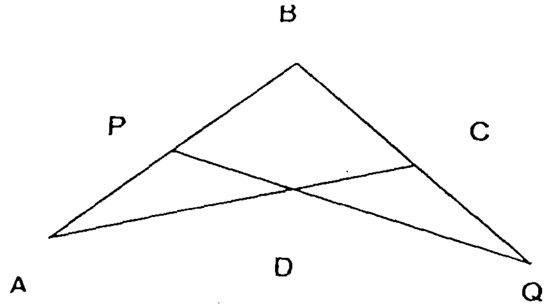
c. $\Delta DAE \cong \Delta BCF$

d. $\Delta ADF \cong \Delta CBE$

Figure 5 Item 12 in the Geometric Achievement Test

Pam was able to do correctly on Item 12 that was about if-then statement in the van Hiele Geometric Thinking Test. She also showed that she understood the if-then statements by responding correctly to Item 6 in the Geometric Achievement Test (Figure 6) which asked her to deal with the conditions for triangles to be congruent. She understood that when $\Delta ABC \cong \Delta QBP$ is true, then $B\hat{C}A = B\hat{P}Q$.

Item 6. From the figure below, if $\Delta ABC \cong \Delta QBP$, then which one is true?



a. $A\hat{B}Q = A\hat{P}Q$

b. $BQ = PQ$

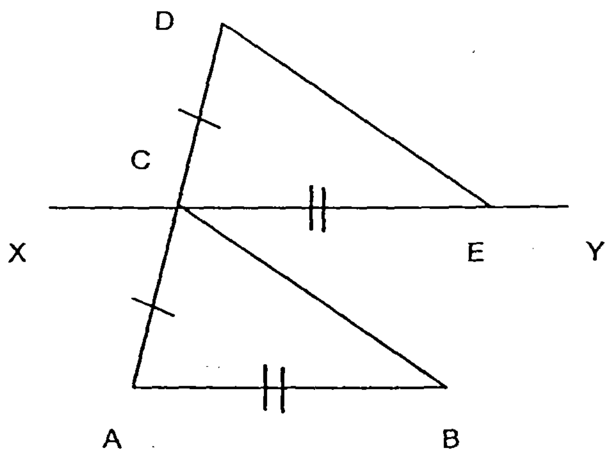
c. $B\hat{A}D = P\hat{D}A$

d. $B\hat{C}A = B\hat{P}Q$

Figure 6 Item 6 in the Geometric Achievement Test

In the Geometric Achievement Test, the researcher designed the items to see the students' ability to follow the given proofs. Thus, the researcher gave two proofs and asked students to select the argument which is missing in the multiple choices. Example of the following proof was given to answer Items 18-20 (Figure 7-9):

Use the given proof to answer the questions 18-20.



Given $\overline{XY} // \overline{AB}$, $AC = CD$ and $AB = CE$

To prove $\overline{BC} // \overline{DE}$

Proof

Statements	Reasons
$\overline{XY} // \overline{AB}$	Given

$C\hat{A}B = D\hat{C}E$	**a**
AC = CD, AB = CE	Given
$\Delta ABC \cong \Delta CED$	SAS
$A\hat{B}C = C\hat{E}D$	The corresponding angles of congruent triangles
$A\hat{B}C = B\hat{C}E$	**b**
Thus $C\hat{E}D = B\hat{C}E$	The same measure as $A\hat{B}C$
That is $\overline{BC} \parallel \overline{DE}$	**c**

Item 18. Which statement determines "a"?

a. Given

b. The corresponding angles of parallel lines are congruent

c. Sum of the interior angles of parallel lines is 180°

d. The alternate interior angles of parallel lines are congruent

Figure 7 Item 18 in the Geometric Achievement Test

Item 19. Which statement determines "b"?

a. The alternate interior angles of parallel lines are congruent

b. Given

c. The corresponding angles of parallel lines are congruent

d. Sum of the interior angles of parallel lines is 180°

Figure 8 Item 19 in the Geometric Achievement Test

Item 20. Which statement determines "c"?

a. Given

b. If two lines are cut by a transversal to form sum of the interior angles of parallel lines is 180°, then the lines are parallel

c. If two lines are cut by a transversal to form the alternate interior angles of parallel lines are congruent, then the lines are parallel

d. none of these a-c is true

Figure 9 Item 20 in the Geometric Achievement Test

Pam was able to answer correctly all items: Item 18 (b), Item 19 (a) and Item 20 (c). This showed that she was able to follow the given proof and select the right arguments. However, she did not demonstrate well enough in the written items which asked her to create own proof. The question is shown in Figure 10.

Suppose $\overline{AB} \parallel \overline{DC}$, AB = EC and $\overline{AC}$ cut $\overline{BE}$ at O, prove that AO = CO.

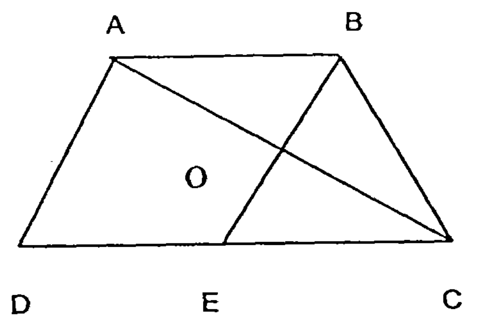


Figure 10 Proof item in the Geometric Achievement Test

Pam's response was given in the Figure 11.

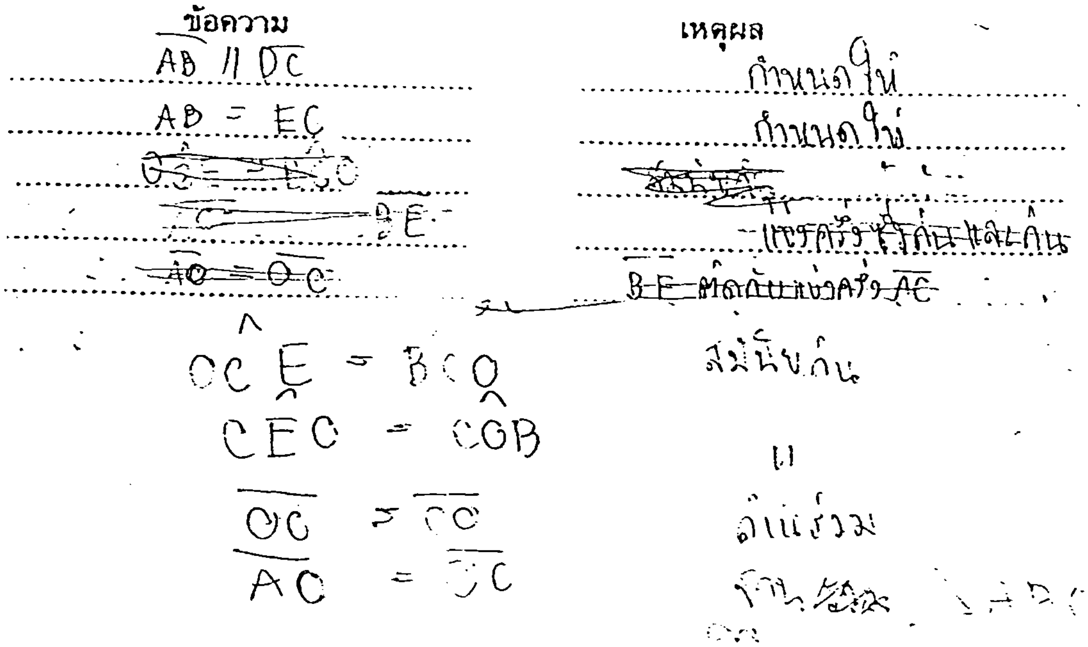


Figure 11 Pam's response in the Proof item in the Geometric Achievement Test

Pam was able to write only the given information, AB // DC and AB = EC but she was not able to given any correct arguments.

The researcher summarized Pam's thinking according to the constructs from the van Hiele Geometric Thinking Test and the Geometric Achievement Test in Table 12. The table presented the indicators of the constructs across levels referred in Table 8.

TABLE 12 PAM'S GEOMETRIC THINKING ON EACH CONSTRUCT BEFORE AND AFTER INSTRUCTION

Time / Construct	Before the instruction	After the instruction
Recognition	- Recognize figures by physical attributes and mathematical properties	- Recognize figures by physical attributes and mathematical properties
Definition	- Able to know sets of properties associated the definition - Unable to understand necessary and sufficient conditions	- Able to know sets of properties associated the definition - Able to understand necessary and sufficient conditions
Classification	- Unable to understand inclusive relationships	- Able to understand more on inclusive relationships
Proof	- Able to apply properties to solve problems - Unable to understand if-then statement	- Able to apply properties to solve problems - Able to understand if-then statement - Able to follow proof - Unable to create own proof

From the summarized table, for recognition, Pam was able to recognize figure by physical attributes and also by mathematical properties which indicate level 2 of geometric thinking. She tended to have more understanding in definition and classification which indicated Level 3 of geometric thinking after the instruction. She was able to understand necessary and sufficient conditions and also the inclusive relationships of the figure. She was able to do several aspects of proof in Level 3 of geometric thinking after the instruct, she was able to apply properties to solve problem, understand if-then statement and follow proof, but was unable to create own proof.

Sand was sometimes the leader of discussion. She responded to a problem and sometimes listened to her friend's comments. The scores on each construct according to the Gutierrez & Jaime Framework and individual items on the van Hiele Geometric Thinking Test for Sand before and after the instruction are shown in Table 13. The table shows the points obtained by Sand for each category and level of items. The total number of points for each category and level is shown in parentheses.

TABLE 13 SAND'S SCORES ON EACH CONSTRUCT FROM THE VAN HIELE GEOMETRIC THINKING TEST BEFORE AND AFTER INSTRUCTION

	Recognition		Definition		Classification		Proof
	Level 1 (3 pt.)	Level 2 (1 pt.)	Level 2 (4 pt.)	Level 3 (1 pt.)	Level 1 (2 pt.)	Level 3 (2 pt.)	Level 3 (2 pt.)
Before	2	1	2	0	1	0	0
After	3	1	2	0	1	2	2

The following table (Table 14) is presented the items which Sand was correct and incorrect for each construct before and after the instruction.

TABLE 14 SAND'S RESPONSES ON EACH ITEM ACCORDING TO EACH CONSTRUCT

	Recognition				Definition					Classification				Proof	
	L1			L2	L2				L3	L1		L3		L3	
Item	2	3	4	10	6	7	8	9	14	1	5	13	15	11	12
Before	x	/	/	/	x	x	/	/	x	/	x	x	x	x	x
After	/	/	/	/	x	x	/	/	x	/	x	/	/	/	/

The following paragraph described the responses of Sand in each item according to the constructs and also her responses in the Geometric Achievement Test.

--**Recognition**. **Before the instruction.** Sand was incorrect on Item 2. Her response was (d) which showed that she might recognize the figures M and L had similar appearance and they were at the same orientation. The figure N was not at the standard orientation which is familiar to her even though it was a rectangle. However, Sand was able to respond correctly to the rest items in Level 1 items and also Level 2 items as Pam did. Therefore, she was using properties to recognize shapes. This implies that she might be able to recognize shapes base on properties.

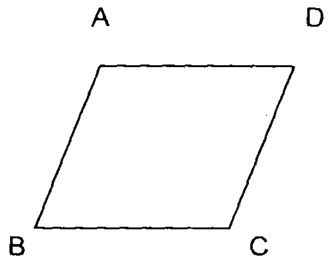
After the instruction. Sand was correct on Item 2 and able to do the same as Pam did. This implies she continued to recognize figures by their properties.

--**Definitions. Before the instruction**. Sand was able to do 2 out of 4 on Level 2 items. Her responses were not similar to Pam. She did Items 6 and 7 incorrect and was correct Items 8 and 9. Interestingly, in Item 6, she selected the answer (e). She did not select the right answer (c) that was the isosceles triangle has at least two angles with the same measure. This showed that she might have difficulty to understand the logical expression "at least" which indicated Level 2 thinking. She also had problem in Item 7, her response was (d) which showed that she might not recognize that PS is a side and QS is a diagonal, so she thought that they have the same length.

Sand was not able to do correctly in Level 3 Item, Item 14. This response indicated that she might not able to understand the necessary and sufficient conditions for parallelograms. She recognized only the familiar figure of parallelogram.

After the instruction. Sand was able do correctly the same items as she did before the instruction in Level 2 items. She still was incorrect in Item 14 but, interestingly, she was able to correct the Items 2 and 5 (see Figure 1 and 2) in the Geometric Achievement Test as Pam did. She also corrected in the Item 7 (figure 12) which requires the understanding of the necessary and sufficient for congruence of triangle. Her response was (a). She was able to understand that the information given was not enough to summarize that ΔABC and ΔADC were congruent. This showed she might be able to understand the necessary and sufficient conditions for congruent triangles.

Item 7. From the given figures, suppose ΔABC and ΔADC are isosceles triangles whose base is $\overline{AC}$. Which one is true?



a. Cannot conclude that $\Delta ABC \cong \Delta ADC$

b. $ABC \cong \Delta ADC$ with SAS

c. $\Delta ABC \cong \Delta ADC$ with ASA

d. $\Delta ABC \cong \Delta ADC$ with SSS

Figure 12 Item 7 in the Geometric Achievement Test

--**Classification**. **Before the instruction**. Sand did the same things as Pam did; that is, she was able to give the correct response in Item 1 but incorrect in Item 5.

Sand was incorrect all items in Level 3 items, Items13 and Item 15. She had no ideas about inclusive relationship between figures.

After the instruction. Sand did the same response in Level 1 items as before the instruction. She was able to do correctly all on Level 3 items. She did Item 13 correctly which showed that she recognized inclusive relationships of a square, a rectangle and a parallelogram. She was also correct on Item 15; this showed that she was able to recognize that the property "diagonals are equal" holds for a rectangle but not for a parallelogram. In the Geometric Achievement Test, Sand was able to reason that a rectangle was a parallelogram because a rectangle has the two pairs of opposite sides the same length. She also understood that diagonals of both a rectangle and a square were the same length. Her response illustrates her understanding of ordering and classification in figure 13.

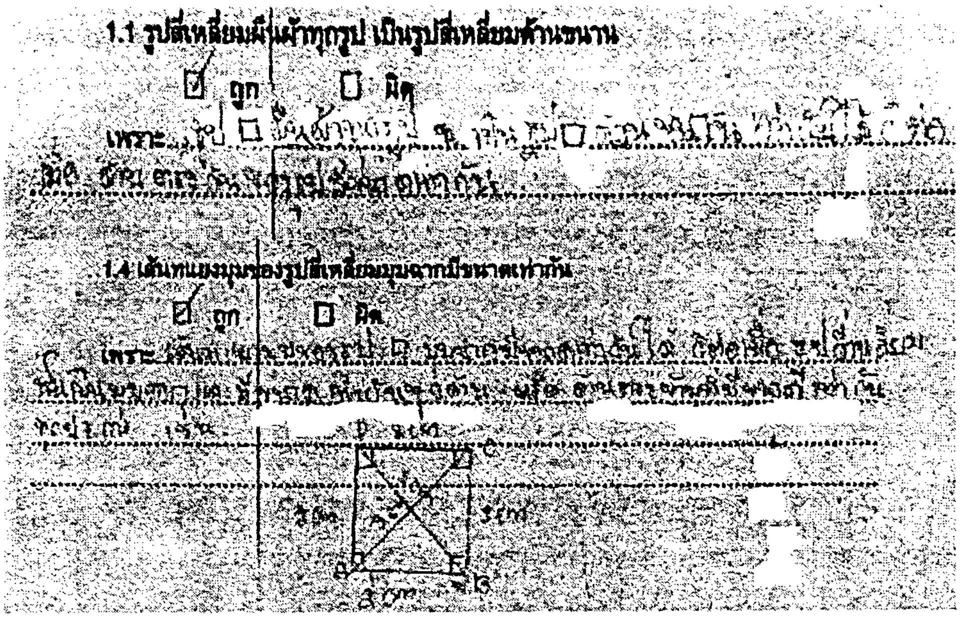
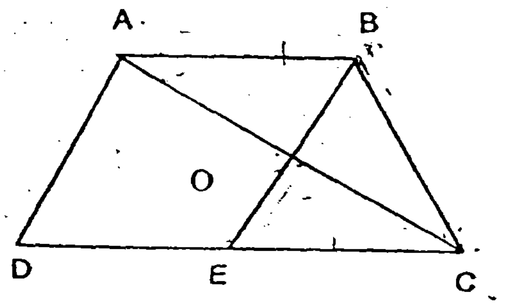


Figure 13 Sand's responses in the Classification items in the Geometric Achievement Test

--**Proof. Before the instruction.** Sand did all incorrect on Level 3 items. She was not able to find the measurement of angle x in Item 11 (Figure 4) and also not able to understand the if-then statements in Item 12 (Figure 5).

After the instruction. Sand was correct on Level 3 items for proof. She was correct in Item 11 (Figure 4) as Pam did. In addition, Sand was able to do correctly 3 out of 4 for the items which asked her to apply the properties to solve problems in the Geometric Achievement Test. For the items asked about the notion of if-then statements, she was correct on Item 12 in the Van Hiele Geometric Test and also Item 6 (figure 6) in the Geometric Achievement Test.

Sand was able to respond correctly all items which asked to follow the given proof in the Geometric Achievement Test (Figure 7-9) as Pam did. This showed that she was able to follow the given proof and select the right arguments. However, she did not demonstrate well in the Proof items (Figure 10) which asked to create her own proof as Pam. Her responses illustrated in Figure 14.



