LEARNING OF A BINARY NUMBER CONCEPT UNDER VARIOUS CONTINGENCIES OF REINFORCEMENT AND LEVELS OF STIMULUS COMPLEXITY AND ITS ANALYSIS BY A MARKOV CHAIN MODEL

Abstract

by

Kenneth Kampe

A dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Education at Srinakharinwirot University

August 1976

Chairman: Dr. Ravipan Somnapan

คณะกรรมการที่ปรึกษาประจำตัวนิสิต ได้พิจารณาปริญญานิพนธ์ฉบับนี้แล้ว เห็นสมควรรับเป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาการศึกษาดุษฎีบัณฑิต ของมหาวิทยาลัยศรีนครินทรวิโรฒได้

.... ประธาน

.... กรรมการ

.... กรรมการ

๑๑ สิงหาคม ๒๕๑๙

ACKNOWLEDGMENTS

The author wishes to express his sincere appreciation for the generous assistance and advice given him by Drs. Ravipan Somnapan, Preeja Dhunna, and Pote Sapianchai. Special thanks are due Dr. Ravipan for numerous discourses which, though not always directly related to the research at hand, were quite instructive.

The author is grateful to Dr. Chaiyaporn Wichawut for suggestions resulting in the overall improvement of the dissertation.

Thanks are given to the Secondary Demonstration School attached to Srinakharinwirot University, Prasanmit Campus, for providing subjects and material assistance.

TABLE OF CONTENTS

TABLE OF CONTENTS

TABLE OF TABLES

TABLE OF TABLES

TABLE OF FIGURES

TABLE OF FIGURES

Chapter I

INTRODUCTION

Statement of the Problem

A major premise of both education and psychology is that human behavior conforms to certain rules, some of which are known and some of which are not. Within the confines of individual differences, these rules ensure that persons tend to respond similarly to the same stimulus complex. Were this not so, there would be no reason for studying the psychology of learning. Neither would there be a reason for grouping students together into classrooms for the purpose of creating similar changes in behavior. The fact that we do group students and expect similar changes in outputs given similar inputs, indicates that we do accept the above premise. Nevertheless, the efforts of educators are more often directed at the study of educational objectives, the arrangement of subject matter, teaching methods, and evaluation methods than at the rules governing learning. Whether this neglect is because learning is not so directly observable and manipulable as, for example, subject matter or because it is expected to follow directly upon changes in areas such as teaching methods, the fact remains that learning - the supposed goal of education[1] - is not yet well enough understood to permit really effective education.

[1] This statement is suspect by more than one researcher. For example, Dr. Ravipan Somnapan claims that, "Education is a schooling business," and not necessarily concerned with learning per se.

Increased understanding of the learning process, then, would seem to be necessary to increased effectiveness in education. Within the diverse range of learning encountered by the child as well as the adult, perhaps the most pervasive is concept learning. As Bruner (1956, p. 9) wrote, "...Categorizing is ubiquitous." Fortunately, there has been extensive research into the field of concept learning. Bourne, et al (1971), Bruner, et al (1956), and Marx (1970) have presented fairly comprehensive summaries of work in this area. Unfortunately, this work has not been sufficient to provide as thorough an understanding as is necessary for an effective application in the classroom. Millward and Wickens (1974, p. 45) observed,

> "Psychologists have only a partial understanding of how concepts are learned - partial in the sense that the definition of the term 'concept' is incomplete, and even within this limited definition, only a partial statement of the learning process has been made."

More specifically, there are a number of important limitations in our understanding of concept learning. First, research studies have neglected complex concept learning and concentrated on the learning of very simple concepts. Carroll (1964, pp. 178-9) questioned this concentration on oversimplified concepts, writing,

> "...there is a gap between the findings of psychologists on the conditions under which very simple concepts are learned in the psychological laboratory and the experiences of teachers in teaching the 'for real' concepts that are contained in the curricula of the schools."

Second, investigations into the nature of concept learning have generally centered on what Bourne, et al (1971, pp. 259-60) called "deterministic class concepts", mostly geometric figures and verbal concepts. Bourne, et al (1971, pp. 259-60) wrote,

> "Clearly these are neither the only, nor even necessarily the most interesting concepts that people learn and use in their everyday lives. Another type of concept of obvious importance is quantitative, i.e. the type of concept that prescribes numerical values for each stimulus....Unfortunately, to date, there have been relatively few attempts to describe empirically the properties of performance in quantitative concept tasks."

Third, research on concept learning has frequently neglected what Hilgard and Bower (1966) have called, "the most important principle in all learning theory", the law of reinforcement. Bourne, et al (1971, p. 273) noted,

> "...only on rare occasions have any inducements - above and beyond informative feedback - been included in a study of concept learning. What studies are available in this issue seem neither to be systematic nor unambiguous in outcome."

Investigations into the three above mentioned areas alone would extend our knowledge of concept learning. Nevertheless, it would be wise not to conduct such studies in isolation of previous research findings, particularly when those findings indicate interactions among variables. One widely studied variable with both obvious importance for education and demonstrated influence on learning is stimulus complexity. Different levels of stimulus complexity have been shown to yield differences in concept learning (Bulgarella and Archer, 1962). This, plus the fact that stimulus complexity varies widely in different concept learning situations, implies the importance of including this variable in concept learning studies aimed at eventual application to education.

Due to the complexity of any learning process, the availability of techniques which offer deeper analyses of the process without greatly increasing the effort involved are most welcome. One of these recent techniques is the employment of mathematical models to represent the learning process. A particularly successful representation of learning is achieved through the use of a Markov chain model.[1] Bower and Trabasso (1964) introduced a relatively simple Markov chain model capable of representing and predicting simple concept and other types of learning. Their model provides a means for formulating and testing a number of hypotheses about the concept learning process and, also, a means of predicting statistics defining this process. Although the Bower and Trabasso model has received a great deal of interest and success, this has generally been in the study of simple and non-mathematical[2] concepts. It is not yet known how the model will fare in the study of more complex (and mathematical) concepts.

---

[1] A Markov chain model is one of several mathematical models (including linear models) used in the analysis of learning. The Bower and Trabasso (1964) Markov chain model is explained in detail in the sections on Related Literature and Theoretical Considerations.

[2] The term "mathematical" is used in place of Bourne, et al's "quantitative" as it seems more representative of the types of concepts not previously studied.

Objectives of the Study

1. To examine concept learning employing a concept which is:

a. mathematical[1]

b. more complex than the simple concepts normally studied

2. To examine the effects of the following variables on behavior in mathematical concept learning:

a. trials

b. contingencies of reinforcement

c. stimulus complexity

3. To employ a Markov model of learning (specifically, the Bower and Trabasso, 1964, model) in the analysis in order to:

a. accomplish a deeper analysis of the data than is generally possible with normal statistical procedures, in that various assumptions about learning and its parameters can be investigated.

b. test the ability of the Bower and Trabasso model to handle data in a mathematical concept learning situation.

Related Literature

Bourne, et al (1971, p. 260) referred to mathematical concepts thus, "To attain the concept the subject must learn to rate each stimulus in a population on some unitary scale." Thus, the concept is determined by a

---

[1] Specifically, a binary number concept. This will be discussed in detail in the section on Procedure.

mathematical interpretation of the stimuli. Although the studies on mathematical concept learning are few, there appear to be a number of similarities between mathematical and verbal or geometric (figures) concept learning.

Azuma and Cronbach (1966) required subjects to respond with numerals to a large number of stimulus patterns, consisting of two objects positioned on a matrix. They found that Ss gradually came to depend only on variations in relevant dimensions (a finding also reported by Bruner, et al, 1956, with non-mathematical concepts) and with proper weighting. However, reports by individual Ss indicated that they did not learn the concept according to the simple mathematical formula governing the solution. Rather, they frequently reported associating stimulus subsets with specific numerical responses. That is, they seemed to learn the concept in parts. This finding was also noted by Hunt (1962) and Trabasso and Bower (1964) in studies involving non-mathematical concepts.

Uhl (1963) found that as the degree of relevance of two dimensions of a concept was varied (a mathematical concept in that the dimensions were weighted) from equality to extreme difference, performance improved. This result is consistent with the findings of others regarding non-mathematical concepts (e.g. Bulgarella and Archer, 1962, and Walker and Bourne, 1961).

Uhl (1966) also reported that the varying of irrelevant stimuli in mathematical concepts interfered with performance. Bourne, et al (1965) and Haygood and Stevenson (1967) also found the same result in non-mathematical concept learning.

The results of the vast literature concerned with the effects on learning of varying the contingencies of reinforcement are not always clear or consistent. For example, Estes (1944), Thorndike (1933), and Thurstone (1931) found that "supposed" punishments sometimes functioned as positive reinforcers. There is a similar ambiguity about the effects of various reinforcement contingencies on concept learning.

In a concept learning experiment by Buss and Buss (1956), <u>S</u>s were divided into three groups receiving the following oral rewards and punishments from the experimenter: I - "right" following a correct response and nothing following an incorrect response; II - "wrong" following an incorrect response and nothing following a correct; III - "right" following a correct and "wrong" following an incorrect. Using trials to solution as the criterion measure, the "right"/"wrong" group (III) performed best. However, the "wrong"/nothing group (II) was not too inferior and was much superior to the "right"/nothing group (I). Buss and Buss concluded that "wrong" had a greater reinforcing effect than "right".

Wallace (1964) employed tones and oral comments derogating the performance of <u>S</u>s as feedback and punishment in a concept learning situation. Overall performance in some situations improved with both increase in tone intensity and level of derogation. Wallace concluded that punishment can have an effect beyond its informational content.

In suspicion of the Buss and Buss and Wallace results, Bourne, et al (1967) conducted an experiment similar to the Buss and Buss one, employing the same three treatment groups, but combined monetary payment with "right" and monetary loss with "wrong". The number of feedback trials to solution was the criterion measure. In contradiction to the previous results, no differences in performance between reinforcement and feedback groups were found. Bourne, et al (1971, p. 273) have concluded that a solution of the problem depended on "some fixed number of trials with feedback" and that "all the motivation necessary to solve was provided by instructions and verbal feedback, the effects of added reinforcement being nil." Nevertheless, they admitted to many substantive and procedural differences between studies in this area, noting that "nothing can be said to be definitive."

The effects of stimulus complexity on concept learning are much clearer. Logically, one would expect more complex stimulus presentations to make learning more difficult and the empirical results bear this out. Increasing the number of stimulus dimensions increases the number of trials to solution, error rate, etc. This effect has been found to hold whether the increased stimulus complexity is along dimensions relevant to concept attainment (Bulgarella and Archer, 1962, and Schvaneveldt, 1966) or along dimensions irrelevant to concept attainment (Walker and Bourne, 1961, and Haygood and Stevenson, 1967).

A problem much less understood and unstudied is the effect of the interaction of reinforcement contingencies and stimulus complexity on concept learning. There appear to be no studies which directly attack this issue. However, a number of investigations in the areas of choice behavior and probability learning shed some light. For example, Siegel (1959) conducted an experiment in which Ss were required to predict which of two lights would illuminate on each trial. The Ss were assigned to three groups, as follows: Risk - monetary gain for correct predictions and loss for incorrect predictions; Reward - monetary gain for correct predictions; No Payoff - no monetary gain or loss. The lights were programmed to illuminate in a random sequence, with one light illuminating on 75 per cent of the trials and the other on 25 per cent of the trials. Siegel's results (see Table I) showed evidence that both learning and overall performance were affected by the different reinforcement contingencies. This, of course, is not classical concept learning, but probability learning.[1] Nevertheless, probability learning can be interpreted as concept learning. The subject is unaware that the sequence of events is determined randomly and, hence, unlearnable. Rather, he is under the impression that he is to learn the rule governing event presentation. Thus, the situation is similar to one in which S is required to learn a very difficult concept, one which is unlikely

---

[1] In probability learning, the occurrence of any specific event (e.g. light illumination) is determined randomly and, therefore, uncertain. S generally learns to match his proportion of predictions of a particular event to the probability of occurrence of that event.

Table I

The Probability of Predicting the More Frequently Illuminating Light in an Uncertain Outcome Situation (from Siegel, 1959)

Trials	No Payoff	Reward	Risk
1 - 100	.69	.78	.95
101 - 200	.74	.85	.95
201 - 300	.75	.86	.95

to be perfectly learned. Under such circumstances, S's behavior is influenced by the reinforcement contingencies under which he operates.

On the other hand, in the concept learning situation employed by Bourne, et al (1967), reinforcement had no effect on S's behavior. If this situation is interpreted as a less complex one than that employed by Siegel (probability learning), then complexity and contingencies of reinforcement interact in their effect on subject behavior in concept learning. Such a conclusion is, of course, hypothetical.