พิสูจน์

ข้อความ	เหตุผล
$\overline{AB} // \overline{DC}$	กำหนดให้
$AB = EC$	กำหนดให้
$AC = BE$	กำหนดให้ ([illegible] O)
$BC = BC$	ร่วม
$\triangle ABC \cong \triangle EBC$	ด้าน - มุม - ด้าน
$AC = CE$	[illegible]
	[illegible]

Figure 14 Sand's response in the Proof item in the Geometric Achievement Test

Sand was able to write only the given information, $\overline{AB} // \overline{DC}$ and AB = EC but she was not able to give the correct arguments.

The researcher summarized Sand's thinking according to the constructs from the van Hiele Geometric Thinking Test and the Geometric Achievement Test in Table 15.

TABLE 15 SAND'S GEOMETRIC THINKING ON EACH CONSTRUCT BEFORE AND AFTER INSTRUCTION

Time / Construct	Before the instruction	After the instruction
Recognition	- Recognize figures by physical attributes and mathematical properties	- Recognize figures by physical attributes and mathematical properties
Definition	- Able to know and use set of properties associated the definition - Unable to understand necessary and sufficient conditions	- Able to know and use set of properties associated the definition - Able to understand necessary and sufficient conditions
Classification	- Unable to understand inclusive relationships	- Able to understand inclusive relationships
Proof	- Unable to apply properties to solve problems - Unable to understand if-then statement	- Able to apply properties to solve problems - Able to understand if-then statement - Able to follow proof - Unable to create own proof

From the summarized table, Sand tended to have more understanding in definition and classification and proof which indicated Level 3 of geometric thinking after the instruction; she was able to understand necessary and sufficient conditions and also the inclusive relationships of the figures. Sand also was able to apply properties to solve the problem, understand if-then statement and follow the proof.

Heart was not normally the leader of discussion in her group. She always listened to her friend's comments on the problem and sometimes responded. The scores on each construct according to the Gutierrez & Jaime Framework and individual items on the van Hiele Geometric Thinking Test for Heart before and after the instruction are shown in Table 16. The table shows the points obtained by Heart for each category and level of items. The total number of points for each category and level is shown in parentheses.

TABLE 16 HEART'S SCORES ON EACH CONSTRUCT FROM THE VAN HIELE GEOMETRIC THINKING TEST BEFORE AND AFTER INSTRUCTION

	Recognition		Definition		Classification		Proof
	Level 1 (3 pt.)	Level 2 (1 pt.)	Level 2 (4 pt.)	Level 3 (1 pt.)	Level 1 (2 pt.)	Level 3 (2 pt.)	Level 3 (2 pt.)
Before	3	1	2	0	1	1	1
After	3	1	3	1	1	2	2

The following table (Table 17) is presented the items which Heart was correct and incorrect for each construct before and after the instruction.

TABLE 17 HEART'S RESPONSES ON EACH ITEM ACCORDING TO EACH CONSTRUCT

	Recognition				Definition					Classification				Proof	
	L1			L2	L2				L3	L1		L3		L3	
Item	2	3	4	10	6	7	8	9	14	1	5	13	15	11	12
Before	/	/	/	/	/	x	/	x	x	/	x	/	x	x	x
After	/	/	/	/	/	x	/	/	/	/	x	/	/	/	x

The following paragraph described the responses of Heart in each item according to the constructs and also the support evidences in the Geometric Achievement Test.

--Recognition. **Before the instruction.** Heart was able to respond correctly to the items in Level 1 and Level 2 items as Pam and Sand did. Hence, this implies that she might move beyond recognizing shapes by their appearance to using properties to recognize shapes.

After the instruction. Heart was able to do correctly on Level 1 items and also in Level 2 item for recognition as she did before the instruction and the same as Pam and Sand. This might show that she continued to recognize figures by their properties.

--Definitions. **Before the instruction**. Heart was able to do 2 out of 4 on Level 2 items. Her responses were similar to Pam. For example, she did Items 6 and 8 correctly. She was incorrect in Item7 and Item 9.

For Level 3 items, Heart was incorrect on Item 14. This response indicates that she might not understand the necessary and sufficient conditions for parallelograms. She recognized only the familiar figure of parallelogram.

After the instruction. Heart was able do correctly 3 out of 4 on Level 2 items. She was correct in Items 6, 8-9 which showed that she might able to know and use the properties associated the definition. For Level 3 item, she was able to select the correct answer in Item 14. This might show that Heart was able to recognize that both S and R have the opposite side parallel which is the necessary and sufficient properties of parallelograms.

However, in the Geometric Achievement Test, Heart demonstrated that she might have difficulties to understand the necessary and sufficient conditions for congruent triangles. She was able to correct the Item 2 (see Figure 1) but incorrect in Item 5 (see Figure 2) and in Item 7 (see Figure 12) as Pam and Sand did.

--Classification. **Before the instruction**. Heart did the same things as Pam and Sand did; that is, she was able to give the correct response in Item 1 but incorrect in Item 5.

For Level 3 items, she was correct in Item 13 and incorrect in Item 15 as was Pam.

After the instruction. Heart showed strong growth in this construct. She was able to do correctly on all Level 3 items even though still do the same thing in Level 1 item. She was correct on Item 15; this implies she recognized the property "diagonals are equal" holds for a rectangle but not for a parallelogram. In the Geometric Achievement Test, Heart got good scores for the items that asked about this construct. Her response in Figure 15 illustrates her understanding of ordering and classification.

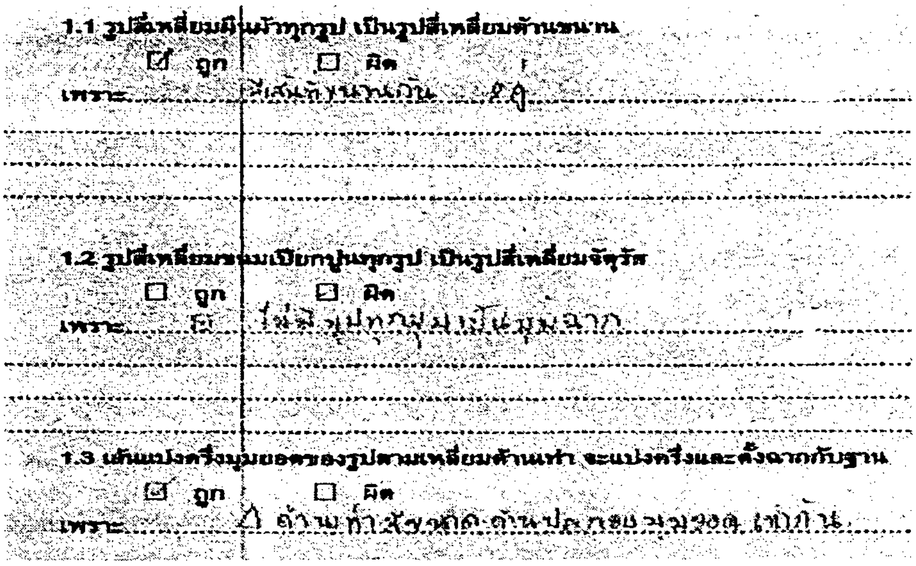


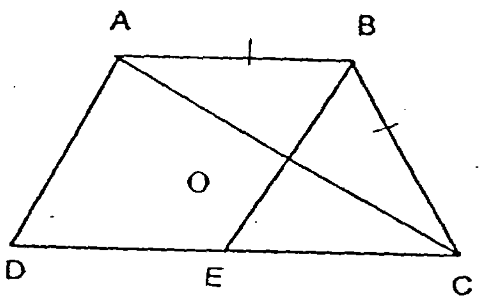
Figure 15 Heart's responses in the Classification items in the Geometric Achievement Test

Heart was able to give a reason why a rectangle was a parallelogram; she stated that the rectangle had two pairs of parallel sides. She also understood that the rhombus was not a square because its angles were not right angles. Another example, asked whether "the bisector of the vertex angle of an equilateral triangle is perpendicular to and a bisected the base." Heart was able to answer that it was true because an equilateral triangle had at least two sides of the same length, so the bisector of the vertex angle of an equilateral triangle was perpendicular to and bisected the base. This showed that she understood inclusive relationships between and equilateral triangle and isosceles, that is, an equilateral triangle is an isosceles triangle, and thus it has the same properties as an isosceles triangle.

--**Proof. Before the instruction.** Heart did correctly only 1 out of 2 on Level 3 items. She was not able to find the measurement of angle x in Item 11. Even though she had difficulty with Item 11, she showed the she was able to understand the if-then statement in Item 12.

After the instruction. Heart was able to find the measure of angle x in Item 11. She was able to apply the properties such as the angles of square, the angles of straight lines and sum of the interior angles of triangles to find the measure of angle x. Heart also did 3 out of 4 for the items which asked to apply properties to solved the problem in the Geometric Achievement Test. She was able to do as Pam and Sand. For the items asked about the notion of if-then statements, Item 12 in the van Hiele Geometric Thinking Test and Item 6 (figure 6) in the Geometric Achievement Test, she was correct both of them.

In the Geometric Achievement Test, Heart was able to respond 1 out of 3 items , Item 18-20 (Figure 7-9) which asked students to follow the given proof. This indicated that she was not able to follow the given proof and also create her own proof in the Proof item (Figure 10). Her response in Figure 16 illustrates that she was not able to create her own proof.



พิสูจน์

ข้อความ	เหตุผล
$\overline{AB}$ // [illegible]	กำหนดให้
AB = [illegible]	กำหนดให้
$\overline{AC}$ ตัดกับ $\overline{BE}$ ที่จุด O	กำหนดให้
AO = [illegible]	[illegible] △ [illegible]

Figure 16 Heart's response in the Proof item in the Geometric Achievement Test

Heart also wrote only given information, $\overline{AB} // \overline{DC}$ and AB = EC as Pam and Sand did. She was not able to give any correct arguments.

The researcher summarized Heart's thinking according to the constructs from the van Hiele Geometric Thinking Test and the Geometric Achievement Test in Table 18.

TABLE 18 HEART'S GEOMETRIC THINKING ON EACH CONSTRUCT BEFORE AND AFTER INSTRUCTION

Time / Construct	Before the instruction	After the instruction
Recognition	- Recognize figures by physical attributes and mathematical properties	- Recognize figures by physical attributes and mathematical properties
Definition	- Able to know and use sets of properties associated the definition - Unable to understand necessary and sufficient conditions	- Able to know and use sets of properties associated the definition - Unable to understand necessary and sufficient conditions
Classification	- Unable to understand inclusive relationships	- Able to understand inclusive relationships
Proof	- Unable to apply properties to solve problems - Able to understand if-then	- Able to apply properties to solve problems - Able to understand if-then statement - Unable to follow proof - Unable to create own proof

From the summarized table, Heart was not able to do well in definition, however, she tended to have understanding in classification which indicated Level 3 of geometric thinking after the instruction; she was able to understand the inclusive relationships of the figures. She did not do perfectly in proof.

In summary, after the instruction, all students exhibited that they were able to recognize the figures based on mathematical properties which indicate Level 2 of geometric thinking. Pam and Sand were able to understand more in the necessary and sufficient conditions which is an indicator for Level 3 thinking for definition. All students were able to understand the inclusive relationship between classes of the figure which indicate Level 3 of geometric thinking and this roughly imply that they exhibit Level 3 of geometric thinking for classification. Pam and Sand were able to show several aspects of proof in Level 3 of geometric thinking, for example, they were able to apply properties to solve problems, understand if-then statement and follow the given proof but they were not able to create their own proof. Heart was able to apply properties to solve problems, understand if-then statement but she was not able to follow the given proof and create her own proof.

The third section: changes in the constructs of each target student during the instruction

In this section, changes were traced during the instruction for each target students' thinking according to the constructs: recognition, definition, classification and proof by analyzing video and audio tapes and students' written works.

The researcher described individual target students' changes in each construct, started with Pam, Sand and Heart by showing words or sentences that students expressed during they were solving the problems in the sessions. Some dialogues were used among the three students because they discussed with each other.

Pam-- Recognition. Pam showed that she recognized figures by their simple mathematical properties which indicated Level 1 thinking according to Gutierrez & Jaime Framework in the early session but she recognized figures by advance mathematical properties in the later sessions. For example, in Session 1, students were asked to classify the given figures to different groups as many ways as they were able to do (see the problem of Session 1 in APPENDIX B). First of all, students helped each other to classify shapes which had three sides into the classes of triangles. With several kinds and sizes of the triangles given, Pam did not concern about size of the given triangles which indicated Level 1 of geometric thinking as the following excerpt:

Pam: I thought this figure (point at isosceles triangle) was triangle and those figure (point at a scalene triangle) as well.

Heart: Was this should be in a class? (She pointed to an equilateral triangle).

Pam: Yes, they were all triangles even that one (equilateral triangle) was bigger than others.

Dialogue 1

After that, all students classified the given figures into several classes: quadrilaterals and circles. This showed that they classified the triangles and quadrilaterals based on their number of sides which indicated Level 1 for recognition according to Gutierrez & Jaime Framework. However, in Session 6 (see the problem of Session 6 in

APPENDIX B), Pam recognized the figure by their mathematical properties which indicated Level 2 of geometric thinking. For example, she recognized the figure which she constructed to be an isosceles triangle because the two rays which were the legs of the triangles have the same measure. With the same session, when students were asked to find properties of an equilateral triangle; Pam still recognized the equilateral triangle by its properties, the equality of sides. She stated that the equilateral must have three sides equal as following dialogue:

Pam: Equilateral, every side was congruent, right?

Sand: Yes.

Heart: I thought it should have the same properties as an isosceles triangle but I did not know how it was.

Pam: It had the same properties as an isosceles triangle because its vertex was bisected. (She referred to the vertex of an equilateral triangle was bisected as an isosceles triangle).

Heart: Can you explain again?

Pam: Because an equilateral triangle was an isosceles triangle, so it had the same properties.

Dialogue 2

Pam also continued to recognize figures by their properties in Session 7 (see the problem of Session 7 in APPENDIX B). For example, Pam recognized that two segments, FC = BC, had the same measure, so the triangle BFC was isosceles triangle. This evidence showed that Pam recognized shapes by their mathematical properties which indicated Level 2 of geometric thinking.

-Definition. Pam showed that she was able to know and use the properties of figures and other geometric configurations and also understand the necessary condition for congruent triangle which indicated Level 2 of geometric thinking. For example, in Session 6, Pam used the properties of symmetry line to indicate the congruence of triangle. She stated that the line which bisected the vertex of an equilateral triangle divided it into two congruent triangles. She did not recognize the necessary and sufficient conditions of congruent triangles. The following excerpt showed her response:

Sand: Why were these two triangles congruent?

Pam: Because this line (line of symmetry) bisected the vertex of this triangle.

Dialogue 3

Pam continued to know and use the property of parallel lines to show her reason while she discussed with her Sand and Heart in Session 7 (see the problem of Session 7 in APPENDIX B), she knew that the distance between the parallel lines were equidistant. Her response was as follow:

Pam: This two lines were parallel (she pointed to the line AB and FC) because if you drew another line between them, that line was perpendicular to them.

Sand: But there was no line between them.

Pam: We were able to draw it. O.k. tried another way, because these two lines (AB and FC) were fit to each other.

Dialogue 4

For the congruence of triangles, Pam showed that she understood only the necessary condition of congruent triangles which indicated Level 2 of geometric thinking in Session 7. While discussion with her friends, she initiated the idea that Δ CGE $\cong$ Δ BGD. Her performance was as follows:

Pam: O.k. this triangle (Δ CGE) was congruent to this (Δ BGD) because this point (point D) was the midpoint of AB and this point (point E) was the midpoint of AE, so DB = EC and then DG = EG

Pam did not mention the angle, so she stated only the necessary conditions for congruent triangles; in other words, she stated only the two corresponding sides.

The researcher led the discussion with the whole class about the types of triangles and how they were related in the early of Session 6, students were asked to construct a triangle with 12 toothpicks. They were able to classify the triangles into three types: the triangle with two sides equal, the triangle with three sides equal and the triangle with no side equal. Students were able to know that the first type was called an isosceles triangle; the second type, an equilateral triangle; and the last type, a scalene triangle. The researcher asked students "Could an equilateral triangle be an isosceles triangle?" students said it could because the equilateral triangle had two sides equal. The researcher asked students "Could an isosceles triangle be an equilateral triangle?", students said that it could not because the isosceles triangle had only two sides equal. The researcher still asked students the same question but comparing between an isosceles triangle and scalene triangle. Students were able to answer correctly. From this discussion, Pam was able to understand the interrelationship between an isosceles triangle and an equilateral triangle, so ,as referred to Dialogue 2, Pam was able to see the interrelationship between classes of figure which indicated Level 3 of geometric thinking.

-Classification. After discussion about types of triangles in the early Session 6, Pam showed that she understood the inclusive relationship between an isosceles and an equilateral triangle. When the researcher asked students to explore the properties of isosceles triangles and asked them to construct an equilateral triangle, the researcher asked them to discuss the question "Do you think that the properties of the equilateral are the same as that of the isosceles triangles? Pam showed her response in Dialogue 2. Her response showed that she got the idea of inclusion of triangles by stating that the equilateral triangle was the isosceles triangle, so the sets of properties of an isosceles triangle could be the same that of an equilateral. This response indicated that she exhibited Level 3 of geometric thinking in this construct. Pam continued to understand the idea of inclusive relationship among the classes of quadrilateral in Session 11, when students were asked to describe the significant properties of parallelogram, square and rhombus and also the similarity and difference properties between square and rhombus, Pam still described those properties correctly all questions. For example, Pam showed that she understood the properties shared between rhombus and square, she stated that both figures were an quadrangle, two pairs of sides were parallel and equal, and sum of the interior angle was

180 °. When the researcher asked to answer the same questions but comparing between the square and parallelogram, Pam also answered correctly and gave her reason that because the square was the parallelogram.

-Proof. Pam exhibited Level 2 of geometric thinking for proof in the early session and still exhibit until the last session. For example, in Session 2, when the researcher gave three pairs of triangles and asked students to identify which pairs of triangles were congruent. Pam initiated the idea of using patty paper to justify the congruent triangle, she justified the first pair of triangles by using the patter paper to copy one triangle and then fit it on top of the other triangle. She still used the same strategies in the rest two pairs. Her idea showed in the following dialogue:

Pam: From the given three pairs of triangles, consider which one was congruent, if so, which part of the triangle was congruent. O.k. gave me some patty paper (She read the instruction and then asked her friend to give her the patty paper).

Pam: O.k. the first pair was fit. (She copied one triangle on the patty paper and then fit it on the other).

Heart: The first pair is fit!

Pam: All right.

Sand: Was the second pair fitting? (Sand asked when she saw Pam fit the triangle on top of the other triangle).

Heart: The second pair was not fit, we were able to justify by looking it.

Pam: No, the second one was also congruent but the third one was not!

Dialogue 5

Pam still used the patty paper to justify the triangle to be congruent in Session 3-5. When the researcher asked students to prove the statement in the later sessions, for example, Session 9-10 and 12, students used drawing and gave several examples to justify the statement. For example, in Session 10, the researcher asked them to justify sum of interior angles of triangle was 180°, Pam had no idea to justify , so she asked Sand to do.

Sand drew different kinds of triangles: an equilateral triangle, an isosceles triangle and a right triangle to justify that sum of interior angles of triangle was 180° (see figure 17). She gave the measure of angles in each triangle, for instance, she gave 60° to all angles in equilateral triangle, Pam helped her to gave 75°, 75° and 30° in isosceles triangle and Heart wrote the symbol of right angle in the right triangle. After that, they conclude that every triangle had sum of the interior angle equal to 180°. The interesting thing was even though students used examples of triangles to verify the truth of statement, they showed their sophisticated understanding by drawing a variety types of triangle rather than drawing several triangles to generalize that sum of the interior angles of triangle equal to 180°.

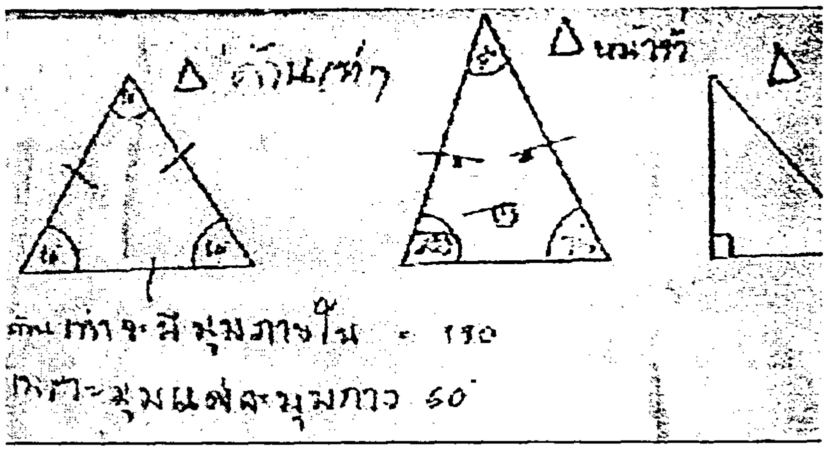


Figure 17 Students' proof for sum of the interior angles of triangle

Sand--Recognition. Sand also recognized the figure by their number of sides which indicated Level 1 of geometric thinking as Pam in Session 1 (see the problem of Session 1 in APPENDIX B), but in the same session, she generated the idea of recognition of the figure by their mathematical properties which indicated Level 2 of geometric thinking. For example, Sand was confused whether the figures were a parallelogram or rhombus. This showed that she concerned about sides of the figures which indicated the properties of the figures. She tried to measure sides of the figure. In session 6 (see the problem of Session 6 in APPENDIX B), she also recognized that the figure on the patty paper was isosceles triangle because it had two sides equal. Sand continued to recognize figures by their properties in Session 7, this showed that Sand recognized shapes by their mathematical properties which indicated Level 2 of thinking in this construct.

-Definition. Sand showed that she knew the properties associated with the figures which indicated Level 2 of geometric thinking as Pam did. For example, in Session 7 (see the problem of Session 7 in APPENDIX B), she was able to stated that the base angles were equal when she identified that those triangles were isosceles triangles. In Session 6 (see the problem of Session 6 in APPENDIX B), she was able to identify that the triangle from folding the patty paper was the isosceles triangle. Sand gave her reason that AB =AC because they were the legs of isosceles triangle. For the congruence of triangle in the same session, Sand stated that $\overline{AD}$ divided the isosceles triangle into two right triangles which were congruent by SAS. She knew that $\overline{AD}$ was a common line and was perpendicular to $\overline{BC}$, so she got a right angle. But from these sides and angles did not show that the triangles were congruent by SAS as she claimed. This seemed to show that Sand was able to state only the necessary conditions for the congruent triangles. However, in the same session, when considered the equilateral triangles with the common line $\overline{AD}$, Sand justified that the two triangles were congruent by SSS. This seemed that she knew from the isosceles that $\overline{BC}$ was bisected, so she was able to know that BD = CD. So she was able to conclude that the triangles were congruent by SSS. This seemed to show that she understood the necessary and sufficient conditions of congruent triangles which indicated Level 3 of geometric thinking.

As referred to Dialogue 3 in Session 4, Sand also showed that she understood the necessary and sufficient conditions for congruent triangle (SSS). She tried to explain when

Pam said that the two triangles were congruent because they had all sides the same measure but it was not enough to say. She would like to emphasize that Pam should concern about the corresponding sides.

From the discussion in the early Session 6, Sand also got the idea of interrelationship between classes of figure which indicated Level 3 of geometric thinking as Pam. For example, Sand was able to describe the similarity and difference between rhombus and square in Session 11 (see the problem of Session 11 in APPENDIX B). She was able to know that the difference between the two figures was the right angles, that was, square had four right angles, on the other hand, rhombus did not have.

-Classification. Sand showed that she understood implicitly the inclusive relationships between classes of figures which indicated Level 3 of geometric thinking in this construct. For example in Session 6, when students were asked to find the properties of equilateral triangle, Sand did not comment that an equilateral triangle was an isosceles triangle like Pam and Heart. She just listened to her friends discussed and accepted it. When students were asked to find the properties of equilateral after found the properties of isosceles triangle, Pam repeated to answer the same properties as were in an isosceles triangle. Sand showed that she understood the inclusive relationship between square and rhombus while discussing with her friends in Session 11, but she still repeated to answer the same questions for parallelogram and square instead of stating that square was parallelogram so the properties of parallelogram were the same of that of square.

-Proof. Sand also exhibited geometric thinking in Level 2 for proof. She still believed that using patty paper to copy the triangle and fit in on the top of another triangle was the best way to justify the congruence of triangles. Sometimes she measured sides and angles instead. In Session 12, Sand was asked by her friends to prove that the diagonal of rhombus was perpendicular to each other. She began by drawing the figure of rhombus and drew the two diagonals and then she measured the angle at the point that two diagonals intersected. She found that the angle was a right angle and she also confirmed by measuring the right angle by the protractor again. Her drawing was in the Figure 18 However, after the exploration and discussion in Session 3-5 (see the problem of Session 3-5 in APPENDIX B), Sand seemed to give the informal argument. For example, in Session

6 and 7, she tried to justify the congruence of triangle by using the theorem of triangle such as ASA, SSS.

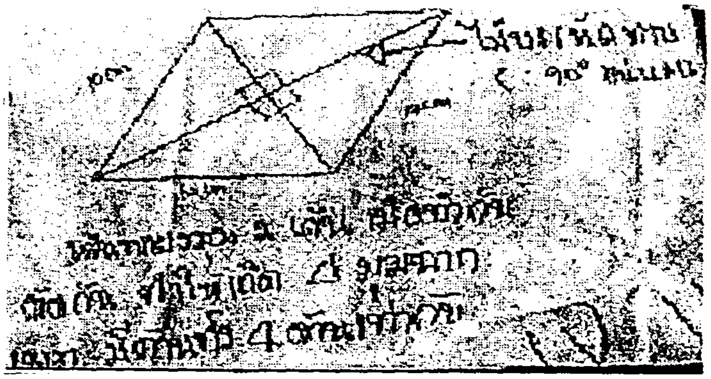


Figure 18 Students' proof for diagonals of rhombus

Heart--Recognition. Heart showed that she recognized the figures by their number of sides which indicated Level of geometric thinking as Pam and Sand in Session 1. But she always did not show her idea, she just listened to her friends' discussion. When Pam and Sand discussed about how to classify the given shapes into classes, and then decided to group the given shapes by number of sides, she agreed with them. Her duty was writing on the paper. She was not confident to even classify the figure. For example, when she picked the figure up, she hesitated that whether it was a parallelogram or not and then she asked Pam to confirm her thought. Heart tried to recognize the properties of isosceles triangle even she had misconceptions in Session 7. Her response was in the following excerpt:

Pam: FC = BC

Heart: Yes, because they were the arms of angle C (She assumed that triangle BGC is an isosceles triangle).

Dialogue 6

Heart learned from her friends to recognizing figures by their properties. Her group helped her to see and made her more confidence.

-Definition. Heart developed her understanding of necessary and sufficient conditions of congruent triangles which indicated Level 3 of geometric thinking from discussing with her friends. She was not able to state by her own, just waiting for her friend to answer and then she might agree or disagree. Heart was the first one to come up with the idea that the two triangles were congruent with SSS with the following excerpt in Session 4 (see the problem of Session 4 in APPENDIX B) :

Sand: You wrote (Sand asked Heart to write), triangles were congruent because....

Heart: What was the reason? Was that Side-Side-Side, wasn't it?

Pam: Was it possible, side-side-side?

Sand and Heart: Yes, it was.

Teacher: How do you know that it was congruent by side-side-side?

Heart: Because if you use this side (point to segment 5), this side (segment 7) or this side (segment 10) to be the base, it was the same, I thought.

Heart: But I did not know the exact reason.

Pam: Because all sides were the same measure, it was enough!

Sand: No, not because all sides the same measure, what did we say? I couldn't remember (Sand tried to use the word "corresponding").

Dialogue 7

-Classification. Heart did salient in this construct. She showed that she understood the inclusive relationship between classes of figures which indicated Level 3 of geometric thinking. For example, in session 6, she was the first one who came up with the idea that the properties of equilateral should be the same as that of an isosceles triangle as seen in Dialogue 2. It seemed that she recognized in her mind that an equilateral was a kind of an isosceles triangle but she was not confident in her thought. Heart continued to show this idea in Session 11. She stated that the properties of parallelogram could be applied to square. This might show that she understood the inclusive relationships among a parallelogram and a square.

-Proof. Heart also exhibited geometric thinking in Level 2 for proof. She still believed that using patty paper to copy the triangle and then fit on the top of another triangle in Session 2-5 was the best way to justify the congruence of triangles. She also used patty paper to show the congruence of segments or angles.

The forth section: summaries of changes in each construct

From data provided in sections 2 and 3, the researcher summarized the changes of each construct as follows:

Recognition. All students coherently and consistently recognized figures from their physical attributes to the use of mathematical properties since before, during and after the instruction.

Definition. Students showed that they were able to know sets of properties associated with definition of figures and the necessary condition of triangle congruence which indicated Level 2 of geometric thinking since before and during the instruction. For example, they were able to know the property of parallel line which was about the distance between the two parallel lines.

Students were able to exhibit Level 3 of geometric thinking in this construct, they were able to understand the interrelationships between classes of figures, for example, students stated and used implicitly properties which were shared by the parallelogram, rhombus and square. They stated that parallelogram had two sides parallel; rhombus had two sides parallel and diagonal perpendicular; square had two sides parallel and four right angles. It seemed to showed that students implicitly understood that both rhombus and square were parallelogram because they have the same properties, two sides parallel.

Students exhibited their understanding about the necessary and sufficient conditions of triangle congruence which indicated the Level 3 of geometric thinking. For example, they were able to justify the two triangles which were formed by dividing isosceles triangles were congruent by SSS. They were able to find three corresponding congruent sides and conclude that two triangles were congruent by SSS which was a necessary and sufficient condition of congruent triangles.

Classification. All students showed that they improved to Level 3 in this construct. Students exhibited the idea of inclusive relationships among classes of triangles and also classes of quadrilaterals. For example, when students were asked to find the properties of an equilateral triangle, they were able to state that set of properties of an equilateral were the same as that of an isosceles triangle because an equilateral was an isosceles triangle.

Proof. Students exhibited at Level 2 of geometric thinking in this construct. They usually used several examples or measurement to verify the truth of statements. For example, when the researcher asked students to verify about sum of interior angles of triangle, they verified by drawing several kinds of triangles such as an equilateral triangle, an isosceles triangle and a right triangle. They also determined the measure of interior angles in each triangle, for instance, each angle 60° in an equilateral triangle; 75°, 75° and 30° in an isosceles triangle; and one right angle in a right triangle. Then they concluded that each triangle had sum of interior angles 180°. Students usually used the patty paper to verify the congruence of triangles; they moved one triangle to fit on top of another triangle. However, students were able to show some aspects of proof in Level 3 of geometric thinking, for example, they were able to state the given part in if-then statement; they were able to apply the properties to solve the problem; they were able to follow the proof by given the informal arguments but they were not able to construct their own proof.

Sociomathematical Norms and Classroom Mathematical Practices

To address the fourth purpose, the researcher examined sociomathematical norms and classroom mathematical practices of discussion in whole class during the instruction. Table 19 shows the summary of sociomathematical norms and their corresponding classroom mathematical practices that emerged during the instruction.

TABLE 19 SOCIOMATHEMATICAL NORMS AND CORRESPONDING CLASSROOM MATHEMATICAL PRACTICES

Socomathematical Norms	Classroom Mathematical Practices
What count as a valid way of showing that triangles are congruent	• use of measurements • use of fit on top of each other • use of reasoning (taken-as-shared)
What constitutes a valid proof	• use of drawing and examples • use of chain of reasoning

The paragraphs below described the emergence of the sociomathematical norms and the classroom mathematical practices accompanied to them.

What count as a valid way of showing that triangles are congruent. The sociomathematical norm "what count as a valid way of showing that triangles are congruent" emerged during the discussion about the valid way to show that triangles are congruent in several sessions, for example, Session 1-5. In Session 1, when the researcher asked students to match the congruent geometric figures and then asked them the reason why those figures were congruent, **the first classroom mathematical practice: the use of measurements** emerged.

Most students said they measured sides and angles of those figures. Some of them fit one figure on top of another figure. When the researcher showed the figures which were on the paper and asked them how to show that those figures were congruent, most students still measured. However, some students stated that they could cut the figure and then fit it on top of another figure; some said using the patty paper to copy one figure and then fit on top of another.

From the idea of using patty paper to verify the congruent triangles emerged during the discussion in Session 1, in Session 2, the researcher gave students three pairs of triangles and asked them which pairs of triangles were congruent. Most students were able to answer that the first two pairs of triangles were congruent because each pair had sides the same length and also angles the same measure. Some students said they still measured sides and angles but some used the patty paper to copy the triangle and fit on top of another triangle.

The idea of using patty paper became to be the second classroom mathematical practice and still continued in Sessions 3-4 and also 5. In session 3, when the researcher asked students to construct the triangles under two conditions : using two given segments and one angle which included between the two segments and using two given segments and one angle which was not included between the two segments and then compared what condition made the triangle congruent. Students constructed and justify the congruent triangles by using the patty paper. During this time, some students noticed the relationship between the two segments and the angle; they said that the two triangles might be congruent because they had two pairs of corresponding sides and one pair of corresponding angle the same measure. When the researcher asked them which condition between the angle included in the two segments and the non-included angle in the two segments made the triangle congruent, the students were able to answer that the first condition which was the SAS theorem made the triangle congruent. In Session 4, the researcher asked students to construct the triangles by using three given segments and using three given angles and then compared which condition made the triangles congruent. Most students agreed that the first condition made triangles congruent. For the second condition, the triangles of some students were congruent but some were not. The researcher showed them the examples of both cases and asked students to discuss why some case was congruent while the other was not. Students discussed the case of congruent triangles and stated that the triangles were congruent because they are made by the same measure of sides. However, in the case that triangles were not congruent, they knew that because the measure of sides were not the same. From this discussion, the students were able to understand that the condition SSS made triangles congruent, on the other hand, AAA did not made congruent triangles. This seemed to show that students initiated to recognize sides and angles of triangles instead of using the patty paper to show the congruence of triangles.

From the idea of recognizing sides and angles of the triangles in Session 4, **the third classroom mathematical practices: the use of reasoning** emerged. A few students still used the patty paper to show the congruence of triangles in Session 5 but most students changed to consider the relationships between sides and angles. They were able to conclude that when given two angels and one side, the triangles were congruent when the side was included between the two angles which was ASA theorem. Students continued to use the theorems of congruent triangles to show that triangles were congruent in Session 6. They were able to give reason that the line of symmetry divided the isosceles into two triangles which were congruent by SSS, SAS and ASA theorems. The congruent triangle theorem became taken-as-shared to show that the triangles were congruent.

What constitutes a valid proof. The second sociomathematical norm "what constitutes a valid proof" emerged during the discussion about what the valid way to prove properties of parallel lines in Sessions 9-10 and 12.