A final area of research germane to the present study is the use of Markov chain models[1] in the analysis of concept learning. From the beginning, two important facts concerning Markov chains and Markov chain models of learning should be understood. First, a Markov chain is not a psychological theory aimed at explaining the learning process, but is a probability theorem which provides a very convenient representation of learning phenomena when certain assumptions are made. Second, a particular Markov chain model of learning is a statement of a particular theory of learning (which may or may not be correct) and is not necessarily derived mathematically. The most important of the assumptions underlying a Markov chain model of learning is that learning is an all-or-none rather than an incremental process. This means that S either learns

[1] A Markov chain is a special case of the more general Markov process. In a Markov chain the transition probabilities are constant over trials, while in a Markov process they are not. A Markov chain model is more easily adaptable to learning phenomena.

completely on any one trial or he does not learn at all (all-or-none), rather than continuously increasing the probability of a correct response until learning is complete (incremental).

A Markov chain model is a stochastic model concerned with the conditional probabilities (p_{jk}) of discrete outcomes (E_1, E_2, ..., E_n) over a sequence of trials. "In the theory of Markov chains we consider the simplest generalization which consists in permitting the outcome of any trial to depend on the outcome of the directly preceding trial, and only on it," (Feller, 1968, p. 372). Given that the outcome, X, on trial n is j, a Markov chain gives the conditional probability that the outcome on trial n+1 is k, without referring to any trials previous to trial n, or

$$P_{jk} = P(X^{(n+1)} = k \mid X^{(n)} = j)$$

Assuming, then, that learning is all-or-none, a Markov chain allows us to state the probability that S will make response k on trial n+1, given he made response j on trial n, without referring to any previous responses.[1]

[1] This "independence of path" (i.e. that predictions about S's response on trial n+1 depend only on his response on trial n, regardless of how he got there) can be misleading. It does not mean that S's response on trial n+1 is independent of responses prior to trial n. S's response on trial n+1 is dependent on his response on trial n, which is dependent on his response on trial n-1, and so on. It does mean that, although S's response on any trial is dependent on all preceding trials, knowledge of responses on trials prior to the immediately preceding trial do not aid the prediction of future responses. It is in this sense that S's response on any trial depends only upon his response on the previous trial (as noted by Feller, 1968).

In line with the many theories employing this type of model, it is convenient to let the discrete valued random variable X, with values j, k, ... represent states of the system or, equivalently, states of the learning process. For example, in a two state theory of learning, j may represent the state "unlearned" and k the state "learned". Therefore, P_{jk} (with $0 \leq P_{jk} \leq 1$) is the conditional probability that S "learns" on trial n+1, given that S was in an "unlearned" state on trial n.

A Markov chain model representation of learning, assumes that in a sequence of trials:

(1) learning is all-or-none and consists of discrete states, i.e. S either transits completely from one state to another or he does not transit at all,

(2) the probability of transition is finite valued and constant, and

(3) the probability that S will be in any one state on a certain trial depends only upon the state that S was in on the previous trial, regardless of how he got there.

Furthermore, a particular model is perfectly described by: (1) listing the states of the chain (e.g. learned and unlearned), (2) listing the initial probabilities of being in each state, and (3) listing the probabilities of transition from one state to another (see Bishir and Drewes, 1970, p. 616).

Such a model of learning may seem oversimplified and naive. However it has been found quite effective in explaining and predicting behavior in simple situations.

One of the earliest attempts at specifying the learning process in terms of a Markov chain model was made by Restle (1962). Soon thereafter, Bower and Trabasso (1964) altered Restle's model by making changes in the assumptions concerning hypothesis making and sampling. Their formulation can be expressed as a three state Markov chain with the following transition probability matrix:

		state on trial n+1			initial
		L	C	E	probabilities
state on trial n	L	1	0	0	0
	C	0	π	$1-\pi$	π
	E	θ	$(1-\theta)\pi$	$(1-\pi)(1-\theta)$	$1-\pi$

where: L is the state where S has learned the concept

C is the state where S has responded correctly (by chance), but has not learned the concept

E is the state where S has responded incorrectly and has not learned the concept

π is the probability of making a correct response before learning the concept

$1-\pi$ is the probability of making an incorrect response before learning the concept

θ is the probability of learning the concept (on one trial)

$1-\theta$ is the probability of not learning the concept (on one trial)

The matrix is read from row (trial n) to column (trial n+1). For example, the probability that S transits from state E on trial n to state L on trial n+1 is $p_{EL} = \Theta$. A feature of this model is that state L is absorbing, meaning that once S has entered state L, he remains in that state.

Two other features of this model (and Markov chain models in general) are of special interest. The first is the states of the learning model (L, C, and E). The second is the parameters of the model (π and Θ). Both the states and the parameters (but not the parameter values) are derived from the assumptions of the model. They correspond to the theory behind the model. The specific assumptions of the Bower and Trabasso (1964) model (as adapted from Millward and Wickens, 1974, pp. 54-63) are:

(1) The set of hypotheses about the correct concept, H, which S has to consider consists of one hypothesis for each value on each dimension of the stimuli presented. For example, two dimensions (color and form) with two values each (red and blue, triangle and square) yield 2x2 = 4 hypotheses.

(2) S samples from this pool of hypotheses, H, which can be divided into hypotheses available for testing, A, hypotheses selected for sampling, S, and hypotheses eliminated earlier and unavailable for further sampling, U.

(a) Whenever s = 0, S samples one new hypothesis from A.

(b) Sampling takes place at the end of a trial and hypotheses considered are consistent with current feedback.

(c) Hypotheses are sampled randomly from A. If h_i is an element of A, $P(h_i \text{ sampled}) = 1/a$, where a equals the number of elements, or hypotheses, in A.

(3) In eliminating hypotheses:

(a) Whenever an overt error is made, all hypotheses in S are entered into U. Whenever a correct response is made, any hypotheses in S inconsistent with current feedback are entered into U.

(b) Hypotheses eliminated from S are replaced immediately into A, i.e. sampling with replacement. This is equivalent to S forgetting following hypothesis elimination.

(4) A response based on an empty sample, $S = \emptyset$, is a random selection of one of the response alternatives, with equal probabilities.

(5) Initially, S contains only incorrect hypotheses. Therefore, at least one error must occur before learning.

The above list of assumptions allows many predictions to be made about S's behavior during the learning process. Following estimation of the parameter values, π and θ, the Bower and Trabasso model permits the prediction of a large number of statistics defining the particular theoretical learning process determined by the parameter values. These predicted statistics can then be compared to the same statistics calculated from the observed data. The fit of the model-generated statistics to the observed statistics is a test of the model's states, parameters, and assumptions. Table II shows both the observed and predicted statistics from one of the Bower and Trabasso (1964) experiments designed

Table II

Observed and Predicted Statistics from a Concept Learning Experiment by Bower and Trabasso (1964)

Statistic	Observed	Predicted
Average total errors (TE)	12.16	—
Standard deviation of TE	12.22	12.18
Errors before first success	.92	.89
Average trial of last error (TLE)	25.70	24.50
Standard deviation of TLE	28.90	25.00
P(error following error)	.47	.46
Success/failure alterations	12.33	12.41
Runs of 1 error	3.62	3.57
2 errors	1.32	1.63
3 errors	.64	.75
4 errors	.40	.35

to test their model. It is readily seen that the predicted statistics fit those calculated from the observed data very well. Bower and Trabasso (1964) report similar closeness of fit in concept learning experiments involving clusters of consonants, colors and shapes, five-letter words and colors, and nonsense syllables. Trabasso and Bower (1964) also employed this model in predicting statistics in a concept learning experiment involving two sub-concepts, shape and color. Bower and Theios (1964) further used this model in analyzing data from several diverse learning situations, including avoidance conditioning, eyelid conditioning, and response reversal in paired associate learning.

The Bower and Trabasso model, however, is not the only model successful in analyzing and predicting concept learning. Chumbley (1972), Falmagne (1970), and Restle (1962), among others, have formulated Markov chain models capable of predicting the learning process in a number of situations. Nevertheless, it is beyond the intent and scope of the present investigation to compare different models. Thus, only the Bower and Trabasso model has been presented in detail, the reasons being: (1) it is the model most widely studied to this date, (2) it is as successful as any other model in a wide variety of situations, and (3) it is quite tractable mathematically.

In addition to the ability to predict statistics of the learning process, some basic assumptions of the model have been supported by the empirical data. One assumption of the model is that learning is all-or-none. This implies stationarity of response probability, which means that the probability of a specific response does not change prior to solution. It also implies independence of response probabilities, which means that the conditional probabilities of a specific response are equal. For example, the probability of an error given a preceding error, and the probability of an error given a preceding correct response should be equal (Laming, 1973, p. 254). Bower and Trabasso (1964) found support for both stationarity and independence in their experiments on simple concept learning.

Another assumption of this model is that the learning parameter, θ, is invariant over trials. This assumption was tested by Bower and Trabasso (1964) by comparing the actual and theoretical distributions of total errors made by Ss in simple concept learning. Since the only parameter of this theoretical distribution is θ, the closeness of fit of the theoretical and actual distributions provides a test of the invariance of θ. Bower and Trabasso found support for this assumption.

Theoretical Considerations

A Markov chain model is not unique in its ability to predict statistics which closely fit those calculated from the data in a well controlled experiment. Linear models of learning can do as well in many situations and they assume an incremental learning process rather than an all-or-none one. The fact that two opposed theories of learning, i.e. incremental and all-or-none, lead to different models which further lead to similar predictions is somewhat surprising. Yet it is not so surprising when one considers that the mathematical models derived from opposing theories are generally tested by predictions of group behavior instead of individual behavior. Since individual Ss learning at different rates are averaged together, it is not strange that linear and Markov chain models both lead to predictions of a "negatively accelerated" exponential group learning curve (Hilgard and Bower, 1966, p. 366). As a result, predictions about mean learning curves and other group statistics do not necessarily discriminate between different models or theories of learning. What are needed, in addition to the predictions of statistics, are tests of assumptions critical to the particular model. In the case of Markov chain models (and an all-or-none theory of learning), the viability of the stationarity and independence assumptions is critical (Laming, 1973, pp. 278-9), with the invariance of θ of lesser importance. Thus, while it is essential that a Markov chain model be able to predict statistics, support of the model also depends on tests of its basic assumptions.

Fortunately, the Bower and Trabasso model has what Greeno and Steiner (1964) call "identifiable" states, and tests of various assumptions are not difficult. An identifiable state means that S's being in that state can be readily identified from inspection of the response data. According to the Bower and Trabasso model, all responses are dichotomized into correct (C) and incorrect (E). When S makes an E response, he is in state E (unlearned, incorrect). When S makes a predetermined criterion number of consecutive C responses, he is considered to be in state L (learned) on each of those C responses. When S makes any C responses other than those above he is considered to be in state C (unlearned, correct).

As for the ability of a mathematical model to fit predicted statistics to those calculated from empirical data, there are two possible tests noted by Hilgard and Bower (1966, pp. 375-6). One is to conduct statistical tests, such as the chi square or t test. Hilgard and Bower noted that this method may not always be appropriate in that a particular model may predict with a small average error of, say, 3 per cent, but with sufficient observations, the 3 per cent can become statistically significant. It may be better, they claim, to acknowledge some small inaccuracy of prediction and determine acceptability of the model by a comparative inspection of models - which is the second method. In that an objective of the present study is to test the ability of the Bower and Trabasso model to make predictions in a new area (mathematical concept learning), the former method will be used.

As for the three basic assumptions of the model, i.e. stationarity, independence, and invariance of θ, it is necessary that appropriate statistical tests be made of each.

Terminology

Learning - an hypothetical construct inferred from a decrease in the proportion of error responses over blocks of trials.

All-or-none learning - learning in which S transits from one learning state (e.g. L, C, and E) to another in only one trial, with $0 \leq P(\text{transition}) \leq 1$.

Incremental learning - learning in which S increases the probability of making a correct response on each trial (or block of trials) up to an asymptotic value of one or less.

Performance - the proportion of correct responses (or one minus the proportion of error responses) made on a certain group of trials, or on all trials.

Concept learning - learning in which a correct response is defined as "that single response made to a certain class of stimuli as required by the experimenter-formulated rules".

Binary number concept learning situation = task - a situation in which S is repeatedly presented with a repeating sequence of 26 stimulus sets (18 positive and 8 negative instances of the concept) and required to decide whether a binary number from one to three is or is not equal to the number of lights from one to three and respond by pressing either a switch labeled "equal" or one labeled "not equal".

Positive instance (of the concept) - a stimulus presentation in which the binary number and the number of lights are equal and the response "equal" is correct.

Negative instance (of the concept) - a stimulus presentation in which the binary number and the number of lights are not equal and the response "not equal" is correct.

Reinforcement contingency - the outcome produced by S's response to a stimulus presentation, classified into:

feedback only (FB) - illumination of one of two lights to indicate whether the response is correct or incorrect.

reward - includes FB and the addition of one point to S's score (the score determining S's possibility of winning a monetary prize).

punishment - includes FB and the subtraction of one point from S's score (the score determining S's possibility of winning a monetary prize).

Stimulus complexity - determined by the number of dimensions along which the stimulus presentation varies, classified into:

low - a stimulus presentation consisting of the illumination of one to three lights and the coincident illumination of a binary number from one to three.

high - the same as "low" above, plus the illumination of one other separate light (i.e. one other dimension) whose position corresponds to the number of lights illuminated (excluding binary number lights).

Experimental Design and Method

In accordance with the objectives of the study, the present experiment was designed as a split-plot type SPF - pru.q (see Kirk, 1968, p. 294), which is a four factor experiment with repeated measures on one factor (one within subjects factor and three between subjects factors). The following experimental variables were assigned the following values:

Blocks of trials (A) - 10 blocks of 52 trials each

Reinforcement contingencies (B) - B_1 = Feedback only

B_2 = Reward

B_3 = Punishment

Stimulus complexity (C) - C_1 = Low stimulus complexity

C_2 = High stimulus complexity

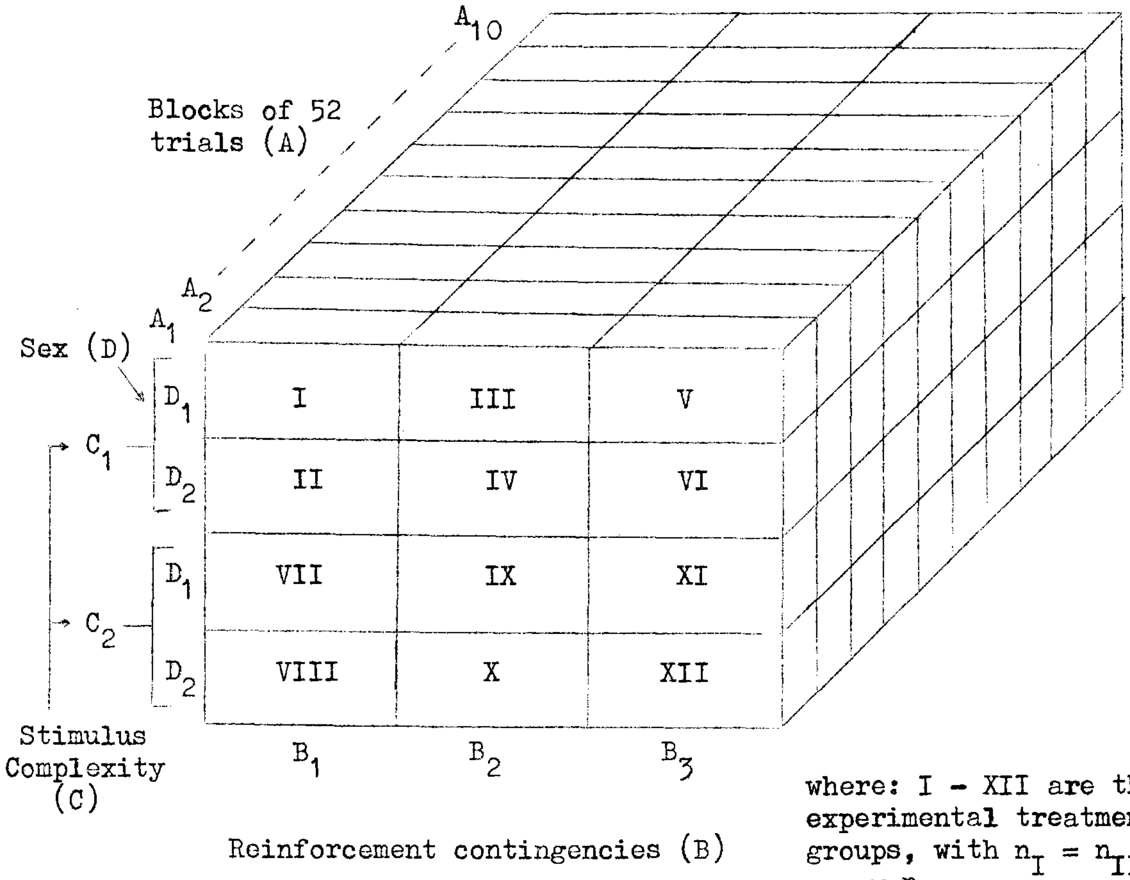
Sex (D) - D_1 = Male

D_2 = Female

The criterion measure employed was the proportion of error responses to total responses (for each block of trials or for all trials, as appropriate). Figure 1 shows a diagram of the experimental design.

Blocks of trials (A) was the within subjects factor, while B, C, and D were the between subjects factors. Factor D (Sex) was employed in the design to increase the precision of the experiment, i.e. D was a control factor.

Procedurewise, a reception paradigm was used in stimulus presentation. This means that E selected and presented (automatically) the stimuli to S. S's task was to respond to the stimuli (by pressing a switch) on the basis of whether they were positive or negative instances of the concept, without being told the exact nature of the concept, i.e. S had to learn to respond to the relevant dimensions without being told which they were. On the basis of a pilot study, it was found that the particular binary number concept employed was difficult to learn. As a result, the ratio of positive to negative instances was set at 18:8



where: I - XII are the experimental treatment groups, with $n_I = n_{II} = \dots = n_{XII}$

Figure 1. The experimental design

in each repeating sequence of 26 trials. A ratio differing from the more common 1:1 was chosen because a combination of positive and negative instances, with positive predominating, is generally more facilitative to concept learning (Bruner, et al, 1956, pp. 164-7).

Hypotheses

The hypotheses of the investigation and their respective rationales were:

Hypothesis 1 If Ss are presented with a task[1], then the probability of making an error on trial n, p_n, over successive trials is a "negatively decelerated" exponential function of the following form:

$$p_n = ae^{-bn} + c \qquad \text{where: } n = \text{trial number}$$
$$e = \text{base of natural logarithms}$$
$$a,\ b,\ c = \text{parameters}$$

This hypothesis simply states that the learning curve is of the same form ("negatively decelerated" exponential) as the learning curves generally characteristic of non-mathematical concept learning. There is no reported evidence that a mathematical concept learning curve differs from the general form of learning curves.

Hypothesis 2

If Ss are presented with a task under different contingencies of reinforcement, i.e. feedback only, reward, and punishment, then the Ss

1 As defined in the section on Terminology under the heading "Binary number concept learning situation = task".

under different contingencies of reinforcement show different performance (as inferred from the total number of errors) and different learning (as inferred from the change in the number of errors over blocks of trials).

This hypothesis is a result of the findings by Buss and Buss (1956), Siegel (1959), and Wallace (1964) that reinforcement over and above feedback affects performance and learning.

Hypothesis 3

If Ss are presented with a task under different levels of stimulus complexity, i.e. low and high, then the Ss under different levels of stimulus complexity show different performance and learning.

This hypothesis is a result of the findings by Bulgarella and Archer (1962) and Schvaneveldt (1966) that increasing stimulus complexity increases the number of errors, trials to solution, etc.

Hypothesis 4

If Ss are presented with a task under both different contingencies of reinforcement and different levels of stimulus complexity, then contingencies of reinforcement and stimulus complexity interact in their effects on performance and learning.

This hypothesis is a result of the conflicting findings that the manipulation of reinforcement contingencies both does and does not affect performance and learning. These differences in findings are probably due to differences in concept complexity.

Hypothesis 5

If Ss are presented with a task, then the Bower and Trabasso (1964) model of concept learning predicts the following statistics:

- Standard deviation of the trial of last error (SD_{TLE})
- Total number of errors (TE)
- Standard deviation of the total number of errors (SD_{TE})
- Number of errors before the first success
- $P(E_{n+1} \mid C_n)$
- $P(E_{n+1} \mid E_n)$
- Runs of errors: total, from 1 to 5, and over 5

This hypothesis is a result of the ability of the Bower and Trabasso model of conept learning to successfully predict response statistics in a wide variety of situations exclusive of mathematical concept learning, and the contention that a binary number concept learning situation is sufficiently similar to the previously studied situations to allow successful predictions. The above statistics are considered to be representative of those generally predicted by the Bower and Trabasso model.

Hypotheses 6a, 6b, and 6c The following hypotheses concern three basic assumptions of the Bower and Trabasso model of concept learning.

Hypothesis 6a

If Ss are presented with a task, then, prior to the trial of last error, the probability of an error is constant, or

$$P(E) = k \qquad \text{where: } E = \text{error response}, \; k = \text{constant}$$

and the probability of an error on trial n+1, given an error on trial n, is constant, or

$$P(E_{n+1} \mid E_n) = k \qquad \text{where: } n = \text{trial number}$$

and the probability of an error on trial n+1, given a correct response on trial n, is constant, or

$$P(E_{n+1} \mid C_n) = k \qquad \text{where: } C = \text{correct response}$$

Hypothesis 6b

If *Ss* are presented with a task, then, prior to the trial of last error, the probability of an error on trial n+1, given an error on trial n, and the probability of an error on trial n+1, given a correct response on trial n, are equal, or

$$P(E_{n+1} \mid E_n) = P(E_{n+1} \mid C_n)$$

Both hypotheses 6a (stationarity) and 6b (independence) are a result of the evidence found in support of stationarity and independence by Bower and Trabasso (1964). Both hypotheses are basic to the theory that learning is all-or-none, which itself is a basic assumption of the Bower and Trabasso model. The four equalities comprising these two hypotheses were suggested by Laming (1973, p. 254).

Hypothesis 6c

If *Ss* are presented with a task, then the learning parameter θ

(of the Bower and Trabasso model) has a constant value over trials, or

$$\theta = k \qquad \text{where: } \theta = p_{EL}$$

This hypothesis is a result of the findings by Bower and Trabasso (1964) that the distribution of total subject errors is successfully predicted with a constant value of θ.

Chapter II

METHOD

Experimental Environment and Apparatus

The experiment was conducted in one part of a large room occupied by the audio-visual staff and equipment of a secondary school. The room was somewhat dimly lit (as required by the visual stimuli employed) and sporadic bursts of noise (children playing) occurred outside. The Ss appeared to take no notice of the noise and, at any rate, treatment conditions were random with respect to the occasional noises. S sat in an armless chair facing a large table, upon which was placed the stimulus display board, including reinforcement and feedback devices, and two response switches. Approximately ten feet in front of S and behind the stimulus display board was another large table containing the control equipment. E was seated at the control table and both E and the control equipment were masked from S's view by an opaque curtain.

The experimental apparatus consisted of the following:

1. A stimulus display board (see Figure 2), 8" x 12" in frontal dimensions, containing the following:

(a) A $3\frac{1}{2}$" x 4" matrix of 26 2/9" diameter red light emitting diodes (LEDs) used for stimulus presentation, with the top three rows of six diodes each labeled "group 1"[1], the fourth

---

[1] All labels, instructions, verbal communications with S, etc. in this experiment were in the Thai language.