The first classroom mathematical practice: the use of drawing and examples emerged in session 9, when students were asked to prove the converse of angle properties of parallel lines, they were not be able to understand the converse statement and they tried to draw the figure to represent the statement. For example, when the researcher asked them to prove the converse of alternate interior angle of parallel lines theorem: if two lines are cut by a transversal to form pairs of congruent alternate interior angles, then the lines are parallel. Most students drew the parallel lines and then measure an alternate interior angles and concluded that the two lines were parallel. In Session 10, when asked students to prove the sum of interior angles of triangles, some students drew several triangles to show the sum of interior angles of triangle was 180°, some drew a variety kinds of triangles such as an equilateral triangle, an isosceles triangle and a scalene triangle and then gave them the measure of angles. Some drew only the equilateral triangle and identified that the each interior angle was 60°, then the sum of interior angles was $3(60) = 180^{\circ}$. In the same session, some students used the idea angle properties of parallel lines to justify the sum of interior angles of triangles. They drew a parallel line and then drew the triangle between the lines. They used the property of alternate interior angles and property of straight line to prove sum of the interior angles of triangle. During discussion, the researcher convinced students to see that the use of chain of reasoning was able to show that sum of the interior angle was true for every triangle. **The second classroom mathematical practice for this sociomathematical norm :use chain of reasoning** emerged. In Session 12, the researcher asked to prove the properties of diagonal of rhombus, most students still proved by drawing the figure of rhombus with the diagonals perpendicular to each other and confirmed that the angles was the right angles by measurement, so the first classroom mathematical practices was become taken-as-shared..

CHAPTER 5

CONCLUSION AND DISCUSSION

This study adopted the open approach to examine the impact on students' levels of geometric thinking and geometric achievement of middle school students and also trace changes in the constructs formulated from Gutierrez and Jaime (1998) across levels of geometric thinking during the instruction as well as to examine the emergence of sociomathematical norms and classroom mathematical practices. This chapter provided summary, discussion, limitation as well as implications of the study.

Summary of the Study

Purposes of the study. This study carried out the open approach and was designed to address the following purposes:

1. to develop instruction based on the open approach and to study its impact on levels of geometric thinking of eighth grade students,
2. to evaluate the effect of the instruction based on the open approach on geometric achievement of eighth grade students,
3. to trace changes in target students' levels of geometric thinking,
4. to examine sociomathematical norms and classroom mathematical practices emerged during the instruction.

Methodology. The participants of this study were eighth grade students and a witness. The students were randomly chosen from two classes among five classes with mixed ability of eighth grade students in the Demonstration School of Chiang Mai University, Thailand. One class with 40 students was randomly assigned to be the experimental group and another class with 42 students was randomly assigned to be the control group. The experimental group was taught by the instruction constructed by the researcher while the control group was taught by another teacher with the conventional method. Both classes used the same contents and exercises. One witness, a mathematics teacher, helped the researcher to observe students in the experimental class.

Two weeks before the teaching, all students in the experimental group were administered the van Hiele Geometric Thinking Test to assign the levels of geometric thinking. Most students were at Level 2 and 3 of geometric thinking. Six students: three from Level 2 of geometric thinking and three from Level 3 of geometric thinking were chosen to be target students. During the instruction, the researcher taught as set in the lesson plans three days a week for eight weeks (twelve 100 minute-sessions). The witness observed the classes. After the instruction, the van Hiele Geometric Thinking Test was administered again to the experimental group. The Geometric Achievement Test was administered to both the experimental and control groups.

Data of this study were from the following sources: (a) the van Hiele Geometric Test, (b) the Geometric Achievement Test, (c) videotapes and audiotapes of classroom events, (d) students' written work, and (e) witness' reflections. There were two parts to the data analyses. The first part was concerned with students' responses in both tests. The second part was concerned with analysis of target students' thinking during the instruction. This analysis focused on students' individual and collective thinking. It also identified the classroom sociomathematical norms and classroom practices emerged during the instruction.

Results of the study

1. The instruction developed according to Taba Model and based on the open approach could be applicable. The impact of the instruction on students' levels of geometric thinking; there was significant difference (χ^2 (df = 2) = 14.10, $p < .05$) on levels of geometric thinking and the instruction based on the open approach. This showed that after the instruction, there were the increase in the number of students exhibiting Level 3 of geometric thinking and the corresponding decrease in the number of students exhibiting Level 2 of geometric thinking.

2. The effect of the instruction on students' geometric achievement; there was no significant difference ($p = .432$) on geometric achievement between the experimental and control group.

3. Changes of all Level 2 target students to Level 3 of geometric thinking; they were traced changes in the following constructs:

Recognition. All three students coherently and consistently recognized figures from their physical attributes which indicated Level 1 of geometric thinking to the use of mathematical properties which indicated Level 2 of geometric thinking in this construct.

Definition. Students showed that they were able to know sets of properties associated with definitions of figures and the necessary conditions of congruent triangles which indicated Level 2 of geometric thinking in this construct. They exhibited Level 3 of geometric thinking, interrelate properties between classes of figures, understand the sufficient conditions, and necessary and sufficient conditions of congruent triangles.

Classification. All students showed that they improved to Level 3 of geometric thinking in this construct. Students exhibited the idea of inclusive relationships among classes of triangles and also classes of quadrilaterals.

Proof. Students exhibited this construct at Level 2 of geometric thinking. They often used several examples or measurements to verify the truth of statements. They also used the patty paper to verify the congruence of triangles. However, students were able to show some aspects of proof in Level 3 of geometric thinking, for example, they were able to understand if-then statements, apply the properties to solve problems, follow the proof but not be able to construct informal proof.

4. The emergence of sociomathematical norms and classroom mathematical practices. Two sociomathematical norms emerged accompanied with their classrooms mathematical practices. The first sociomathematical norm was "what count as a valid way of showing that triangles are congruent". The classrooms mathematical practices accompanied with this sociomathematical norm were the use of measurements, the use of fit on top, and the use of reasoning. The second sociomathemacal norm was "what constitutes a valid proof" accompanied with the classroom mathematical practices were the use of drawing and examples, and chain of reasoning.

Discussion of the Study

Development of geometric thinking level

Regarding to students' geometric thinking performance, the findings of this study revealed that in the experimental group, most students including target students developed their levels of geometric thinking to the higher level, from Level 2 to Level 3 of geometric thinking. As Gutierrez and Jaime (1998) stated that the van Hiele levels of thinking were integrated by several processes: recognition, definition, classification and proof, thus, it is possible that students may progress in the mastering of some processes but not of others, so he or she progresses in some process on the higher level. In accordance to this study, the researcher examined each process and found that target students mastered on some processes: definition and classification but not on proof in Level 3 of geometric thinking.

Another factor that possibly made students changed to the higher level is the open –ended problem and the processes of the instruction based on the open approach. The open-ended problems provided students to see a variety of solutions; after students solved the open-ended problems, they had to present and discuss with the whole class. This kind of processes helps students develop their thinking. For example, in teaching the SSS Theorem of congruent triangles, students were asked to construct the triangle with three given segments and then compared their triangle with those constructed by their friends whether the triangles were congruent. After that, they were asked to construct the triangles with the three given angles as well as compared them. When student presented their solutions, in case of the triangles which were constructed by the three given segments, they agreed to conclude that the triangles were congruent, that was, by Side-Side-Side (SSS) Theorem. On the other hand, in case of the triangles which were constructed by the three given angles, some group showed that their triangles were congruent but some groups were not. From this point, students had to discuss why the results were not the same. After discussion with the teacher in whole class, students were able to conclude that when they had three angles congruent, the triangles may or may not be congruent.

In addition, the factor which is possible to help student progress to the higher level is the social factors, sociomathematical norms. From the environment provided by the researcher, students had an opportunity to discuss what counts as a valid way to show the triangles be congruent and what count as a valid proof. Followed from their discussion, the classroom mathematical practices which were compatible to this sociomathematical norm were evolved to the sophisticated level, that was, from using measurement to the use of reasoning. This might be because students have opportunity to negotiate with each other during the discussion that makes the evolution of sophisticated level. The result of this researcher is consistent with the study of Borrow (2000). She identified sociocultural factors that contributed to students' progress to the higher levels while using Shape Makers to teach geometry. A classroom justification norm and healthy mathematical skepticism were two of these factors. As mathematical interactions occurred in the classroom, details about what constituted an acceptable and appropriate justification evolved through several levels of sophistication. As a result, students would attempt a more rigorous argument, refine and revise inferences or justification, or restrict the domain of application for the inference. The students moved from relate property inference and justify these inferences based on empirical evidence to justify inference using local logic. That showed student progress to the sophisticated level.

However, from the result of the study showed that there were a few students who decreased to the lower level and the majority of students were at the same level before and after the instruction. The possible factors which impeded students' progress are students' previous experiences and the classroom culture (Nodha. 2000 : 47). There are two factors that influence student's previous experiences: their learning style and teaching strategies, and the use of mathematics textbooks. In Thailand, most students are familiar with teacher-center approach, that is, teacher teaches rules and principles and let students practice those rules in the exercise (Inprasitha. 1997 : 233). Students are not familiar with learning by themselves or learning by problem solving. They do not see the importance of exploring the concepts, they just wait for what the teacher teach. This learning style does not support students' thinking. For Thai mathematics textbooks analyzed by Inprasitha (1997) , he stated that that most of the exercises and word problems in the textbooks are merely routine exercises and have one correct answer. Students were practiced by these kinds of exercises since they were young, so students may not familiar with the use of open-ended problems. In the aspect of classroom culture, students are not familiar with the environment of discussion with each other in the whole class. The researcher was usually leading the discussion. Students were waiting for the explaining and summary from the researcher.

Geometric thinking level according to constructs

With regard to "**definition**", performances of target students presented in the situation of defining a parallelogram, a rhombus and a square were interesting. They defined the parallelogram, rhombus and square by what de Villiers (1994) called *hierarchical* definition. Students defined a parallelogram, a rhombus as a *hierarchical* definition, that is, they defined a parallelogram; two pairs of parallel sides; a rhombus; two pairs of parallel sides and two diagonals intersected made the right triangles (this showed that two diagonals are perpendicular). From their defining, they define by providing the inclusion of special cases, that is, for parallelogram, rectangle, rhombus, and square can be included in its class of parallelogram because they were all have two pairs of parallel sides. For rhombus, students defined rhombus by providing square to be included in its class. de Villiers (1998) suggested the advantages of defining with hierarchical definition are students can see that definition should be economical and when providing with the hierarchical definition, all theorems proved for that concept then automatically apply to its special cases. For example, if we prove that the diagonals of a parallelogram bisect each other, we can immediately conclude that it is also true of rectangles, rhombi and squares. However, students sometimes defined a *partitional* definition. In the same session, they defined a parallelogram as it has two longer and two shorter sides which is accordance with de Villiers (1994) that sometimes students preferred to define a partitional definition even they understood the hierarchical definition.

With regard to "**classification**", target students showed their understanding the idea of inclusive relationship between classes of figures consistent with the state of ordering properties of Matsuo (2000 : 273). For example, students were able to identify the similar and different properties between square and rhombus which is consistent with the forth state ,State d : the explanation about the similarities and differences between the two figures. Students also showed that square is rhombus, that is consistent with the final stage, state e : one figure is subordinated to the other figure.

With regard to '**proof**', target students exhibited Level 2 of geometric thinking for this construct, they usually verified the statement by examples and measurements and also used the patty paper to verify the congruence of triangles which identified in Level 2 of geometric thinking. (Nesser. 1991). Student were not able to exhibit Level 3 of geometric thinking for proof, it might be because they did not recognize the importance of proof as de Villiers (1987 : 117) stated that students in Level 1 or 2 do not doubt the validity of their empirical observation, so proof is meaningless for them, they see it as justifying the obvious.

However, in topic of congruence of triangles, Nesser (1991) studied congruence of triangles according to the van Hiele levels of thinking and stated that the study of congruence requires reasoning at least at Level 3, since the object of study are the relations between shapes. Students who were able to establish and understand the sufficient conditions for the congruent triangles were at Level 3 and students who were able to reason logically by using the cases of congruent or transformation were assigned to Level 4. Thus, in contrast, target students who exhibited Level 2 for this construct were able to understand and used the sufficient conditions to reason for congruent triangles which indicated Level 3 and 4 of geometric thinking according to Nesser's study.

Geometric Achievement

As a result that geometric achievement of students in the experimental group and the control group was not different, that means instruction based on the open approach had not affected to the geometric achievement. However, if considering the means and standard deviation between the experimental and the control group, the means and standard deviation of the experimental group was slightly higher and smaller than the control group. This showed that the students in the experimental group who got scores near mean scores was more than that of in the control group. This showed that the abilities of students in the experimental group were not much different. Because the open approach served individual differences (Nodha. 2000 :40), so the low-ability students were able to understand the concepts of congruent triangles and parallel lines when they had discussion with teacher or their friends.

Other remarks

From observation in class, the researcher found some interesting information about construction in geometry. When the researcher asked students to construct a triangles by given segments or angels by their own, most students used protractor to measure segments and angles, but few students were not able to use protractor. They tried to find semicircle to measure the angles. A few students used compass and the geometric constructions.

The study of Chamnankit (2001) provided some information that most students, eighth grade students and even the college students, were not able to justify the congruent triangles while they were not the same direction. In this study, students did not face this problem, they were able to justify the congruence of triangles even they were not the same directions. This might be possible that students learned the basic of transformations, rotate, flip and turn the figures before learning congruence. They were able to understand the conditions of congruent figures;, they were the same figures which preserved their shapes, so they were able to rotate, flip and turn. Different directions of the two figures have not affected their thinking. As suggested by Nasser (Kilpatrick, J. et al (2003); citing Nasser. 1992), student should be taught to use transformation to learn about congruence and this approach were made student advance to a level of van Hiele.

Students were able to apply the theorems of congruent triangles to solve the problems in parallel lines and parallelogram, as suggested by Kilpatrick et al (2003) that concepts are taught more effectively in sets of two or more related concepts than one at a time.

Limitation of the Study

The limitation of this study was time allocation. In this study, the researcher taught 100 minutes (two periods) in each session, but in the normal class took only 50 minute per period. As suggested by Becker and Shimada (1997 : 33) that the teacher needs to allow enough time to explore the problem and discussion, the teacher may use two class periods for one problem; one for solving the problem and another for summarizing and discussing. It may be possible that this setting might effect to the progress of level of geometric thinking.

Implications of the Study

This study provided the information of students' geometric thinking according to the constructs: recognition, definition, classification and proof and how they developed these constructs. By this information, teachers or curriculum developers should design the instruction which served students' individual differences, and also based on students' thinking for developing their geometric thinking.

Further research should be carried out in several directions. Gutierrez & Jaime framework can be used as a tool for assessing the students' level of geometric thinking, it helps teacher to understand mathematical processes of students. Several advantages of the open approach are identified, for example, it provides student to fully express their thinking and helps teacher to understand them; it does not take time to teach because the several concepts can be grouped in one problem, so the use of open approach need to widely use in other areas of Thailand and other fields of mathematics.

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APPENDIX

APPENDIX A

ASSESSMENT INSTRUMENTS

van Hiele Geometric Thinking Test

Name.......................................Surname.............................Class.............Number............

Instructions

1. This test consists of 20 multiple choices.
2. Choose your answer and put X in the box.

Thank you for your cooperation

Answer Sheet

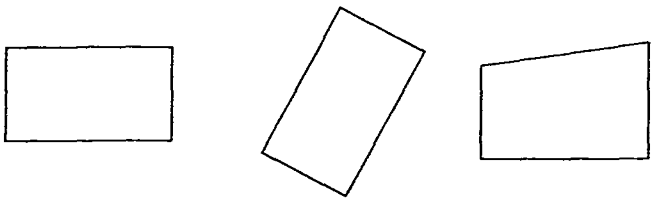
item	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
a																				
b																				
c																				
d																				
e																				

1. Which of these are triangles?

A B C D

a. B only
b. C only
c. C and D only
d. B and D only
e. None of these are triangles

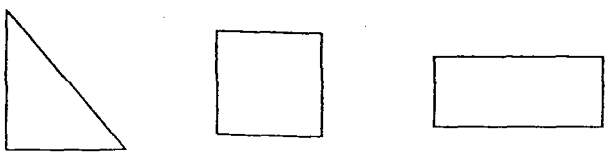
2. Which of these are rectangles?



M N L

a. M only
b. N only
c. M and N only
d. M and L only
e. All are triangles

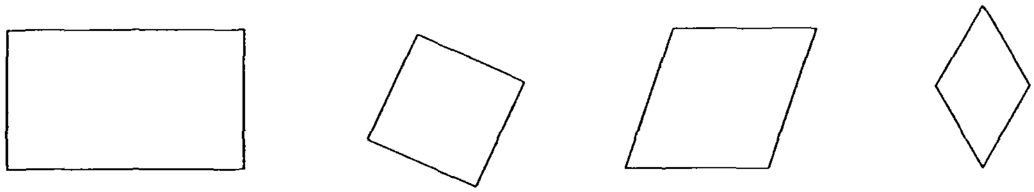
3. Which of these are squares?



G H K

a. G only
b. H only
c. K only
d. H and K only
e. All are squares

4. Which of these are squares?



C D E F

a. D only

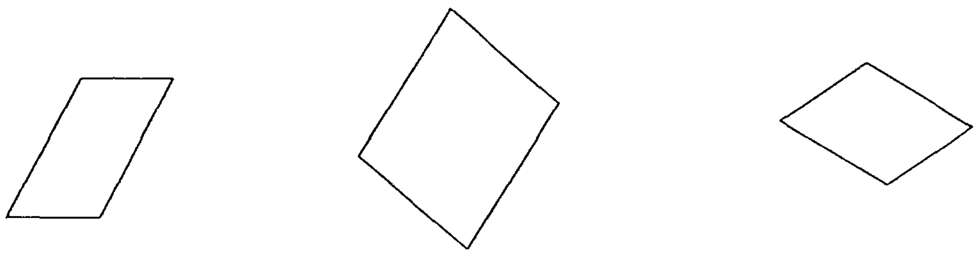
b. C and D only

c. D and F only

d. All of these are squares

e. None of these are squares

5. Which of these are parallelograms?



P Q R

a. P only

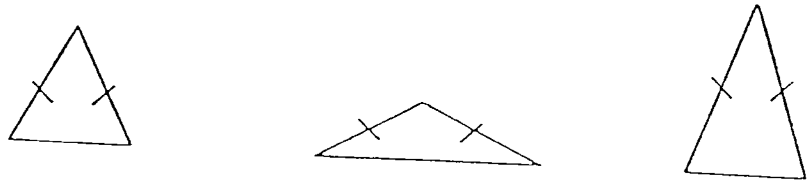
b. R only

c. P and Q only

d. None of these are parallelograms

e. All of these are parallelograms

6. An isosceles triangle is a triangle with two sides of equal length. Here are three examples.



Which of a – d is true in every isosceles triangles?

a. The three sides must have the same measure.