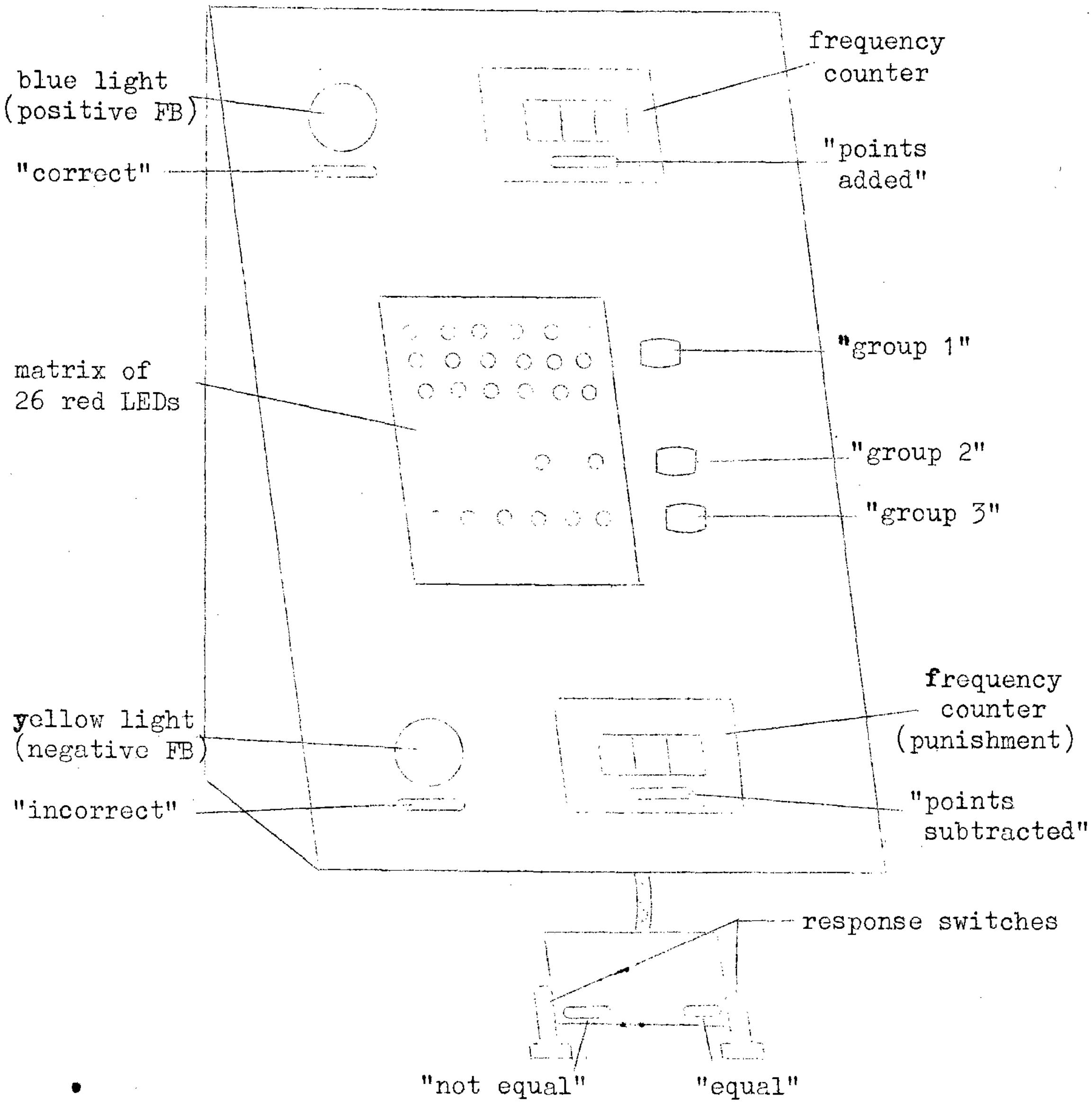


Figure 2. Display board for stimulus presentation and presentation of feedback, reward, and punishment, and response switches. Words in quotation marks are translations of Thai labels affixed to the equipment. Frontal dimensions of the display board are 8" x 12".

row of two diodes labeled "group 2", and the fifth row of six diodes labeled "group 3".

(b) Two $\frac{1}{2}$" diameter lights used to indicate to S whether his responses were correct or incorrect (feedback). The blue light above was labeled "correct" and provided positive feedback. The yellow light below was labeled "incorrect" and provided negative feedback.

(c) Two 2" x $2\frac{1}{2}$" frequency counters used to indicate the cumulative frequencies of rewards and punishments. The counter above was labeled "points added" and added one point for each correct response (if the experimental treatment so required). The counter below was labeled "points subtracted" and added one minus point for every incorrect response (if the experimental treatment so required).

2. Two micro switches mounted 3" apart on a wooden block used as response switches. The switch on the right was labeled "equal" and the switch on the left was labeled "not equal".

3. A control panel which automatically controlled the timing, sequencing, and presentation of stimuli, as well as the presentation of feedback, reward, and punishment. The control panel also provided for E's monitoring of S's trial by trial responses.

4. A portable tape recorder/player used to present the pre-recorded instructions to S.

Procedure

S was invited into the experimental room and asked to sit in the chair before the table containing the display board and response switches. S was then asked to listen to the pre-recorded instructions. The full text of the instructions appears in Appendix B. There were three sets of instructions, one for each of the three different reinforcement contingency groups. The different stimulus complexity and sex groups received the same instructions. The following is a summary of the instructions given to the Reward groups (III, IV, IX, and X) and differs from the instructions given to the other groups where marked with an asterisk (*):

> Today you will be required to solve a simple problem about a new number system. If you get a high enough score, you will win a prize of 20 or 30 baht.* You will be given 50 points to begin with.* Each time you answer correctly, one point will be added to your score.*
>
> Before you is a board in the center of which are many little red lights arranged in rows. From one to three of the lights in group 1 will illuminate and you must notice how many. At the same time, one or two lights in group 2 will illuminate. The lights in group 2 represent a number which is not necessarily the same as the number of lights illuminated. You must guess what number the lights in group 2 represent and decide if it equals the number of lights illuminated in group 1. (An example is given) If you think that group 1 and group 2 are equal, you should press the switch labeled equal. If not, you should press the switch labeled not equal. You needn't watch the lights in group 3.
>
> After you press one of the switches, the little red lights will go out. If the blue light on the top of the board illuminates, your answer is correct. If the yellow light on the bottom of the board illuminates, your answer is incorrect. If your answer is correct, the counter labeled points added will add one point to your score.* Then the red

> lights will illuminate again, but not the same as before. You must decide again if group 1 equals group 2 and press one of the switches. There is no time limit and you may decide as long as you like before pressing one of the switches. You should continue responding until the teacher tells you to stop.

The entire text of the instructions required approximately six minutes to present. As for the asterisked sections in the above example, the two other reinforcement contingency instructions differed as follows:

(1) Punishment groups (V, VI, XI, and XII) began with 150 points and added one minus point for each incorrect response.

(2) Feedback only groups (I, II, VII, and VIII) received no instructions about prizes or points added and subtracted, but were told instead that they would receive five baht for participating, whether or not they solved the problem.

Following the tape recorded instructions, S was asked to explain the procedure to E to ensure his understanding. If S could not, then E orally explained the procedure again and S was once more asked to explain the procedure to E. Then S was left to respond at his own rate until he reached a criterion of 26 consecutive correct responses or 520 total responses, whichever came first. The approximate minimum time interval between stimulus presentations was 2.05 seconds, with feedback and reinforcement requiring 1.55 seconds and switching time plus S's reaction time requiring about .50 seconds. The maximum time interval was determined by S himself and rarely exceeded 12 seconds.

The stimulus presentations consisted of a repeating sequence of 26 separate sets[1] of which 18 were positive instances of the concept (group 1 = group 2) and 8 were negative instances of the concept (group 1 $\neq$ group 2). The concept itself was the equality of a number of lights from one to three (group 1) and the binary representation of a number from one to three (group 2). S was considered to have learned the concept (Bower and Trabasso's state L) when he made 26 consecutive correct responses (thus responding correctly to each of the 18 positive and 8 negative instances of the concept). The number of lights illuminated in group 1 on any trial was either one, two, or three, with 9 ones, 9 twos, and 8 threes in the 26 trial sequence. The particular lights illuminated in the 18 light matrix in group 1 (three rows of six each) was determined randomly to prevent memorization of light positions.

The number of lights illuminated in group 2 on any one trial was either one or two, with the positions of the illuminated lights such that they binarily represented the numbers one (0 ●), two (● 0), or three (● ●). In the 26 trial sequence, the number represented in group 2 equalled the number of lights illuminated in group 1 on 18 trials and did not equal it on 8 trials. There were 8 ones, 9 twos, and 9 threes represented binarily in group 2 in each 26 trial sequence.

As for group 3, the number of lights illuminated on each trial was either one (for the High stimulus complexity groups) or none (for the

---

[1] It was found, in a pilot study, that Ss were not relying on memory in responding to individual recurring stimulus presentations, therefore permitting the use of a repeating sequence.

Low stimulus complexity groups). The position of the one light illuminated in group 3 corresponded to the number of lights illuminated in group 1. Thus, if three lights illuminated in group 1, then the third light (from the left) illuminated in group 3. Light illumination in group 3 was considered to increase stimulus complexity for the following reasons: (1) it created one more varying stimulus dimension in the presentation, (2) although perfectly correlated with light illumination in group 1, it provided information superfluous to concept attainment (the instructions required S to attend only to groups 1 and 2), and (3) although S was informed in the instructions that he needn't pay attention to the lights in group 3, it was expected that he would (and found to be so).

Figure 3 illustrates a typical stimulus presentation. Three lights in group 1 are illuminated. Two lights in group 2 are illuminated (the binary representation of the number three). One light in group 3 is illuminated, its position corresponding to the number of lights illuminated in group 1. Since group 1 = group 2, this is a positive instance of the concept, requiring S to press the "equal" switch to be correct. In the Feedback only groups, a correct response would produce positive feedback and an incorrect response would produce negative feedback. In the Reward groups, a correct response would produce positive feedback and cause one point to be added to S's score, while an incorrect response would produce negative feedback. In the Punishment groups, a correct response would produce positive feedback, while an incorrect reponse

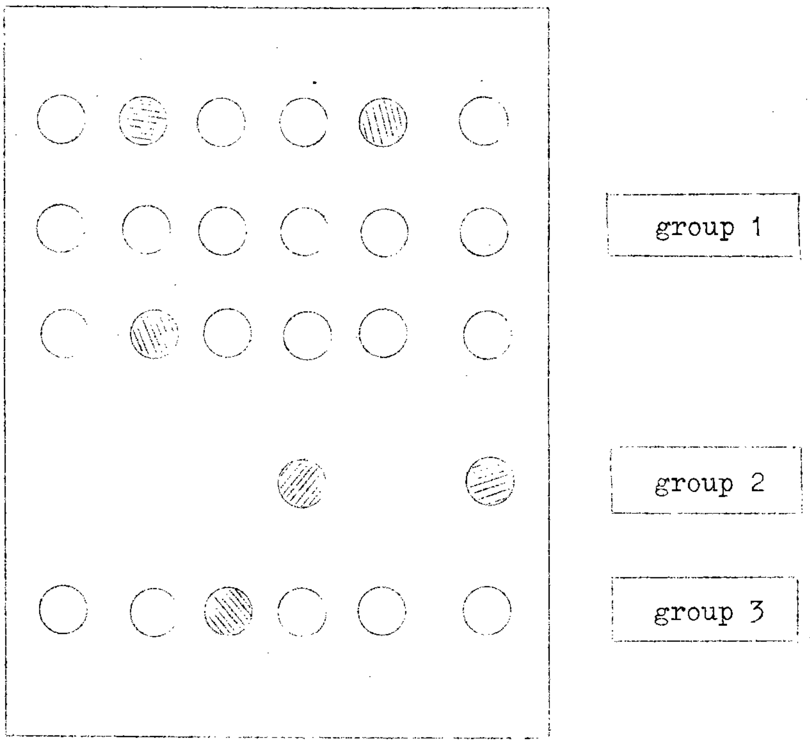


Figure 3. An example of a typical stimulus presentation (positive instance, i.e. group 1 = group 2) for a High stimulus complexity group (group 3 light illuminated).

would produce negative feedback and cause one minus point to be added to S's score. The illuminated light in group 3 indicates that Figure 3 is an example of a stimulus presentation for a High stimulus complexity group. Low stimulus complexity groups would have no lights illuminated in group 3.

In summary, S's task was to learn to press the "equal" switch when the binary representation of a number from one to three equalled a number of lights from one to three, and to press the "not equal" switch otherwise. This task was performed under the following experimental conditions:

(1) Feedback only/Low stimulus complexity (Groups I, Male, and II, Female) - S received instructions to the effect that he would receive five baht for participating, whether or not he solved the problem; positive or negative feedback was given on every trial; the lights in group 3 were not employed.

(2) Reward/Low stimulus complexity (Groups III, Male, and IV, Female) - S received instructions to the effect that he could win a 20 or 30 baht prize with a high enough score, with one point added to his initial score of 50 for each correct response; positive or negative feedback was given on every trial; the lights in group 3 were not employed.

(3) Punishment/Low stimulus complexity (Groups V, Male, and VI, Female) - S received instructions to the effect that he could win a 20 or 30 baht prize with a high enough score, with one point

subtracted from his initial score of 150 for each incorrect response; positive or negative feedback was given on every trial; the lights in group 3 were not employed.

(4) Feedback only/High stimulus complexity (Groups VII, Male, and VIII, Female) - the same as in (1) above except that the lights in group 3 were employed.

(5) Reward/High stimulus complexity (Groups IX, Male, and X, Female) - the same as in (2) above except that the lights in group 3 were employed.

(6) Punishment/High stimulus complexity (Groups XI, Male, and XII, Female) - the same as in (3) above except that the lights in group 3 were employed.