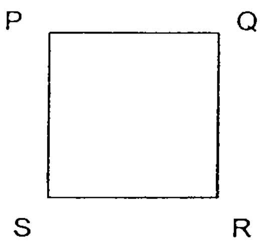
b. One side must have twice the length of another side.

c. There must be at least two angles with the same measure.

d. The three angles must have the same measure.

e. None of these is true in every isosceles triangle.

7. PQRS is a square. Which relationship is true in all squares?



a. PR and RS have the same length.

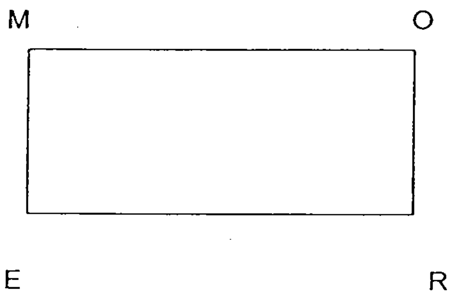
b. QS and PR are perpendicular.

c. PS and QR are perpendicular.

d. PS and QS have the same length.

e. Angle Q is larger than angle R.

8. MORE is a rectangle. Which of a – d is not true in every rectangle?



a. There are four right angles.

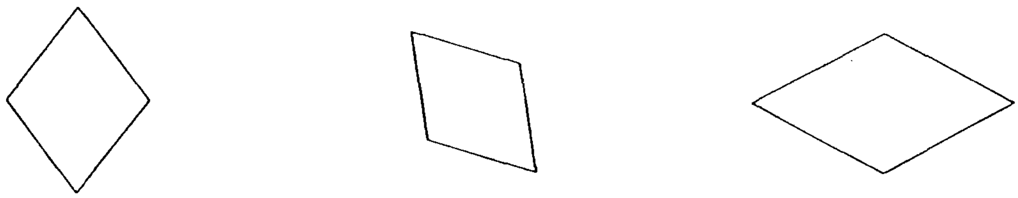
b. The diagonals have the same length

c. The four sides have the same length.

d. The opposite sides have the same length.

e. The opposite sides are parallel.

9. A rhombus is a 4-sided figure with all sides of the same length. Here are three examples



Which of a – d is not true in every rhombus?

a. The opposite angles have the same measure.

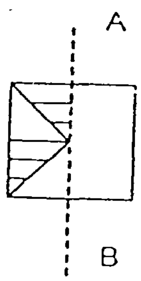
b. The two diagonals have the same length.

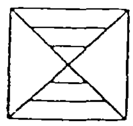
c. Each diagonal bisects two angles of the rhombus.

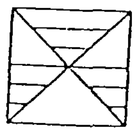
d. The two diagonals are perpendicular.

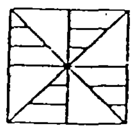
e. The opposite sides are parallel.

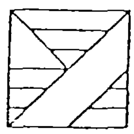
10. Nida designs the patterns of quilts as figure. AB is the line of symmetry, which one is the other sides of AB?

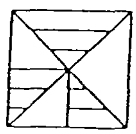


a. 

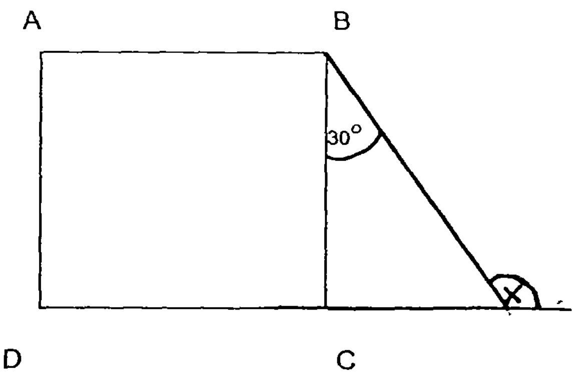
b. 

c. 

d. 

e. 

11. ABCD is a square. What is the measure of angle "x"?



a. 60 °

b. 90 °

c. 110 °

d. 120 °

e. 150 °

12. Here are two statements.

Statement S : ΔABC has three sides of the same length.

Statement T : In ΔABC , angle B and angle C have the same measure.

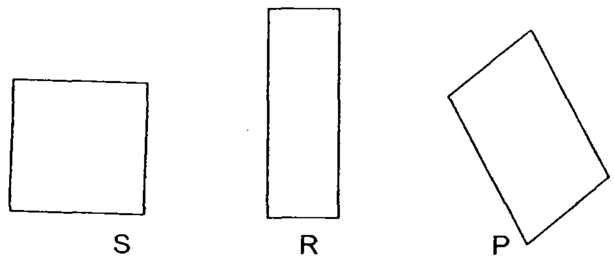
Which is correct?

a. Statement S and T cannot both be true.
b. If S is true, then T is true.
c. If T is true, then S is true.
d. If S is false, then T is false.
e. None of a – d is correct.

13. Which one is true?

a. Every square is rectangle.
b. Every rectangle is square.
c. Every parallelogram is rectangle.
d. Every parallelogram is square.
e. None of a – d is correct.

14. Which of these can be called parallelograms?



a. All can
b. S only
c. P only
d. S and R only
e. R and P only

15. What do all rectangles have that some parallelograms do not have?

a. Opposite sides equal
b. Diagonal equal
c. Opposite sides parallel
d. Opposite angles equal
e. None of a - d

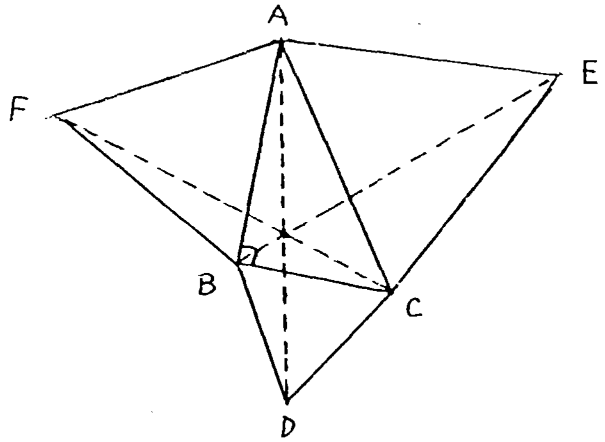
16. Here are three properties of a figure.

Property A : It has diagonals of equal length.
Property B : It is a square.
Property C : It is a rectangle.

Which is true?

a. A implies B which implies C.
b. A implies C which implies B
c. B implies C which implies A
d. C implies A which implies B
e. C implies B which implies A

17. Here is a right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.



From this information, one can prove that $\overline{AD}$ BE and CF have a point in common. What would this proof tell you?

a. Only in this triangle drawn can we be sure that $\overline{AD}$ $\overline{BE}$ and $\overline{CF}$ have a point in common.
b. In some but not all right triangles, $\overline{AD}$ $\overline{BE}$ and $\overline{CF}$ have a point in common.
c. In any right triangles, $\overline{AD}$ $\overline{BE}$ and $\overline{CF}$ have a point in common.
d. In any triangles, $\overline{AD}$ $\overline{BE}$ and $\overline{CF}$ have a point in common.
e. In any equilateral triangles, $\overline{AD}$ $\overline{BE}$ and $\overline{CF}$ have a point in common.

18. In geometry;

a. Every term can be defined and every true statement can be proved true.
b. Every term can be defined but it is necessary to assume that certain statements are true.
c. Some terms must be left undefined but every true statements can be proved true.
d. Some terms must be left undefined and it is necessary to have some statements which are assumed true.
e. None of a – d is correct.

19. Here are two statements.

I : If a figure is a rectangle, then its diagonals bisect each other.

II : If the diagonals of a figure bisect each other, the figure is a rectangle.

Which is correct?

a. To prove I is true, it is enough to prove that II is true.
b. To prove II is true, it is enough to prove that I is true.
c. To prove II is true, it is enough to find one rectangle whose diagonals bisect each other.
d. To prove II is false, it is enough to find one non-rectangle whose diagonals bisect each other.
e. None of a – d is correct.

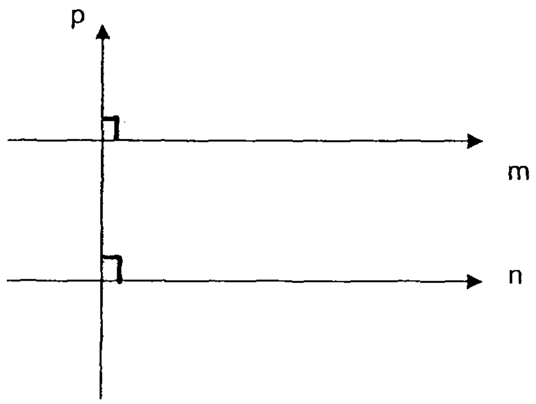
20. Examine these sentences.

(1) Two lines perpendicular to the same line are parallel.

(2) A line that is perpendicular to one of two parallel lines is perpendicular to the other.

(3) If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines m and p are perpendicular and lines m an d p are perpendicular.



Which of the above sentences could be the reason that line m is parallel to line n?

a. (1) only
b. (2) only
c. (3) only
d. Either (1) or (2)
e. Either (2) or (3)

Geometric Achievement Test

Congruence of triangles and Parallel lines

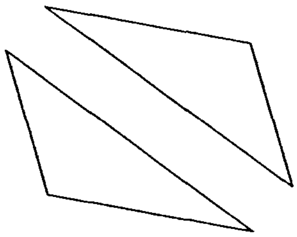
This test is divided into 2 parts. (40 points)

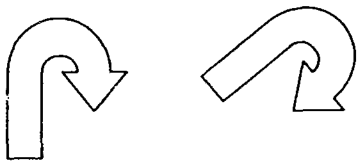
Part 1 Twenty items of multiple choices. (20 points)

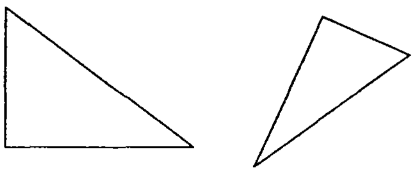
Part 2 Three written items. (20 points)

Part 1 Fill X in each item on the answer sheet (20 points)

1. Which pair of figures is not congruent?

a. 

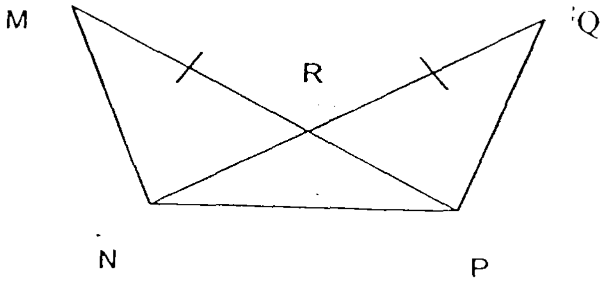
c. 

b. 

d. 

2. Which one is not true?
   a. Two figures are congruent when one figure is fitting on top of the other.
   b. Two segments are congruent when they have the same length.
   c. If two lines are intersecting, the opposite angles are congruent.
   d. Two triangles which have 2 angles the same measure and 1 side the same length are congruent.

3. Suppose MR = RQ and $\Delta MNR \cong \Delta QPR$ with SAS Theorem. Which one should be determined?



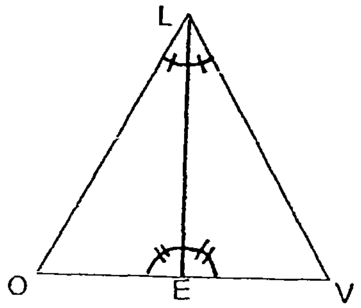
a. MN = QP

b. $M\hat{N}R = Q\hat{P}R$

c. $N\hat{M}R = P\hat{Q}R$

d. NR = PR

4. Suppose ΔLOV, $O\hat{L}E = V\hat{L}E$ and $L\hat{E}O = L\hat{E}V$. Which one is not true?



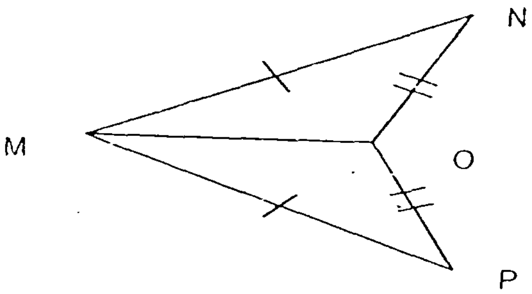
a. LO = LV และ OE = VE

b. LE = OV

c. $L\hat{O}V = L\hat{V}O$

d. ΔLOV is an isosceles triangle

5. Suppose NO = OP and $N\hat{O}M = P\hat{O}M$. If ΔMNO and ΔMPO are congruent by SSS Theorem, which one should be determined?



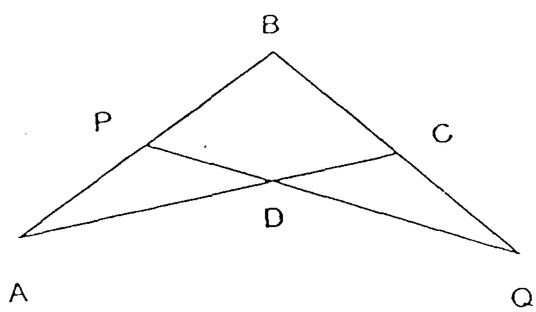
a. $M\hat{N}O = M\hat{P}O$

b. $N\hat{M}O = P\hat{M}O$

c. MN = MP

d. none of these is true

6. Suppose $\Delta ABC \cong \Delta QBP$. Which one is true?



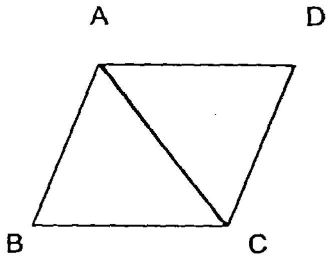
a. $A\hat{B}Q = A\hat{P}Q$

b. BQ = PQ

c. $B\hat{A}D = P\hat{D}A$

d. $B\hat{C}A = B\hat{P}Q$

7. From the given figures, suppose ΔABC and ΔADC are isosceles triangles whose base is $\overline{AC}$. Which one is true?



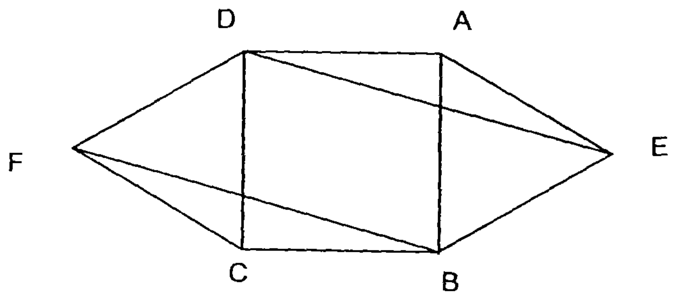
a. Cannot conclude that $\Delta ABC \cong \Delta ADC$

b. $\Delta ABC \cong \Delta ADC$ with SAS

c. $\Delta ABC \cong \Delta ADC$ with ASA

d. $\Delta ABC \cong \Delta ADC$ with SSS

8. $\square ABCD$ is a square. $\Delta ABE \cong \Delta CDF$ and both triangles are equilateral triangles. For proving that $D\hat{E}A = B\hat{F}C$, the sequence of proof is as follow:



(1) AD = CB and AE = CF

(2) $D\hat{A}E = B\hat{C}F$

(3) $\Delta DAE \cong \Delta BCF$

(4) $D\hat{E}A = B\hat{F}C$

From the given proof,

(3) $\rightarrow$ (4) means the statement (3) can be conclude to the statement (4)

(2) $\rightarrow$ (4) means the statement (2) and (3) can be conclude to the statement (4)
(3) $\nearrow$

Consider that which one is accordance to the proof above?

a.
(2) $\longrightarrow$ (4)
(1) $\longrightarrow$ (3) $\nearrow$

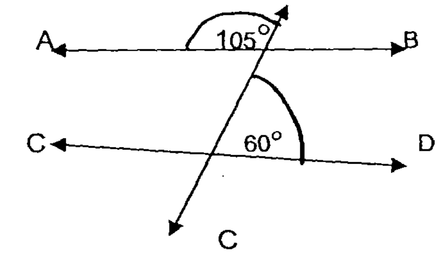
b (1) $\longrightarrow$ (2) $\longrightarrow$ (3) $\longrightarrow$ (4)

c. (1) $\longrightarrow$ (3) $\longrightarrow$ (4)

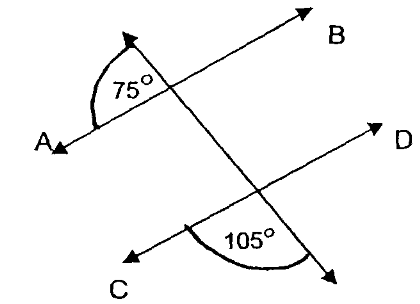
d.
(1) $\longrightarrow$ (3) $\longrightarrow$ (4)
(2) $\nearrow$

9. Which one determines $\overleftrightarrow{AB} // \overleftrightarrow{CD}$?

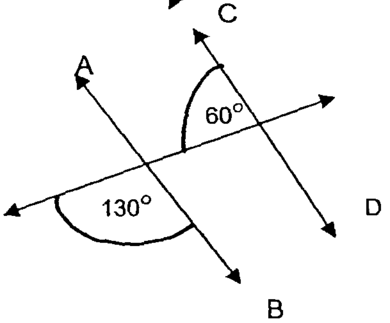
a.



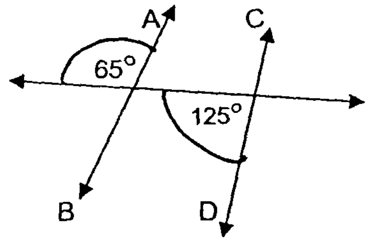
c.



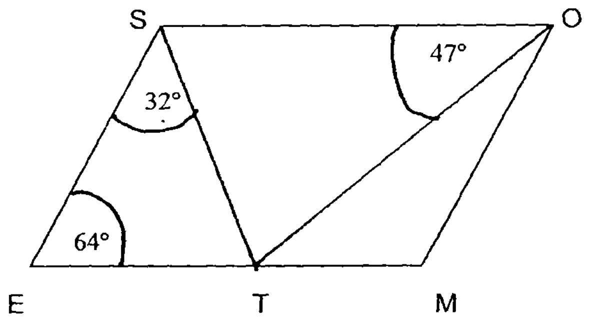
b.



d.



10. ▱SOME is a parallelogram. Find the measure of $S\hat{T}O$.



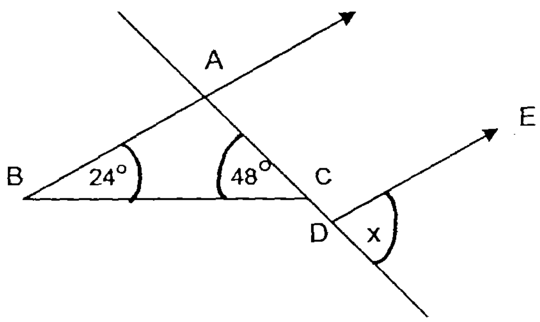
a. 32°

b. 49°

c. 84°

d. 116°

11. Suppose $\overrightarrow{BA} // \overrightarrow{DE}$. Find the measure of x.



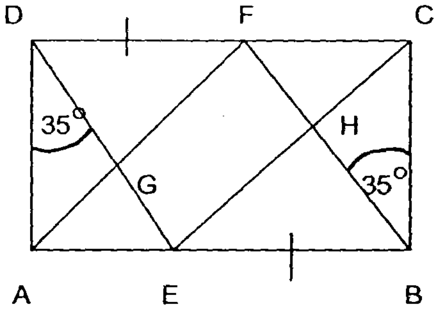
a. 24°

b. 48°

c. 72°

d. 108°

12. ABCD is a rectangle, DF = BE, $A\hat{D}E = 35^{\circ}$ and $C\hat{B}H = 35^{\circ}$. Which one is not true?



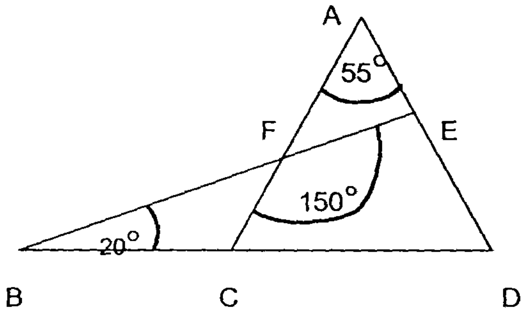
a. $\overline{DE} \,//\, \overline{FB}$

b. $B\hat{F}A = 35^{\circ}$

c. $\Delta DAE \cong \Delta BCF$

d. $\Delta ADF \cong \Delta CBE$

13. From the figure, $C\hat{A}D = 55^{\circ}$, $E\hat{B}D = 20^{\circ}$ and $C\hat{F}E = 150^{\circ}$. Find the measure of $F\hat{E}D$?



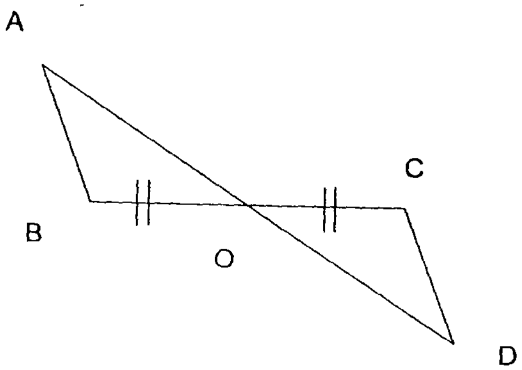
a. 12.5°

b. 25°

c. 75°

d. 85°

14. From the figure, $\overline{AD}$ intersects $\overline{BC}$ at O and BO = OC. $\Delta ABO \cong \Delta DCO$ by AAS. Which one should be determined?



a. AO = OD

b. AB = CD

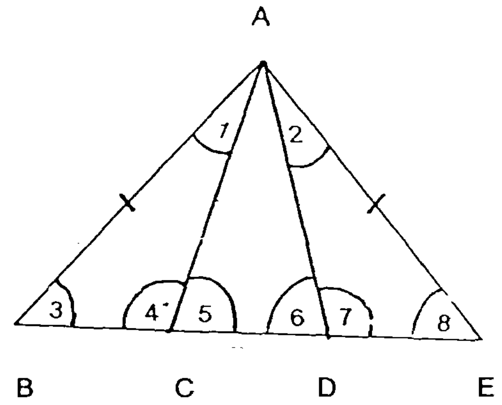
c. $A\hat{B}O = D\hat{C}O$

d. $B\hat{A}O = C\hat{D}O$

Use the figure and the given proof to answer the questions 15-17.

Given AB = AE , $\hat{1} = \hat{2}$

To prove $\hat{4} = \hat{7}$



Proof

	Statements	Reasons
1.	AB = AE	Given
2.	$\hat{3} = \hat{8}$	**a**
3.	$\hat{1} = \hat{2}$	Given
4.	$\Delta ABC \cong \Delta AED$	**b**
5.	$\hat{4} = \hat{7}$	**c**

15. Which statement determines "a"?

a. Given

b. base angles of isosceles triangle are equal

c. The vertex of an isosceles triangle

d. The angle of straight line

16. Which statement determines "b"?

a. SAS

b. AAS

c. ASA

d. SSS

17. Which statement determines "c"?

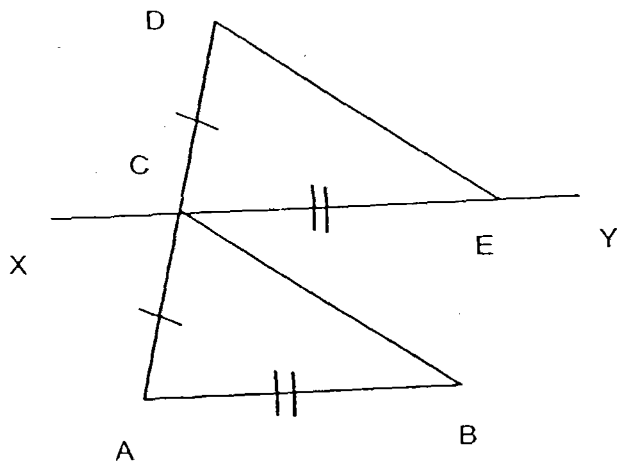
a. Given

b. The corresponding angles of congruent triangles are equal

c. The base angles of an isosceles triangle

d. The angel of straight line

Use the given proof to answer the questions 18-20.



Given $\overline{XY} // \overline{AB}$, AC = CD และ AB = CE

To prove $\overline{BC} // \overline{DE}$

Proof

Statements	Reasons
1. $\overline{XY} // \overline{AB}$	Given
2. $C\hat{A}B = D\hat{C}E$	**a**
3. AC = CD, AB = CE	Given
4. $\Delta ABC \cong \Delta CED$	SAS
5. $A\hat{B}C = C\hat{E}D$	The corresponding angles of congruent triangles
6. $A\hat{B}C = B\hat{C}E$	**b**
7. $C\hat{E}D = B\hat{C}E$	The same measure as $A\hat{B}C$
8. $\overline{BC} // \overline{DE}$	**c**

18. Which statement determines "a"?

 a. Given

 b. The corresponding angles of parallel lines are congruent.

 c. Sum of the interior angles of parallel lines is 180°

 d. The alternate interior angles of parallel lines are congruent

19. Which statement determines "b"?

a. The alternate interior angles of parallel lines are congruent

b. Given

c. The corresponding angles of parallel lines are congruent

d. Sum of the interior angles of parallel lines is 180°

20. Which statement determines "c"?

a. Given

b. If two lines are cut by a transversal to form sum of the interior angles of parallel lines is 180°, then the lines are parallel

c. If two lines are cut by a transversal to form the alternate interior angles of parallel lines are congruent, then the lines are parallel

d. none of these a-c is true

Part 2 Three written items. (20 points)

1. Consider the following statements, true or false, and then fill / in and give the reasons. (2 points per each, 8 points)

1.1 Every rectangle is a parallelogram.

true false

reason...

....

....

....

1.2 Every rhombus is a square.

true false

reason...

....

....

....

1.3 The line which bisects the vertex angle of equilateral triangle is bisect and perpendicular to the base.

true false

reason..

....

....

....

1.4 The diagonals of a rectangle have the same measure.

true false

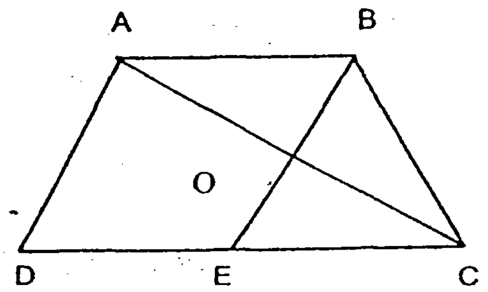
reason..

....

....

....

2. Suppose $\overline{AB} \parallel \overline{DC}$, AB = EC and $\overline{AC}$ cut $\overline{BE}$ at O, prove that AO = CO. (4 points)



Proof

Statements	Reasons
....	....
....	....
....	....
....	....
....	....

3. Prove that two pairs of the opposite angles of a parallelogram have the same measure. (8 points)

Drawing

Given ....
....

To prove ....

Proof

Statements	Reasons
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APPENDIX B

EXAMPLES OF LESSONS PLAN AND OPEN-ENDED PROBLEMS

EXAMPLES OF LESSON PLANS

Session 3

Fundamental Mathematics **Mattayom Suksa 2**

Topic : SAS Theorem **Time : 100 minutes**

Concepts

Side-Angle-Side Theorem

In the early session, to verify the congruence of triangles, we need to measure and compare six parts of both triangles, three corresponding sides and three corresponding angles. However, in some cases, if we know only three parts: two sides and one angle, two angles and one side, three sides and three angles of each triangle, it is sufficient to conclude that the two triangles are congruent.

There are six different ways for the combination between sides and angles, that are, three pairs of congruent sides (SSS), two pairs of congruent sides and one pair of congruent angles (angles between the pairs of sides : SAS), two pairs of congruent angles and one pair of congruent sides (sides between the pars of angles: ASA), two pairs of congruent angles and one pair of congruent sides (sides not between the pairs of angles : SAA), two pairs of congruent sides and one pair of congruent angles (angles not between the pairs of sides : SSA), three pairs of congruent angles (AAA).

We begin to investigate the cases of two sides and one angle: SAS and SSA. After the investigation, we can conclude that if we have two pairs of congruent sides and one pair of congruent angles and the angle includes between the pairs of sides, the rest of corresponding angles and corresponding sides must be congruent. Therefore, the two triangles are congruent.

SAS Theorem

"If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent".

Learning objective

Students can conclude that the two triangles which have two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. (SAS Theorem)

Instructional activities

Reviewing the previous lesson.

Teacher and students review the definitions of congruent triangles.

Presenting and understanding the problems.

1. Students discuss about the sufficient three parts of triangles which make the triangles congruent.
2. Each group takes the Problem 3 and the materials.

Solving the problems by students.

Each group solves the Problem 3.

Presenting and discussing the solutions.

1. Random 2-3 groups to present their solutions about 10 minutes for each.
2. Teacher and students discuss the following issues:
   2.1 Are the triangles you constructed and those constructed by your friends congruent? How do you justify that they are congruent?
   2.2 How do you conclude after doing the activities in Part 1 and Part 2?
   2.3 What condition does make the triangles congruent when given two segments and one angle?

Summarizing the lesson.

Teacher guides students to discuss that if the two triangles have two pairs of congruent sides and one pair of congruent angle, the two triangles are congruent when the congruent angle is included between the two pairs of congruent sides.

Working individually.

Students work in Exercise 1.3 individually.

Materials

1. Papers
2. Patty papers

3. Color pencils
4. Protractor
5. Compass

Evaluation

Students' evaluation

1. Students use the definition of congruent triangles to justify two triangles congruent.

2. Students understand the necessary and sufficient condition (SAS) for the congruence of triangles.

Teacher evaluation

Role of teacher in supporting and facilitating students' thinking when discuss about the condition of congruent triangles when given two segments and one angle.

Expected responses

Students can give the reason that we need only three parts of triangles to conclude the congruence of triangles. They can understand the condition of SAS theorem that when they have two pairs of congruent sides and one pair of congruent angle, the triangles will be congruent when the congruent angle is included between the two congruent sides.

Problem 3

Fundamental Mathematics **Mattayom Suksa 2**

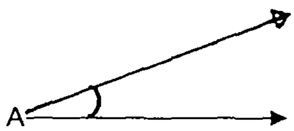
Topic : SAS Theorem **Time 30 minutes.**

Directions

Part 1 Given the following segments and angle:

8 centimeters

5 centimeters



1. Construct a triangle on your own paper using these two segments as sides and the angle as the angle between them.

2. Compare your triangle with the triangles constructed by others in your group. Are they congruent? Discuss why the triangles are congruent.

Part 2 Construct another triangle using these two segments as sides and the angle as the angle is not between them.

If you can not construct, follow these steps:

1) Construct $\hat{P}$ congruent to $\hat{A}$.

2) At point P, measure one arm 8 centimeters and determine point Q.

3) Set the compass to a radius of 5 centimeters and place the point on Q. Draw an arc to cut another arm of $\hat{P}$. Label the intersecting point R and form the triangle.

4) Answer the following questions:

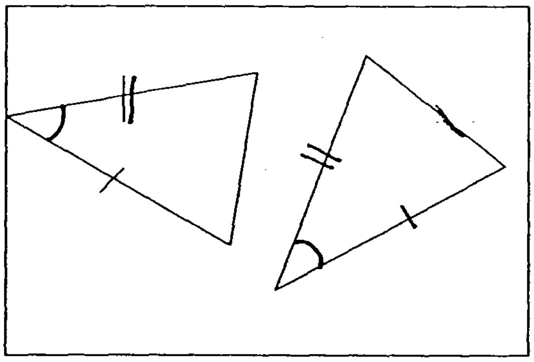
4.1 How many times does your arc cut the line?

4.2 How many different triangles can you construct? Which triangle is congruent to the triangle constructed in Part 1.

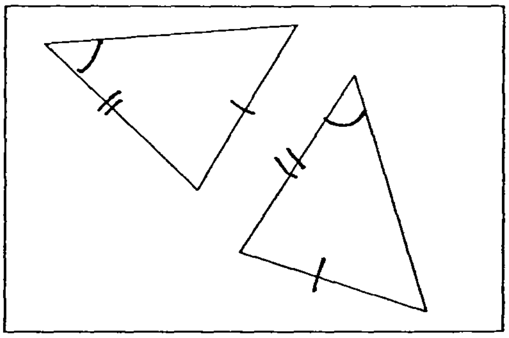
4.3 Discuss why the triangles in Part 1 are congruent but why the triangle in Part 2 are not congruent.

Exercise 1.3

1. Given two sets of triangles.



A



B

In which of the two sets of triangles above can you conclude that the pairs of triangles are congruent? Explain!

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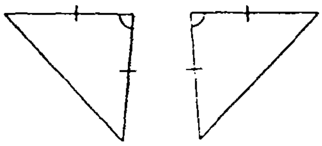
2. Fill / in front of the item which two triangles are congruent by SAS Theorem.

______1)



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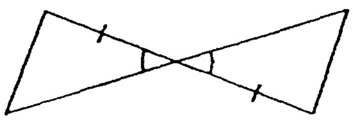
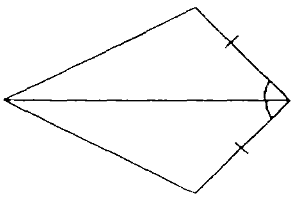
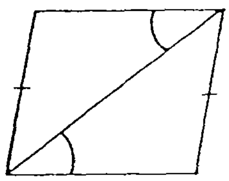


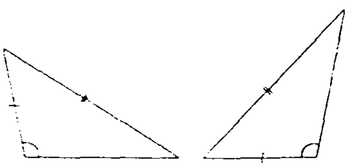
______2)



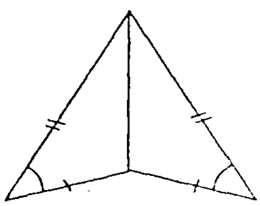
______5)



______3)

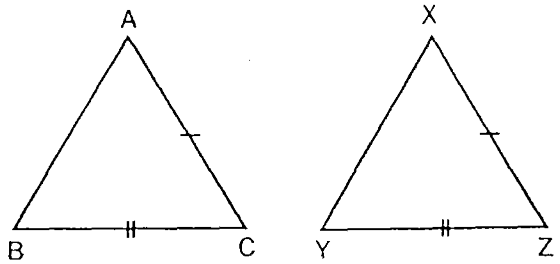


______6)



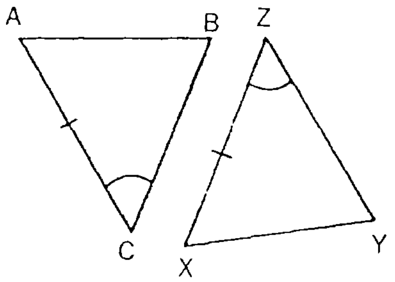
3. Suppose $\Delta ABC \cong \Delta XYZ$ by SAS Theorem, find the other corresponding part which makes the triangle be congruent.