<u>Subjects</u>

A total of 54 male and 54 female students served as <u>S</u>s in the experiment. The <u>S</u>s comprised two and one-half of the six ninth grade classes in the Secondary Demonstration School attached to Srinakarinwirot University, Prasanmit Campus, Bangkok, Thailand. Three of the six classes were randomly selected and one half of the students in the third class were randomly discarded, giving the desired 108 <u>S</u>s. Of the total of 108 <u>S</u>s, nine males and nine females were randomly assigned to each of the six experimental treatments (as listed in the section on Procedure).

Chapter III

RESULTS

Results and Test of Hypothesis 1

In order to test Hypothesis 1 (i.e. the probability of an error is a "negatively decelerated" exponential function of the number of trials), it is necessary to first construct an exponential function, with suitable parameter values, which represents the empirical data. Then a test of the "goodness of fit" of the fitted curve to the observed curve can be conducted.

Lewis (1960, pp. 72-76) has suggested a procedure for fitting an exponential function of the form

$$Y = ae^{-bn} + c$$

where: Y = (here) the number of errors in a block of trials

e = base of natural logarithms

n = (here) the number of blocks of trials

a, b, c = parameters

to empirical data. Lewis' procedure results in the function

$$\hat{Y} = 952.58\, e^{-.116n} + 108.66$$

Figure 4 shows a plot of the observed and predicted (from the above function) errors against blocks of trials.

Lewis (pp. 375-379) also has suggested a procedure for testing the goodness of fit of the fitted curve to the observed curve. Since the

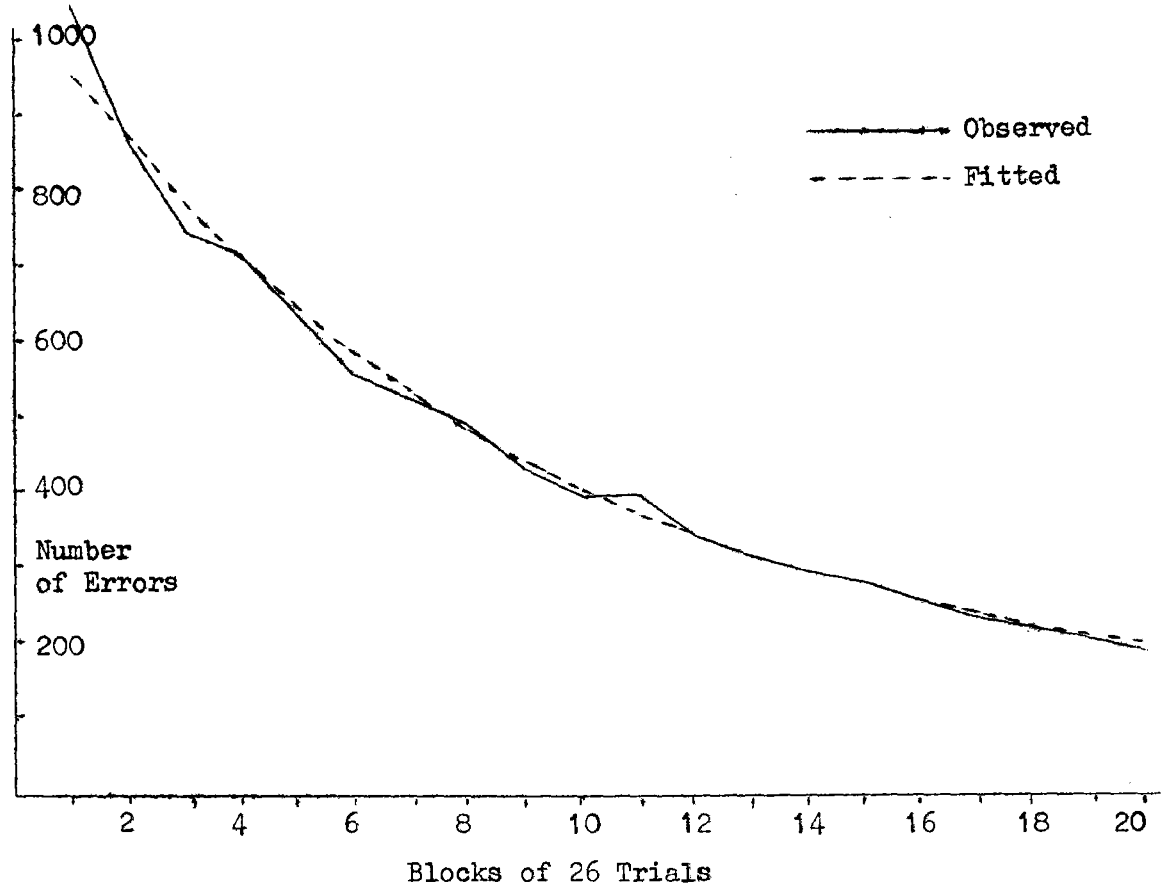


Figure 4. Observed and fitted distributions of errors over the 20 blocks of 26 trials.

blocks of trials are not independent (observations are made of the same Ss in each and every block of trials), a chi square test is not appropriate and an F test employing a within-subjects error term is necessary. The results of the F test ($F = .32$, $df = 18/864$, $p > .05$) showed that the observed and predicted values of errors did not significantly differ.[1] Therefore, Hypothesis 1 was somewhat supported (i.e. not rejected).[2]

Results and Tests of Hypothesis 2

Hypothesis 2 is composed of two parts. One states that if Ss are under different contingencies of reinforcement, then their performance differs. The other part states that their learning differs. Thus, two tests are required.

Table III shows the results of the analysis of variance of errors over the ten blocks of trials for the different reinforcement contingency groups. The F test of differences among reinforcement contingency groups (F_B) did not support the "differences in performance" part of Hypothesis 2. Nevertheless, inspection of Table IV shows that the different reinforcement contingency groups obtained different mean error scores. Inspection of Figure 5 further shows that, while the three

---

1 The .05 level of confidence was adopted as the criterion for retaining and rejecting all statistical hypotheses in this study.

2 In that the research hypothesis here is also a null hypothesis, failure to reject it, with a consequent unknown probability of Type II error, does not necessarily support it.

Table III

Summary of the Analysis of Variance of Errors Over the Ten Blocks of Trials (A) for the Different Reinforcement Contingency (B), Stimulus Complexity (C), and Sex (D) Groups

Source of Variance	df	MS	F
Total Between Subjects	107	389.81	
Reinforcement Contingencies (B)	2	615.75	1.96
Stimulus Complexity (C)	1	1,357.89	4.32*
Sex (D)	1	5,183.90	16.48*
Rein X Complex (BC)	2	15.84	.05
Rein X Sex (BD)	2	233.53	.74
Complex X Sex (CD)	1	1,046.00	3.33
Rein X Complex X Sex (BCD)	2	1,094.35	3.48*
error (between)	96	314.62	
Total Within Subjects	972	36.82	
Blocks of Trials (A)	9	2,127.74	128.28*
Blocks X Rein (AB)	18	28.59	1.72*
Blocks X Complex (AC)	9	12.12	.73
Blocks X Sex (AD)	9	28.55	1.72
Blocks X Rein X Complex (ABC)	18	36.73	2.21*
Blocks X Rein X Sex (ABD)	18	15.84	.95
Blocks X Complex X Sex (ACD)	9	4.60	.28
Blocks X Rein X Complex X Sex (ABCD)	18	24.46	1.47
error (within)	864	16.58	
Total	1,079	71.78	

* $p < .05$

Table IV

Mean Number of Errors for the Various Groups

General Mean		84.64		
Reinforcement Contingencies		99.08	81.14	73.67
		Reward	Punishment	Feedback only
Stimulus Complexity		95.83	73.43	
		Low	High	
Sex		106.57	62.69	
		Female	Male	
Rein and Complexity	Low	108.78	91.44	87.28
	High	89.39	70.83	60.06
		Reward	Punishment	Feedback only
Rein and Sex	Female	125.11	93.78	100.83
	Male	73.06	68.50	46.50
		Reward	Punishment	Feedback only
Complexity and Sex	Female	107.93	105.22	
	Male	83.74	41.63	
		Low	High	

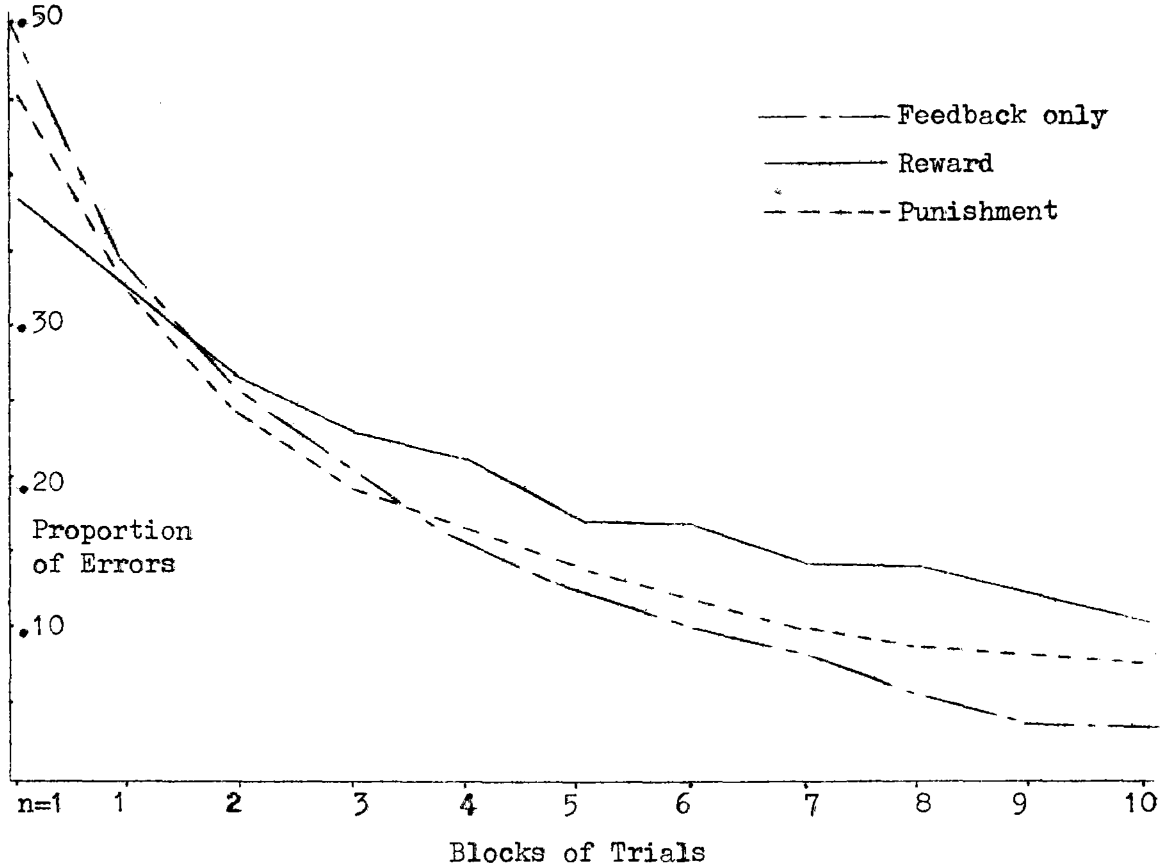


Figure 5. Proportion of errors over the ten blocks of 52 trials for the three reinforcement contingency groups.

reinforcement contingency groups did not differ greatly in the proportion of errors in the first half of the experiment, differences became more apparent during the last half. For these two reasons, an analysis of variance of error totals in the last five blocks of trials was conducted and appears in Table V. In this case, the F test of differences among the reinforcement contingency groups supported the "differences in performance" part of Hypothesis 2. Therefore, the Tukey (HSD) test was performed on the mean error differences between each pair of reinforcement contingency groups. Table VI shows that the Feedback only and Punishment group means were significantly higher than the Reward group mean, and the Feedback only and Punishment group means did not differ.

Thus, the "differences in performance" part of Hypothesis 2 was supported for the last half of the experiment, with the Feedback only and Punishment groups showing better performance than the Reward group.

Table III shows that the F test[1] of the Blocks X Reinforcement contingencies interaction (F_{AB}) supported the "differences in learning" part of Hypothesis 2. Further evidence of these differences can be seen in Figure 5 in that the three curves are not parallel and each intersects the other.

---

[1] The Cochran test for homogeneity of variance of both the between-subjects ($C = .16$, $p > .05$) and within-subjects ($C = .03$, $p > .05$) measures gave no reason to suspect the homogeneity of variance assumptions of the F tests involved.