3.1



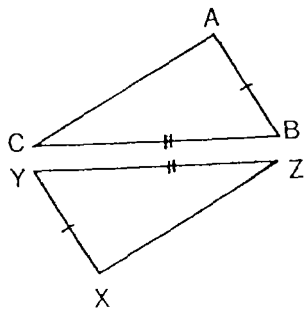
Answer.....................................

3.2



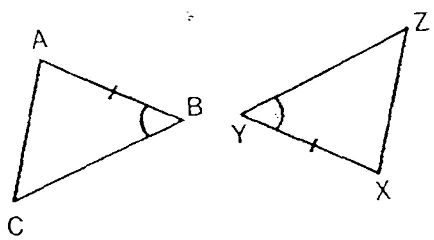
Answer.....................................

3.3



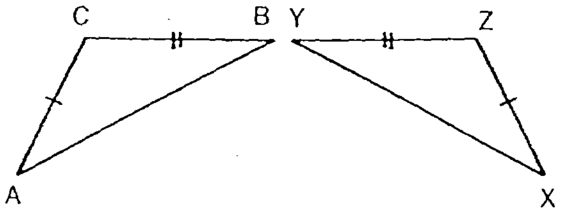
Answer.....................................

3.4



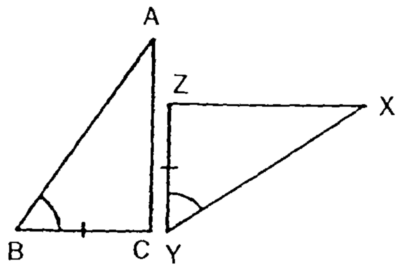
Answer.....................................

3.5



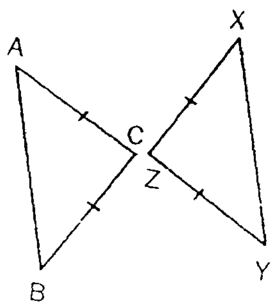
Answer......................................

3.6



Answer.......................................

3.7

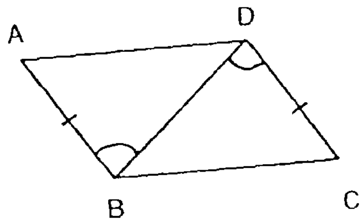


Answer..

4. Consider the flowchart proof in item 4.1. Then, fill in the blank.

4.1 Given AB = CD and $A\hat{B}D = C\hat{D}B$.

Prove that $\Delta ABD \cong \Delta CDB$.



1 AB = CD

Given

2 $A\hat{B}D = C\hat{D}B$

Given

3 DB = DB

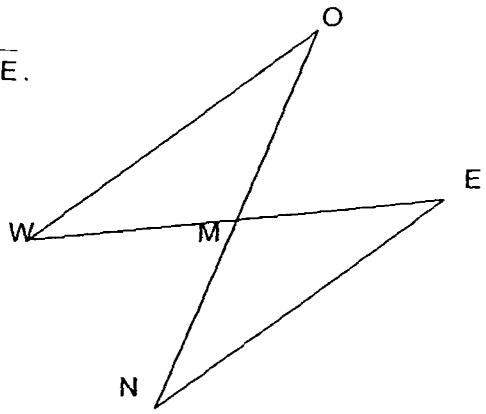
Common side

4 $\Delta ABD \cong \Delta CDB$

SAS Theorem

4.2 Given NM = OM. M is the midpoint of $\overline{WE}$.

Prove that $\overline{WO} \cong \overline{NE}$.



1 NM = OM

Given

2

Vertical opposite angles

3 WM = EM

4 $\Delta WMO \cong \Delta MNE$

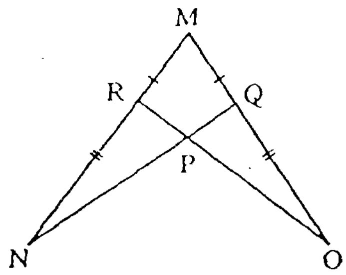
....

5

The corresponding sides of congruent triangles

....

5. Given MR = MQ and NR = OQ. From the given figure, which pairs of triangles are congruent? Explain.



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Session 7

Fundamental Mathematics | **Mattayom Suksa 2**

Topic : Isosceles triangle and its application | **Time : 100 minutes**

Learning objective

Students can apply the theorems of congruent triangles, isosceles triangle and its properties to solve the problems.

Instructional activities

Reviewing the previous lesson.

Teacher and students review the conditions of congruent triangles, isosceles triangle and its properties.

Presenting and understanding the problems.

Each group takes Problem 7 and materials.

Solving the problems by students.

Each group solves Problem 7.

Presenting and discussing the solutions.

1. Every group presents their solutions about 10 minutes each.
2. Teacher and students discuss the solutions.

Summarizing the lesson.

Teacher and students summarize what they find in the solutions.

Working individually.

Students work in the extra exercise individually.

Materials

1. Papers
2. Patter papers
3. Color pencils
4. Protractor
5. Compass

Evaluation

Students' evaluation

1. Students can use the definition of congruent segments, angles and triangles.
2. Students can verify the congruent triangles by using the theorem of congruent triangles.
3. How students work in their group.
4. How students present their solutions.

Teacher evaluation

Role of teacher in supporting and facilitating students' discussion in class.

Expected responses

Viewpoints	Expected responses
Side	1. AD = AE, BD = CE
	2. BE = CD
	3. BG = CG, DG = EG
Angle	
Vertically opposite angle	4. $D\hat{G}B = E\hat{G}C$, $D\hat{G}E = B\hat{G}C$
	5. $A\hat{E}G = F\hat{E}C$, $A\hat{E}F = G\hat{E}C$
Base angle	6. $A\hat{B}E = C\hat{B}E = A\hat{C}D = B\hat{C}D$
	7. $A\hat{B}C = A\hat{C}B$
Other angles	8. $A\hat{D}G = A\hat{E}G$, $B\hat{D}G = C\hat{E}G$
	9. $B\hat{G}C = A\hat{C}H$

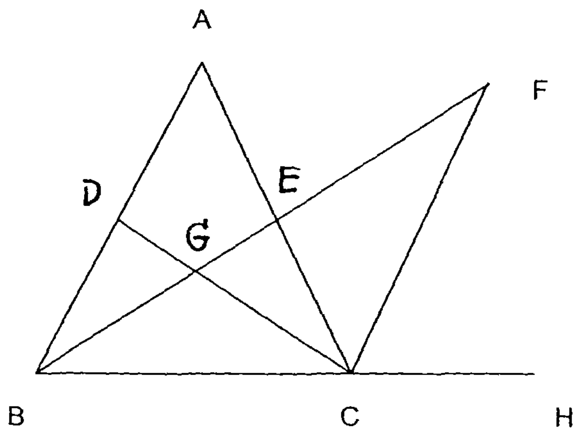
Viewpoints	Expected responses
Shapes and sizes	
Area	10. $\Delta ABE = \Delta ACD$
	11. $\Delta BDC = \Delta CEB$
	12. $\Delta BDG = \Delta CEG$
Shape	13. ΔGBC is an isosceles triangle.
	14. ΔADE is an isosceles triangle.
	15. ΔFBC is an oblique triangle.
	16. ΔGBC is an oblique triangle.
	17. ΔFGC is a right triangle.
Congruence	18. $\Delta ABE \cong \Delta ACD$
	19. $\Delta BDC \cong \Delta CEB$
	20. $\Delta BDG \cong \Delta CEG$
Similarity	21. $\Delta DBC \sim \Delta DGB$
	21. $\Delta ECB \sim \Delta EGC$
	22. $\Delta DEG \sim \Delta CBG$
Others	23. $DE \parallel BC$

Problem 7

Fundamental Mathematics **Mattayom Suksa 2**

Topic : Isosceles triangle and its applications **Time 30 minutes.**

Direction

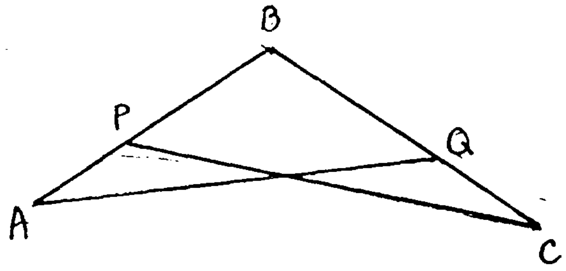


In the figure above, $\overline{BF}$ and $\overline{CD}$ are the bisectors of the base angle of an isosceles triangle ABC. $\overline{CD}$ intersects $\overline{AB}$ at point D; $\overline{BF}$ intersects $\overline{AC}$ at point E and the bisector of the exterior angle $A\hat{C}H$ at point F.

Study this figure various viewpoints and find as many relations as you can and give your reason.

Extra exercises

1. P and Q are the points on $\overline{AB}$ and $\overline{BC}$ respectively, AP = CQ and BP = BQ. Show that AQ = CP and $B\hat{A}Q = B\hat{C}P$



Given...

To prove...

Proof

Statements **Reasons**

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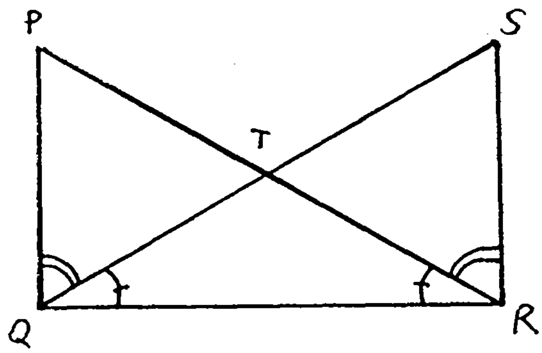
2. Suppose $P\hat{R}Q = S\hat{Q}R$ and $P\hat{Q}S = S\hat{R}P$. Show that PQ = SR.



Given...

To prove...

Proof

Statements	Reasons
....	....
....	....
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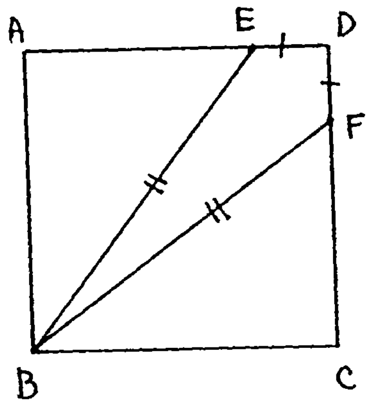
3. ABCD is a square, ED = DF and BE = BF. Show that $A\hat{B}E = C\hat{B}F$.



Given..

To prove..

Proof

Statements	Reasons
....	....
....	....
....	....
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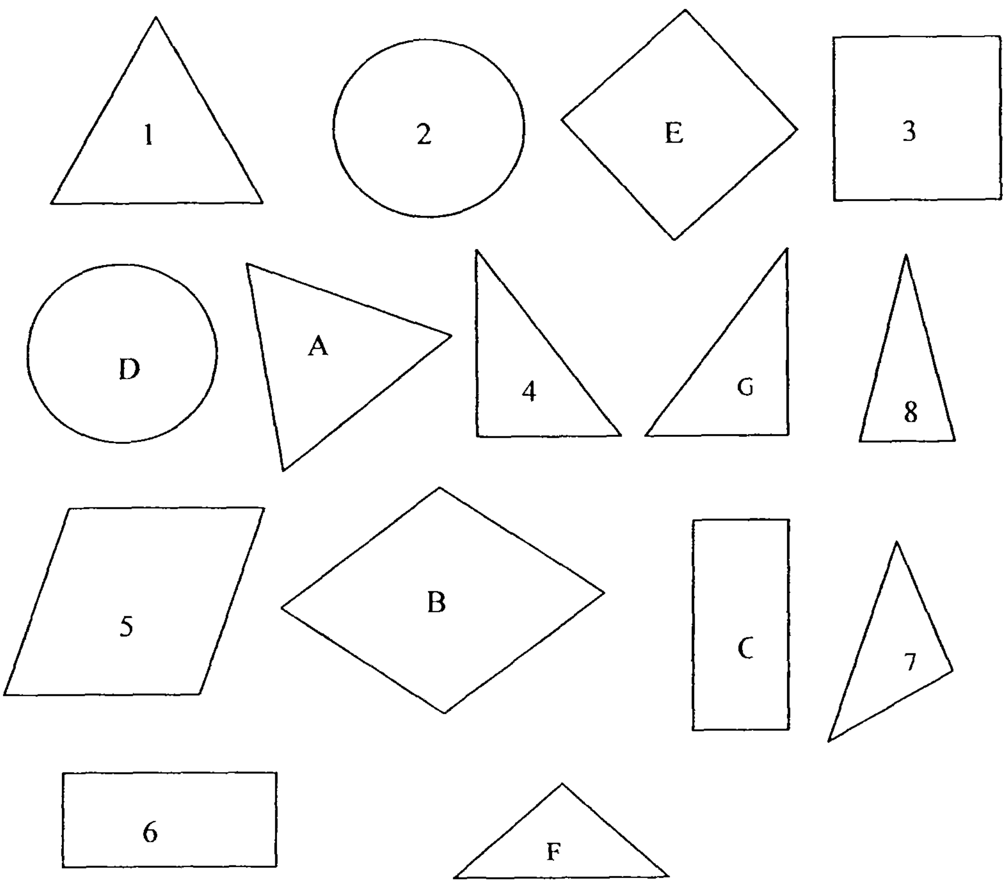
....	....

OPEN-ENDED PROBLEMS

Problem 1

Fundamental Mathematics **Mattayom Suksa 2**

Topic : Congruent Figure **Time 30 minutes.**



Direction Several figures are shown above. Do the following activities:

Sort the figures which have the same characteristics as many ways as you can, then present the way you use and give the reason.

Which figures are fit with each other? Give the reason why they are fit.

Problem 2

Fundamental Mathematics **Mattayom Suksa 2**

Topic : Congruent lines, angles and triangles **Time 30 minutes.**

Directions

Part 1 1. One of your group draws one line, then other members construct the line which has the same measure as the first one.

2. One of your group draws one angle, then other members construct the angle which has the same measure as the first one.

3. Discuss how to construct the line and angle with the same measure as your friends and how to justify that the line and angle you construct have the same measure as that of your friends.

Part 2 1. Each one draws two intersecting lines onto patty paper or tracing paper. Label the angles.

2. Fold the paper by making one pair of intersecting line congruent.

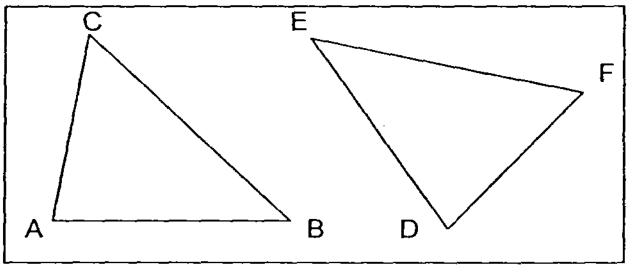
3. Repeat this investigation with another pair of intersecting line.

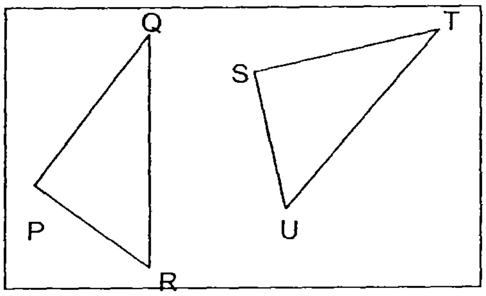
4. Compare your results with the results of others. Discuss how do you conclude.

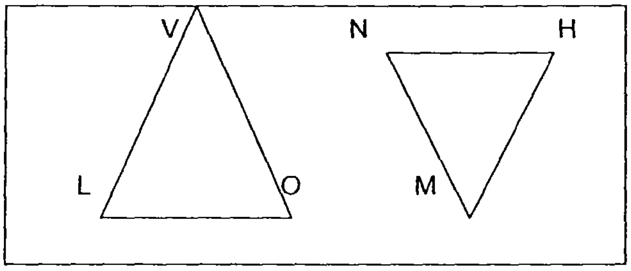
Part 3 Given three pairs of triangles, answer the following questions:

1. Is each pair of triangles congruent?

2. If the triangles are congruent, which parts of these triangles are congruent?







Problem 3

Fundamental Mathematics **Mattayom Suksa 2**

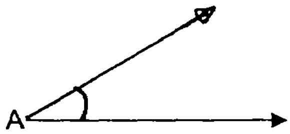
Topic : SAS Theorem **Time 30 minutes.**

Directions

Part 1 Given the following segments and angle:

8 centimeters

5 centimeters



1. Construct a triangle on your own paper using these two segments as sides and the angle as the angle between them.

2. Compare your triangle with the triangles constructed by others in your group. Are they congruent? Discuss why the triangles are congruent.

Part 2 Construct another triangle using these two segments as sides and the angle as the angle is not between them.

If you can not construct, follow these steps:

1) Construct $\hat{P}$ congruent to $\hat{A}$.

2) At point P, measure one arm 8 centimeters and determine point Q.

3) Set the compass to a radius of 5 centimeters and place the point on Q. Draw an arc to cut another arm of $\hat{P}$. Label the intersecting point R and form the triangle.