Table V

Summary of the Analysis of Variance of Errors in the Last Five Blocks for the Different Reinforcement Contingency (B), Stimulus Complexity (C), and Sex (D) Groups

Source of Variance**	df	MS	F
Reinforcement (B)	2	332.76	4.14*
Stimulus Complexity (C)	1	321.22	3.99*
Sex (D)	1	913.01	11.35*
Rein X Complex (BC)	2	72.98	.91
Rein X Sex (BD)	2	124.66	1.55
Complex X Sex (CD)	1	226.87	2.82
Rein X Complex X Sex (BCD)	2	345.30	4.29*
error (between)	96	80.45	
Total (Between Subjects)	107	389.81	

* $p < .05$

** Within-subjects sources of variance are not listed.

Table VI

Results of Tukey (HSD) Tests of the Differences between the Mean Error Scores of the Different Reinforcement Contingency Groups in the Last Five Blocks of Trials

Reinforcement Contingency Groups	$\bar{X}_1 - \bar{X}_2$	$\sqrt{\frac{MS_w}{n}}$	q
Reward and Feedback only	35.55 - 16.39	2.99	6.40*
Punishment and Feedback only	24.64 - 16.39	2.99	2.76
Reward and Punishment	35.55 - 24.64	2.99	3.65*

* $p < .05$ (two-tailed test)

A significant Blocks X Reinforcement contingencies interaction suggests that, at least, one pair of these three reinforcement contingencies shows significant Blocks X Reinforcement contingencies interaction, i.e. that the interaction can be accounted for by, at least, two of the three reinforcement contingencies. Table VII, which shows the results of the analysis of variance of errors over the ten blocks of trials for each pair of reinforcement contingencies, shows that the Blocks X Reinforcement contingencies interaction for the Feedback only/Reward pair was the only significant one.

Results and Tests of Hypothesis 3

Hypothesis 3 is composed of two parts. One part states that if Ss are under different levels of stimulus complexity, then their performance differs. The other part states that their learning differs. Thus, two tests are required.

Table III shows the results of the analysis of variance of errors over the ten blocks of trials for the two stimulus complexity groups. The F test of differences between the two stimulus complexity groups (F_C) supported the "differences in performance" part of Hypothesis 3. Table IV shows that the mean number of errors was higher (95.83) for the Low stimulus complexity group than it was for the High stimulus complexity group (73.43). Figure 6 shows that this overall difference also holds for each of the ten blocks of trials.

Table VII

Summary of the Analysis of Variance of Errors Over the Ten Blocks of Trials (A) for Each Pair of Reinforcement Contingency Groups (B) - Trend Comparisons

Source of Variance**	df	MS	F
Feedback only and Reward Groups			
Total Between Subjects	71	382.85	
Total Within Subjects	648	38.69	
Blocks X Rein (AB)	9	53.17	3.01*
error (within)	576	17.66	
Total	719	72.67	
Feedback only and Punishment Groups			
Total Between Subjects	71	327.24	
Total Within Subjects	648	38.42	
Blocks X Rein (AB)	9	25.75	1.80
error (within)	576	14.28	
Total	719	66.94	
Reward and Punishment Groups			
Total Between Subjects	71	456.43	
Total Within Subjects	648	29.72	
Blocks X Rein (AB)	9	9.26	.66
error (within)	576	13.98	
Total	719	71.86	

* $p < .05$

** All sources of variance are not listed.

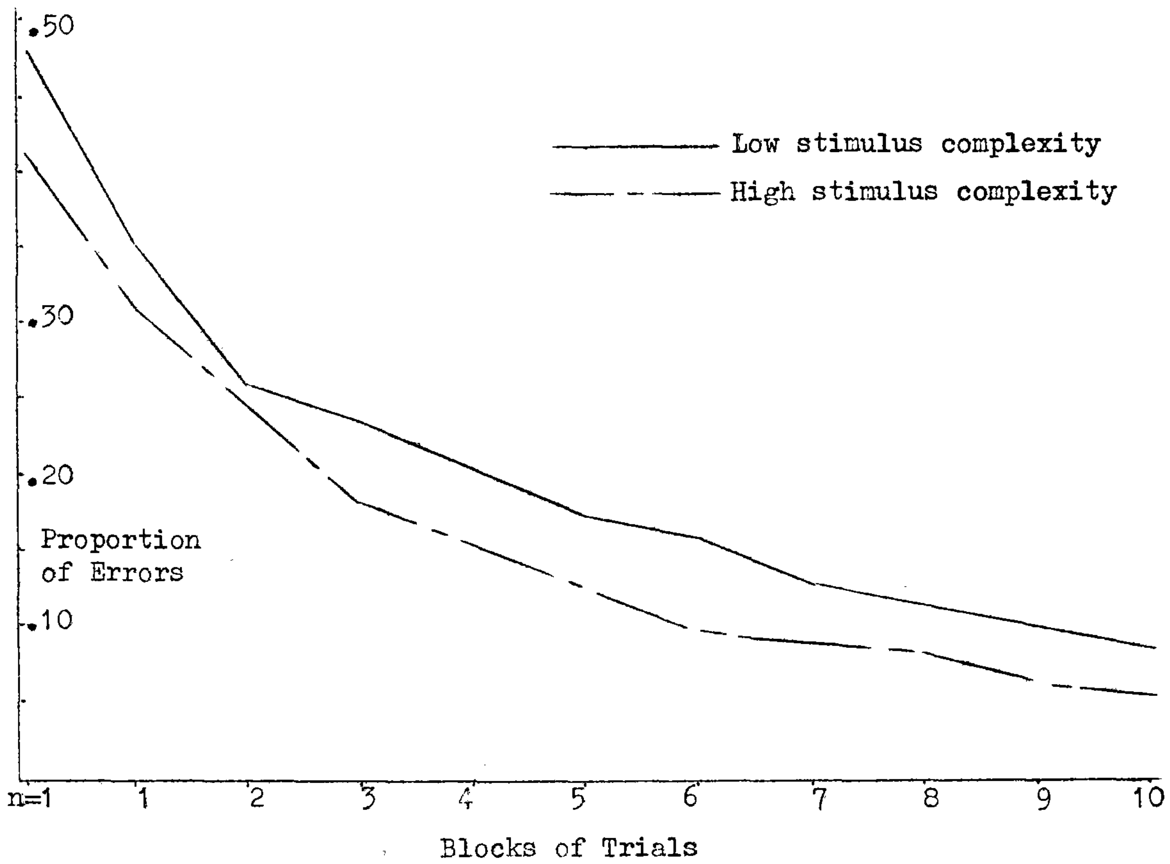


Figure 6. Proportion of errors over the ten blocks of 52 trials for the two stimulus complexity groups.

Table III shows that the F test of the Blocks X Stimulus complexity interaction (F_{AC}) did not support the "differences in learning" part of Hypothesis 3.

Results and Tests of Hypothesis 4

Hypothesis 4 is composed of two parts. One part states that reinforcement contingencies and stimulus complexity interact in their effects on performance. The other part states that they interact in their effects on learning. Thus, two tests are required.

As shown in Table III, the F test of Reinforcement contingencies X Stimulus complexity interaction (F_{BC}) did not support the "performance" part of Hypothesis 4.

Table III shows that the F test of Blocks X Reinforcement contingencies X Stimulus complexity interaction (F_{ABC}) supported the "learning" part of Hypothesis 4 (i.e. that reinforcement contingencies and stimulus complexity interact in their effects on learning).

The presence of a three-way interaction (ABC) does not indicate which "simpler" interactions are present. To test for the presence of these simpler interactions it is necessary to test the interactions of Blocks (A) with one of the other two factors (B or C) at each level of the remaining factor (C or B). For example, the AB interaction is tested both at the Low stimulus complexity level (C_1) and at the High stimulus complexity level (C_2). The AC interaction is tested in the same manner.

Table VIII shows the results of the analysis of variance of the simple Blocks X Reinforcement contingencies interactions (AB) at the two levels of stimulus complexity (C). The AB interaction at the Low stimulus complexity level (AB/C_1) was not significant, while the AB interaction at the High stimulus complexity level (AB/C_2) was significant. This is equivalent to "differences in learning" among the different reinforcement contingency groups at the High stimulus complexity level, but no differences at the Low level. These results are shown graphically in Figure 7.

To test even simpler interactions present in the AB interaction at the High stimulus complexity level, it is necessary to test simple AB interactions for each pair of reinforcement contingencies. Table IX shows that each of these three AB interactions were significant. Therefore, at the High stimulus complexity level, each reinforcement contingency resulted in different learning from each other reinforcement contingency.

Table X shows the results of the analysis of variance of the simple Blocks X Stimulus complexity interactions (AC) at each of the three levels of reinforcement contingencies (B). The AC interactions at the Feedback only (AC/B_1) and Punishment (AC/B_3) levels were significant, while the AC interaction at the Reward (AC/B_2) level was not. This is equivalent to "differences in learning" between the stimulus complexity levels at the Feedback only and Punishment levels, but no differences

Table VIII

Summary of the Analysis of Variance of Errors Over the Ten Blocks of Trials (A) at the Two Levels of Stimulus Complexity (C) - Trend Comparisons

Source of Variance**	df	MS	F
Low Stimulus Complexity Group			
Total Between Subjects	53	394.06	
Total Within Subjects	486	39.18	
Blocks X Rein (AB)	18	7.62	.40
error (within)	432	19.20	
Total	539	74.08	
High Stimulus Complexity Group			
Total Between Subjects	53	367.66	
Total Within Subjects	486	34.36	
Blocks X Rein (AB)	18	57.85	5.31*
error (within)	432	10.90	
Total	539	67.13	

* $p < .05$

** All sources of variance are not listed.

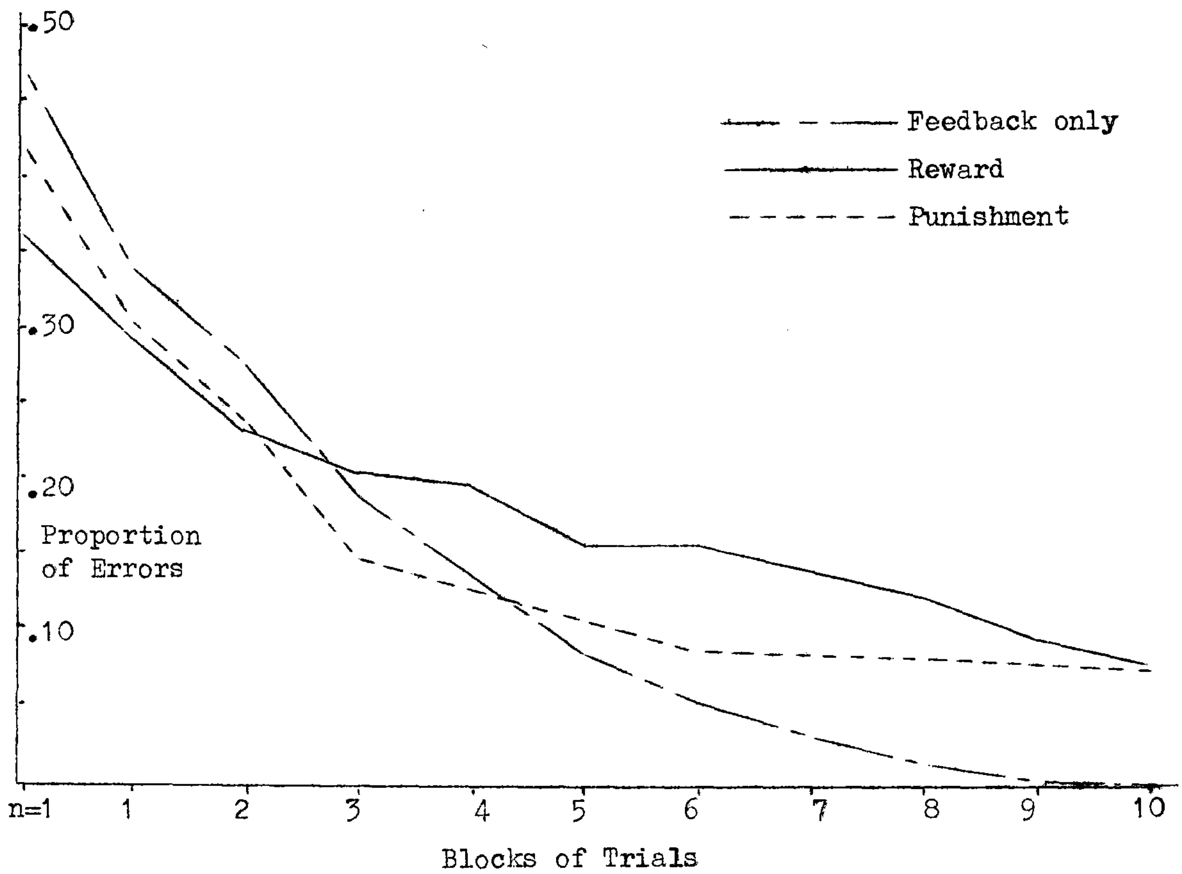


Figure 7. Proportion of errors over the ten blocks of 52 trials for the three reinforcement contingency groups at the High stimulus complexity level.

Table IX

Summary of the Analysis of Variance of Errors Over the Ten Blocks of Trials (A) for Each Pair of Reinforcement Contingency Groups (B) at the High Stimulus Complexity Level - Trend Comparisons

Source of Variance**	df	MS	F
Feedback only and Reward Groups			
Total Between Subjects	35	353.34	
Total Within Subjects	324	38.13	
Blocks X Rein (AB)	9	89.52	8.18*
error (within)	288	10.95	
Total	359	68.86	
Feedback only and Punishment Groups			
Total Between Subjects	35	258.95	
Total Within Subjects	324	39.66	
Blocks X Rein (AB)	9	62.23	5.18*
error (within)	288	12.02	
Total	359	61.04	
Reward and Punishment Groups			
Total Between Subjects	35	489.89	
Total Within Subjects	324	25.28	
Blocks X Rein (AB)	9	21.79	1.94*
error (within)	288	11.20	
Total	359	70.58	

* $p < .05$

** All sources of variance are not listed.

Table X

Summary of the Analysis of Variance of Errors Over the Ten Blocks of Trials (A) for the Three Reinforcement Contingencies (B) - Trend Comparisons

Source of Variance**	df	MS	F
Feedback only Group			
Total Between Subjects	35	247.77	
Total Within Subjects	324	44.14	
Blocks X Complexity (AC)	9	42.55	3.38*
error (within)	288	12.60	
Total	359	63.99	
Reward Group			
Total Between Subjects	35	496.14	
Total Within Subjects	324	33.24	
Blocks X Complexity (AC)	9	12.79	.70
error (within)	288	18.32	
Total	359	78.37	
Punishment Group			
Total Between Subjects	35	413.17	
Total Within Subjects	324	32.93	
Blocks X Rein (AC)	9	30.46	2.27*
error (within)	288	13.44	
Total	359	70,00	

* $p < .05$

** All sources of variance are not listed.

at the Reward level. Figures 8 and 9 illustrate these results.

Results and Tests of Hypothesis 5

Of the 108 Ss who participated in the experiment, only 76 learned to the criterion of 26 consecutive correct responses within the total of 520 trials. Since estimation of the learning parameter, θ, and the calculation of some observed statistics (e.g. runs of errors) require that all Ss eventually learn (i.e. enter the L state), only the data from the 76 Ss who learned were employed in the results and tests of Hypotheses 5 and 6.

Hypothesis 5 states that the Bower and Trabasso (1964) model of concept learning predicts the following statistics:

- SD_{TLE}
- TE
- SD_{TE}
- Number of errors before the first success
- $P(E_{n+1} | E_n)$
- $P(E_{n+1} | C_n)$
- Runs of errors (including total, from 1 to 5, and over 5)

These predictions depend, of course, on the prior estimation[1] of the model's two parameters, θ and π. Restle (1971, pp. 136-137)

[1] The formulas employed in the estimation of parameters and the prediction of statistics are listed in Appendix A.

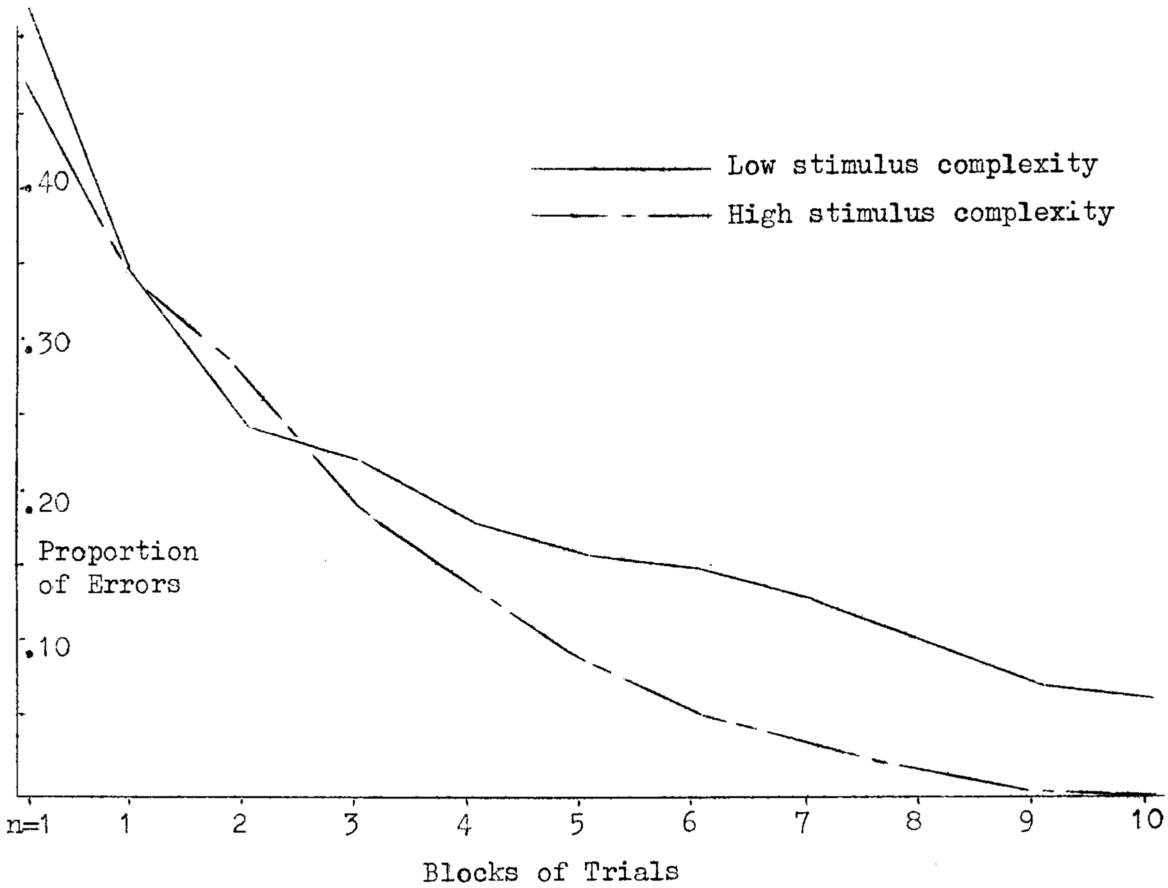


Figure 8. Proportion of errors over the ten blocks of 52 trials for the two stimulus complexity groups at the Feedback only level.

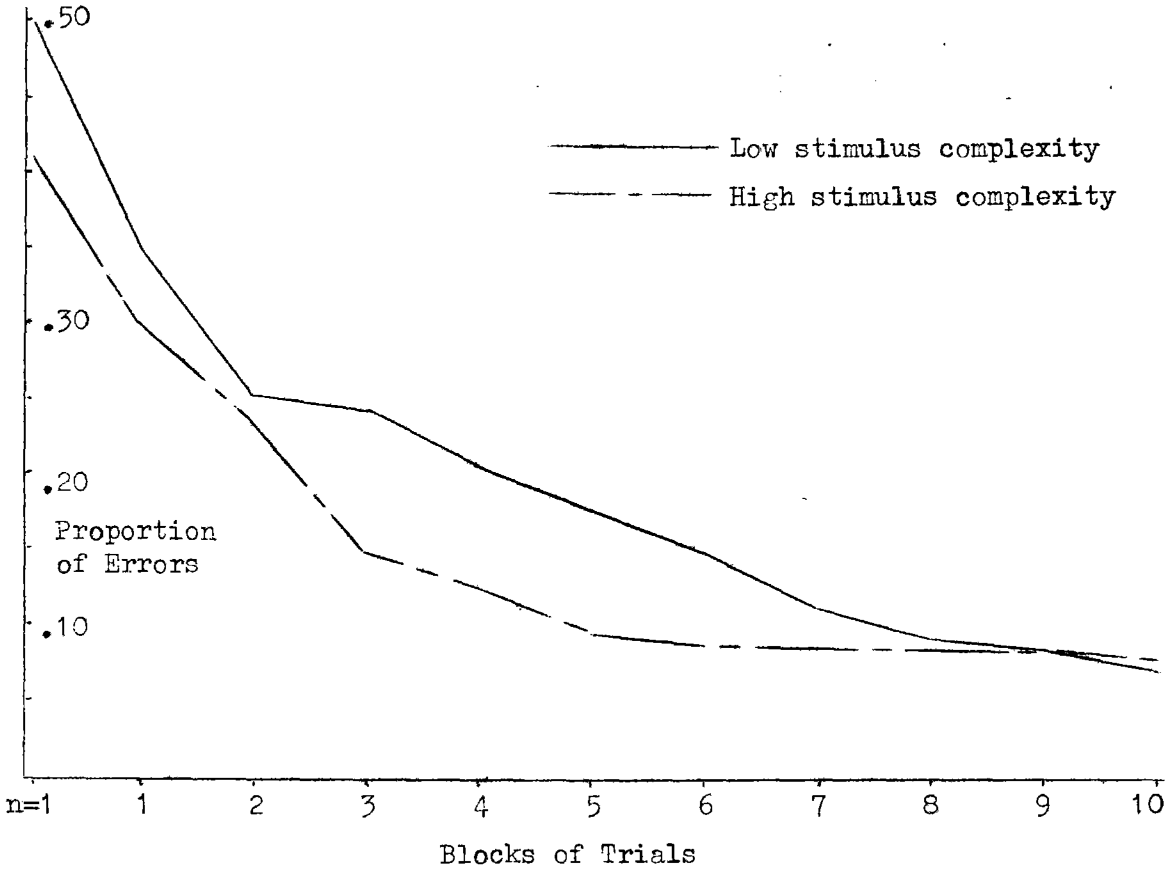


Figure 9. Proportion of errors over the **ten** blocks of 52 trials for the two stimulus complexity groups at the Punishment level.

suggested that a good estimate of π is the proportion of total correct responses to total responses, up to and including the trial of last error (TLE). The value of this estimate was $\hat{\pi} = .741$. The value of θ is estimated by the formula $\hat{\theta} = 1/(1-\pi)E(TLE)$, as given by Millward (1971). The value of this estimate was $\hat{\theta} = .0188$. In that the trial of last error (TLE) was used in estimating the value of θ, this statistic was not one of those predicted.

Table XI lists both the statistics calculated from the observed data and those predicted from the Bower and Trabasso model. Table XII summarizes the results of the statistical tests of the "goodness of fit" of the predicted statistics to those calculated from the observed data. Where appropriate, hypotheses were tested by means of t tests. Otherwise, chi square tests were used.

Table XII shows that the Bower and Trabasso model correctly predicted only the following statistics:

- TE
- $P(E_{n+1} \mid C_n)$
- Runs of errors (including total, from 1 to 5, and over 5)

It should be noted that, given the value of θ is estimated from TLE (as was the case in this analysis), it is very likely that TE will be predicted correctly, since $E(TLE) = E(TE)/(1-\pi)$, see Appendix A.

Since four of the statistics were not accurately predicted by the model, Hypothesis 5, as a whole, was not supported.

Table XI

Summary of the Observed and Predicted Values of Statistics

Statistic		Observed Value	Predicted Value
Trial of last error (TLE)		205.48	*
Standard deviation of TLE		151.81	204.99
Total number of errors (TE)		53.19	53.19
Standard deviation of TE		41.80	53.06
Number of errors before first success		.64	.35
$P(E_{n+1} \mid E_n)$		.204	.254
$P(E_{n+1} \mid C_n)$		.264	.259
Runs of errors:	Total	41.47	38.51
Of length	1	32.91	28.71
	2	6.28	7.30
	3	1.74	1.86
	4	.37	.47
	5	.09	.12
	over 5	.09	.04

* Not predicted because this statistic was used in the estimation of the learning parameter θ.

Table XII

Summary of the Results of Tests of the "Goodness of Fit" of the Observed to the Predicted Values of Statistics

Statistic		Result of Statistical Test
Trial of last error (TLE)		—
Standard deviation of TLE		$X^2 = 41.13$*
Total number of errors (TE)		t = 0.00
Standard deviation of TE		$X^2 = 46.55$*
Number of errors before first success		t = 2.40*
$P(E_{n+1} \mid E_n)$		t = 2.92*
$P(E_{n+1} \mid C_n)$		t = .35
Runs of errors:	Total	t = .78
Of length	1	t = 1.34
	2	t = 1.67
	3	t = .53
	4	t = 1.65
	5	t = .73
	over 5	t = 1.22

* $p < .05$ where: t test is two-tailed

X^2 (chi square) is tested against the confidence interval $X^2_{\alpha=.025} \leq X^2 \leq X^2_{\alpha=.975}$

Results and Tests of Hypotheses 6a, 6b, and 6c

Hypothesis 6a

Hypothesis 6a states that, if Ss are presented with a task, then prior to the trial of last error,

$$P(E) = k,$$

$$P(E_{n+1} \mid E_n) = k, \text{ and}$$

$$P(E_{n+1} \mid C_n) = k$$

Figures 10 and 11 show the proportions of errors and conditional errors over Vincent quartiles. The procedure of using Vincent quartiles rather than the same blocks of trials for all Ss was employed to control for differences in the learning rates of individual Ss.[1]

Tests of Hypothesis 6a are provided by F tests of the equality of the mean number of errors in each Vincent quartile. Table XIII summarizes the results of the analysis of variance of these mean numbers of errors over Vincent quartiles. None of the three F tests in Table XIII supported Hypothesis 6a (the hypothesis of stationarity).