4) Answer the following questions:

4.1 How many times does your arc cut the line?

4.2 How many different triangles can you construct? Which triangle is congruent to the triangle constructed in Part 1.

4.3 Discuss why the triangles in Part 1 are congruent but why the triangle in Part 2 are not congruent.

Problem 4

Fundamental Mathematics **Mattayom Suksa 2**

Topic : SSS Theorem **Time 30 minutes.**

Directions

Given the following segments:

5 centimeters

7 centimeters

10 centimeters

Part 1 1. Use the three segments to construct a triangle on your own paper.

2. Compare your triangle with those constructed by others in your group. Are they all congruent? Discuss.

Part 2 1. Construct the triangles with the following angles: 30° 45° and 105°.

2. Compare your triangle with those constructed by others in your group. Are they all congruent? Discuss.

From the activities in Part 1 and 2, discuss which condition makes the triangles congruent.

Problem 5

Fundamental Mathematics **Mattayom Suksa 2**

Topic : ASA and AAS Theorems **Time 30 minutes.**

Directions

Given the following segment and angles:

7 centimeters

Part 1 1. Construct a triangle on your own paper using these two angles and the segment as the side between them.

2. Compare your triangle with those constructed by others in your group. Are they all congruent? Discuss.

Part 2 1. Construct a triangle on your own paper using these two angles and the segment as the side not between them.

2. Compare your triangle with those constructed by others in your group. Are they all congruent? Discuss

From the activities in Part 1 and 2, discuss which condition makes the triangles congruent.

Problem 6

Fundamental Mathematics **Mattayom Suksa 2**

Topic : Isosceles triangle and its properties **Time 30 minutes.**

Directions

Part 1 1. Each group starts with 12 toothpicks. Arrange any number of toothpicks, end-to-end, to form a triangle. Identify each different solution as a number triple. For example, use 3,3,4 for the triangle with two congruent sides 3 toothpicks long and a base 4 long. Don't count 3,4,3 or 4,3,3 separately as they give the same triangle as 3,3,4.

2. Each group makes a list of all possible triangles that can be formed with any number up to 12 toothpicks.

Part 2 1. Each one draws an angle on patty paper. Label $\hat{A}$. Place a point B on one ray of $\hat{A}$.

2. Fold your patty paper so that the two rays match up. Trace point B onto the other ray. Label the point on the other ray point C.

3. Draw $\overline{BC}$ and draw the dot line at the crease.

Answer the following questions:

1. What kind of figure you construct? Explain how you know.
2. From the figure,

Notice the dot line, how many triangles do you have? Are those triangles congruent? Why?

Which angles are congruent? Why?

How the dot line relates to the angles in 2.2?

How the dot line relates to $\overline{BC}$?

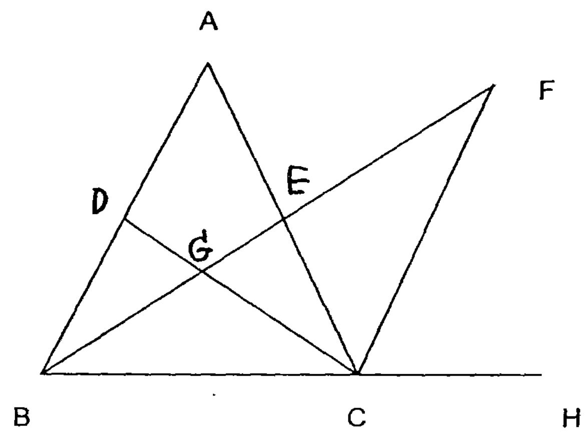
Construct an equilateral triangle, the repeat the questions 2.1-2.4. Do you think they have the same answer? Why?

Problem 7

Fundamental Mathematics **Mattayom Suksa 2**

Topic : Isosceles triangle and applications **Time 30 minutes.**

<u>Direction</u>



In the figure above, $\overline{BF}$ and $\overline{CD}$ are the bisectors of the base angle of an isosceles triangle ABC. $\overline{CD}$ intersects $\overline{AB}$ at point D; $\overline{BF}$ intersects $\overline{AC}$ at point E and the bisector of the exterior angle $A\hat{C}H$ at point F.

Study this figure various viewpoints and find as many relations as you can and give your reason.

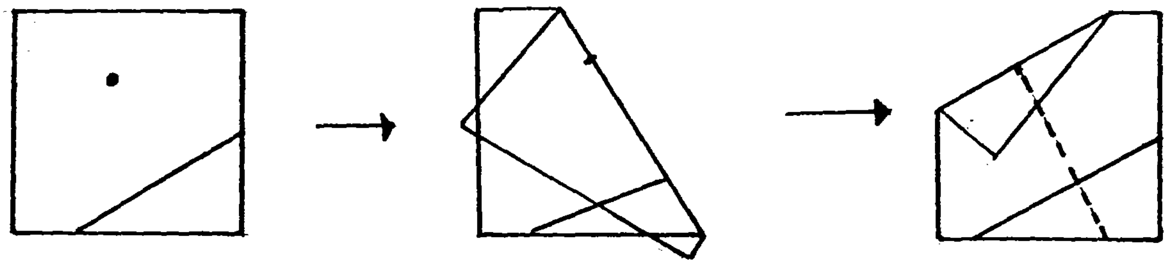
Problem 8

Fundamental Mathematics **Mattayom Suksa 2**

Topic : Parallel lines Theorems **Time 30 minutes.**

Directions

Part 1 1. Draw a line and a point on patty paper, then fold the paper to construct a perpendicular so that the crease runs through the point as shown.

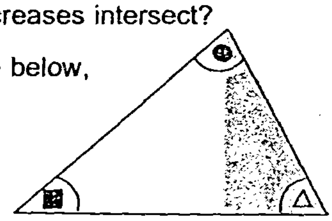


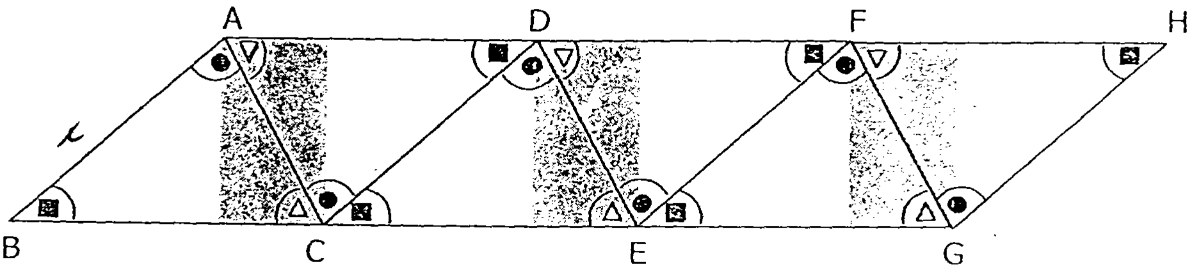
2. Through the point, make another fold that is perpendicular to the first crease.

3. Notice how the two creases relate to each other and how the distance between the creases is.

4. Do the two creases intersect?

Part 2 From the figure below,





1. Find other segments which relate to segment l as many as you can and then discuss how they are related.

2. How the Δ angel, angle and • angle are related?

3. How segment l and its related segments relate to those three angles?

4. Find angles which relate to $\hat{B}$ as many as you can and then discuss how they are related.

Problem 9

Fundamental Mathematics **Mattayom Suksa 2**

Topic : Converses of angle of parallel lines Theorems **Time 30 minutes.**

Directions

Prove the following statements:

1. If two lines are cut by a transversal to form the alternate interior angles of parallel lines are congruent, then the lines are parallel.

2. If two lines are cut by a transversal to form sum of the interior angles of parallel lines is 180°, then the lines are parallel.

3. If two lines are cut by a transversal to form the corresponding angles of parallel lines are congruent, then the lines are parallel.

Problem 10

Fundamental Mathematics — **Mattayom Suksa 2**

Topic : Theorems of triangle — **Time 30 minutes.**

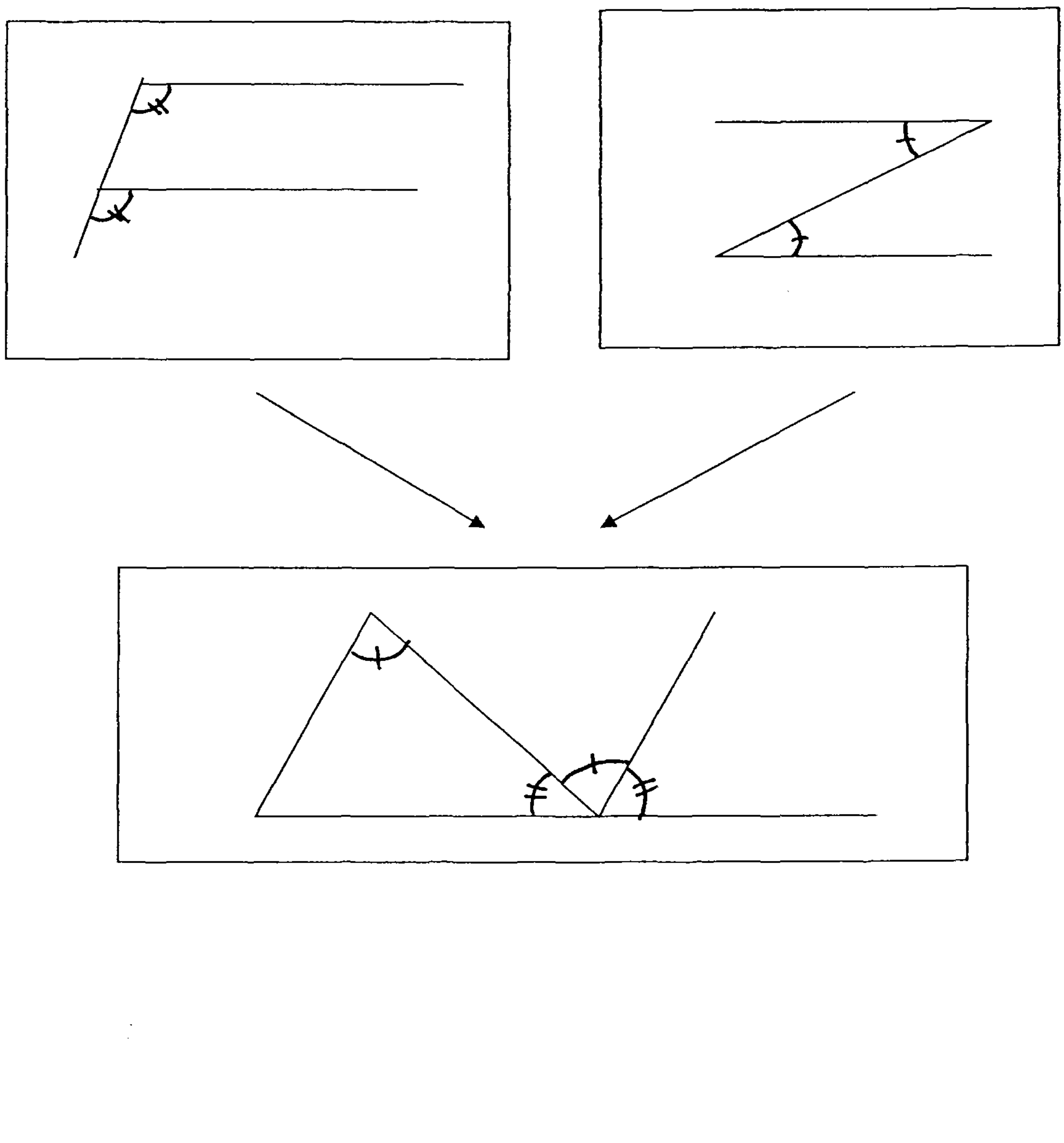
Directions

Part 1 Justify the statement "**Sum of the interior angles of triangle is 180°**"

Part 2 Justify the following statement and fill in the blank.

"If two angles of one triangle are equal in measure to two angles of another triangle, the third angle in each triangle……………………………………………"

Part 3 Discuss the relationships of the following diagrams:

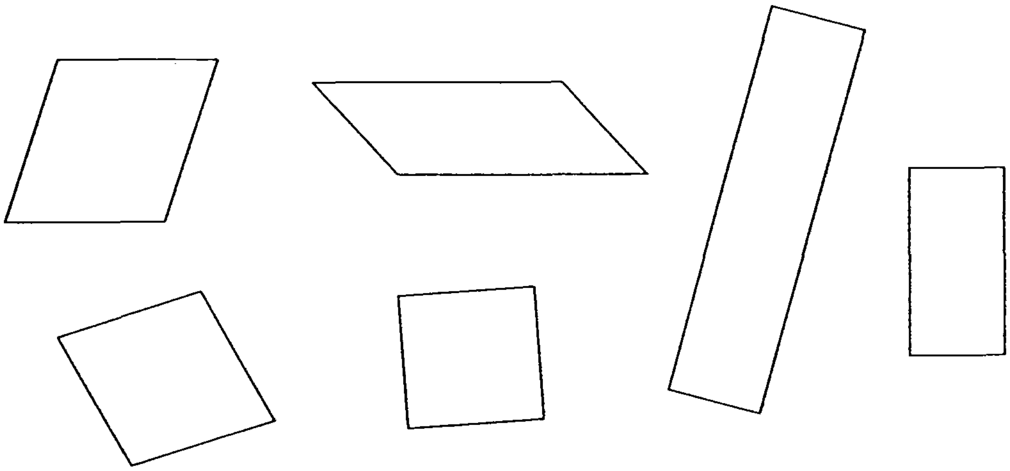


Problem 11

Fundamental Mathematics **Mattayom Suksa 2**

Topic : Parallelograms **Time 30 minutes.**

Directions



1. Write the important properties of parallelogram, rhombus and square.
2. Write all the important properties which are shared by squares and rhombi.
3. Write all the important properties which are true for rhombi but not for squares.
4. Write all the important properties which are true for squares but not for rhombi.
5. Answer the same questions as in items 2-4 for parallelograms and squares.

Problem 12

Fundamental Mathematics **Mattayom Suksa 2**

Topic : Application of parallel lines and parallelograms **Time 30 minutes.**

Prove that "**The diagonals of rhombus are perpendicular**"

APPENDIX C

RELIABILITY AND ITEM ANALYSIS OF TEST

The reliability of the Geometric Achievement Test

The researcher used Kuder-Richardson method (KR-20) for indicating the reliability of the Geometric Achievement Test. The formula is:

$$r_{tt} = \frac{k}{k-1}\left[1 - \frac{\sum p_i q_i}{s_t^2}\right]$$

$$r_{tt} = \frac{20}{20-1}\left[1 - \frac{3.6}{11.24}\right]$$

$$r_{tt} = 0.72$$

where $$s_t^2 = \frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2$$

$$s_t^2 = \frac{12487}{55} - \left(\frac{808}{55}\right)^2$$

$$= 11.24 .$$

TABLE 20 THE DIFFICULTIES INDEX AND THE DISCRIMINATION INDEX OF THE GEOMETRIC ACHIEVEMENT TEST

item	R_h	R_l	p	D
1	27	16	0.78	0.20
2	20	6	0.47	0.25
3	26	10	0.65	0.29
4	25	12	0.67	0.24
5	26	10	0.65	0.29
6	26	8	0.62	0.33
7	26	6	0.58	0.36
8	21	4	0.45	0.31
9	26	11	0.67	0.27
10	24	7	0.56	0.31
11	25	9	0.62	0.29
12	26	9	0.64	0.31
13	27	8	0.64	0.35
14	23	10	0.60	0.24
15	27	13	0.73	0.25
16	26	13	0.71	0.24
17	26	10	0.65	0.29
18	24	7	0.56	0.31
19	27	10	0.67	0.31
20	24	8	0.58	0.29

APPENDIX D

OUTPUTS FROM STATISTICAL TESTS

Output from Analysis of Variance (ANOVA)

Before selecting the sample of the study, the researcher examined students' abilities among five classes by using Analysis of Variance (ANOVA) to compare the mean scores from final scores of the fundamental mathematics achievement test (MA 102). The test results are shown in the following table:

TABLE 21 RESULTS OF MEAN SCORES AMONG FIVE CLASSES WITH ANOVA

Source	df	SS	MS	F	Sig.
Between group	4	639.329	159.832	.562	.690
Within group	202	57429.84	284.257		
Total	206	58059.17			

From the table, there was no significant difference on mean scores among five classes (F (.05 : df 4,202) = 0.562). Therefore, the mathematics abilities of students among five classes were not different.

Output from chi-square test

To test that the levels of geometric thinking and the instruction based on the open approach was independent, the researcher used the Chi-Square Test. The result showed in the following table:

TABLE 22 OBSERVED AND EXPECTED VALUES OF CHI-SQURE

	Observed N	Expected N	Residual
Level 1	1	1	0
Level 2	16	7	9
Level 3	23	32	-9
total	40		

The test statistic was shown in the following table:

TABLE 23 RESULTS OF CHI-SQUARE TEST

	Instruction
Chi-square	14.103
df	2
Asymp. Sig.	.001

The result from the chi-square test showed that there was a significant difference (χ^2 (df =2) = 14.10, p< .05) on levels of geometric thinking and the instruction based on the open approach. Therefore, after the instruction, there were the increase in the number of students exhibiting Level 3 of geometric thinking and the corresponding decrease in the number of students exhibiting Level 2 of geometric thinking.

Output from the t-test

To evaluate the effect of the instruction based on the open approach on students' geometric achievement, the researcher used t-test to test mean scores from the Geometric Achievement Test of both the experimental and the control groups. The test result is shown in the following table:

TABLE 24 RESULTS OF THE T-TEST

	Levene's Test for Equality of Variance		t-test for Equality of Means		
	F	Sig.	t	df	Sig. (2-tailed)
Equal variance assumed	1.778	.186	.790	80	.432

From the table, there was no significant difference ($p = .432$) on geometric achievement between the experimental and control group. This showed that the instruction based on the open approach has not affected on geometric achievement.

VITAE

VITAE

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