Hypothesis 6b

Hypothesis 6b states that, if Ss are presented with a task, then prior to the trial of last error,

$$P(E_{n+1} \mid E_n) = P(E_{n+1} \mid C_n)$$

[1] Suppes and Ginsberg (1963) found this method to give a more accurate test of response stationarity. For further discussion on the use of Vincent quartiles, see Kling and Ross (1971, pp. 611-613).

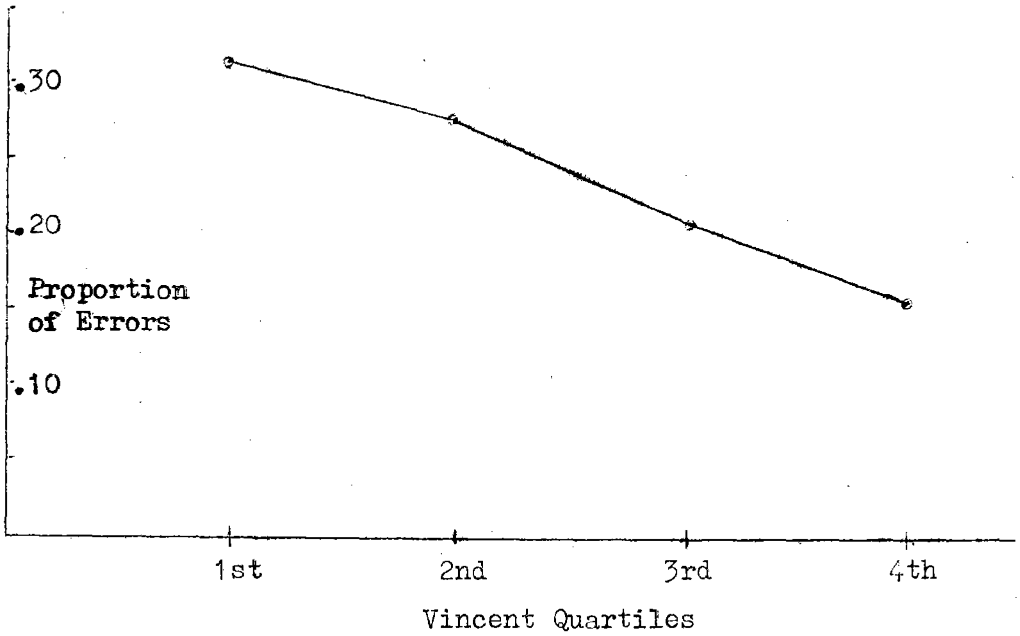


Figure 10. Proportion of errors over Vincent quartiles.

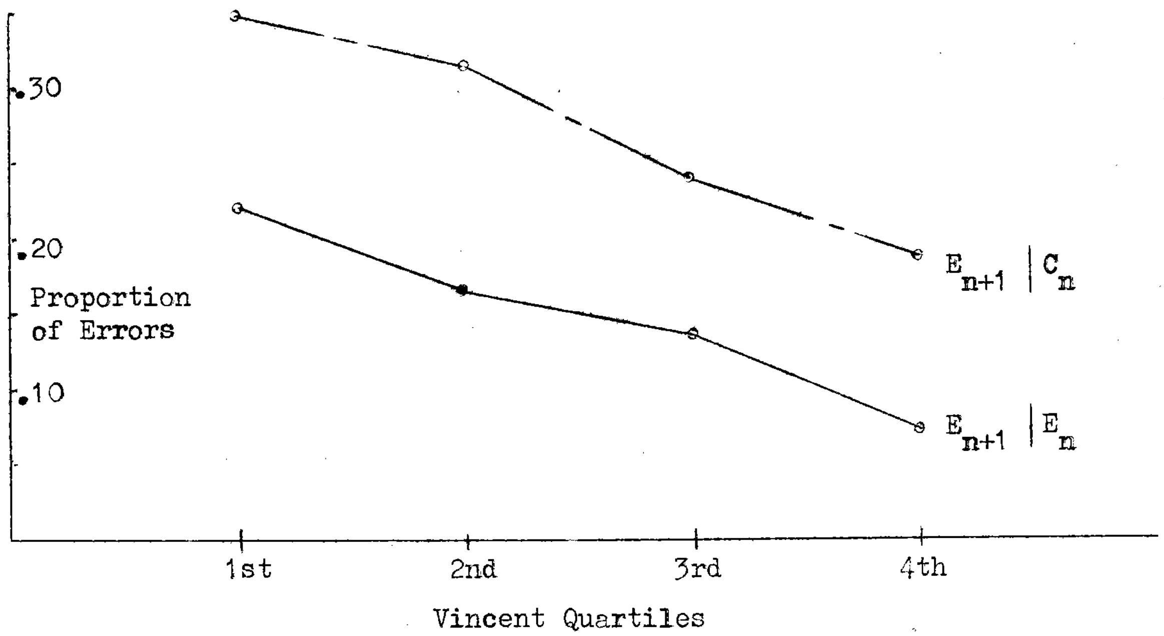


Figure 11. Proportion of conditional errors over Vincent quartiles.

Table XIII

Summary of the Analysis of Variance of Errors Over Vincent Quartiles

Source of Variance	df	MS	F
For E			
Total Between Subjects	75	.29	
Total Within Subjects	228	.22	
Vincent quartiles	3	4.36	26.32*
Quartiles X Subjects	225	.17	
Total	303	.24	
For $E_{n+1} \mid E_n$			
Total Between Subjects	75	.26	
Total Within Subjects	228	.19	
Vincent quartiles	3	3.80	26.04*
Quartiles X Subjects	225	.15	
Total	303	.21	
For $E_{n+1} \mid C_n$			
Total Between Subjects	75	.31	
Total Within Subjects	228	.23	
Vincent quartiles	3	4.67	27.48*
Quartiles X Subjects	225	.17	
Total	303	.25	

* $p < .05$

Table XI shows the mean values of $P(E_{n+1} | E_n)$ and $P(E_{n+1} | C_n)$. Figure 11 shows these values over Vincent quartiles. Table XIV shows the results of the t test of the mean difference between the proportions of $E_{n+1} | E_n$ and $E_{n+1} | C_n$.

The results of the t test in Table XIV did not support Hypothesis 6b (the hypothesis of independence).

Hypothesis 6c

Hypothesis 6c states that, if Ss are presented with a task, then the learning parameter has a constant value over trials, or

$$\Theta = k$$

This hypothesis can be tested by comparing the observed distribution of total errors (for each of the 76 Ss) and the theoretical distribution of total errors (employing a constant value of Θ in the calculations) as suggested by Laming (1973, p. 279). Since the observations are independent (i.e. the observations come from different Ss), the Kolmogorov-Smirnov one-sample test is an appropriate test of the "goodness of fit" of the theoretical to the observed distribution. Table XV shows the results of this test. Figure 12 shows both the observed and theoretical distributions of total errors.

The results of the Kolmogorov-Smirnov test in Table XV showed that the observed and theoretical distributions were not significantly different. Therefore, Hypothesis 6c was somewhat supported (i.e not rejected).

Table XIV

Result of the t Test of the Mean Difference between the Values of the Proportions of $E_{n+1} \mid E_n$ and $E_{n+1} \mid C_n$

Mean Difference between the Proportions of $E_{n+1} \mid E_n$ and $E_{n+1} \mid C_n$	df	$\frac{S_{X_1-X_2}}{\sqrt{n}}$	t
.0594	75	.0153	3.88*

* $p < .05$ (two-tailed test)

Table XV

Results of the Kolmogorov-Smirnov One-Sample Test of the "Goodness of Fit" of the Theoretical to the Observed Distribution of Total Errors

D Value Observed	Critical Value of D	N	p
.123 at TE = 50-59	.156 at $\alpha = .05$	76	$> .05$

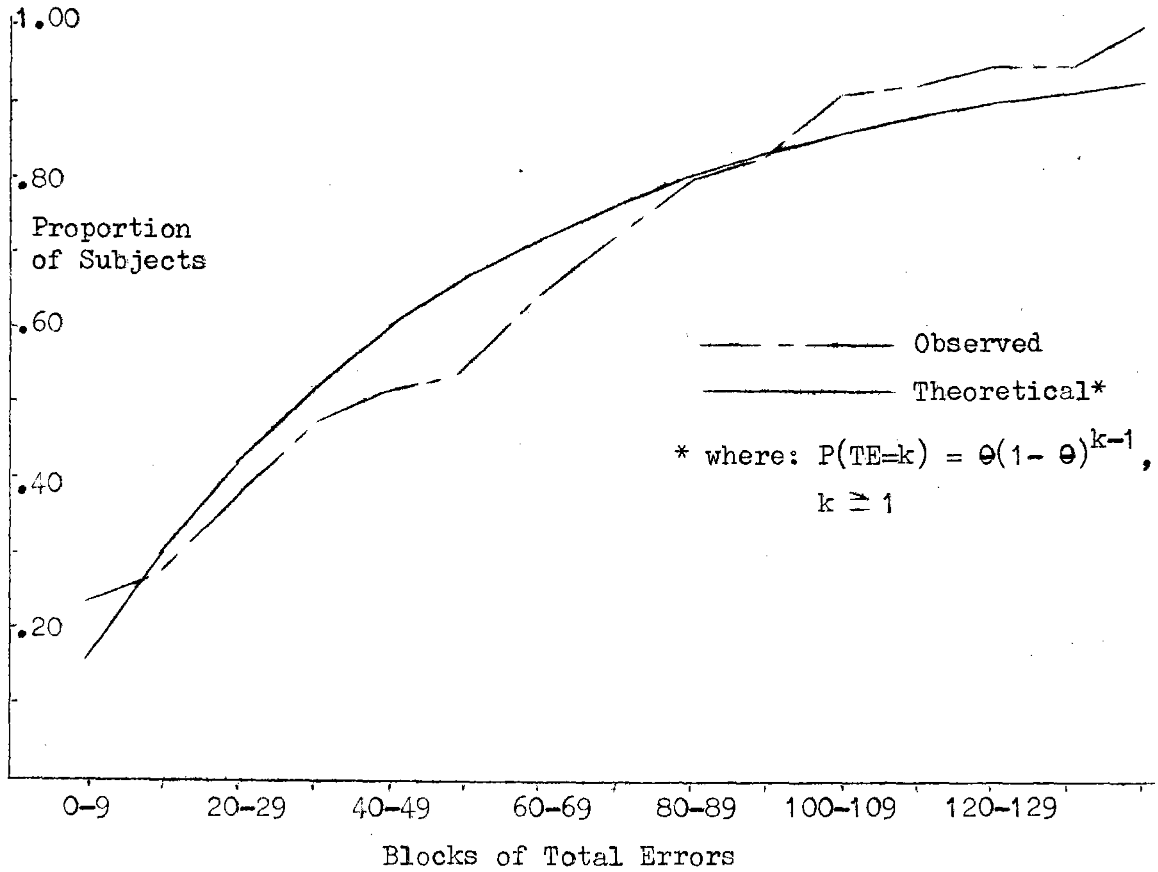


Figure 12. Observed and theoretical cumulative distributions of total errors (TE).

Chapter IV

DISCUSSION

Hypothesis 1 (Exponential Learning Curve)

The result of the test of Hypothesis 1 somewhat supported the hypothesis that the probability of an error on trial n was a "negatively decelerated" exponential function of the number of trials. Thus, the learning curve in the present binary number concept learning situation followed the form generally characteristic of the non-mathematical concept learning situations previously studied (as reported by Hilgard and Bower, 1966, and others). As a result, there appears to be some similarity between the learning processes involving mathematical and non-mathematical concepts, at least, as far as the change in error probability over trials is concerned.

Hypothesis 2 (Effects of Reinforcement Contingencies)

Differences in performance among the three reinforcement contingency groups were found to occur during the last half of the 520 learning trials, though not for the entire 520 trials. Differences in learning among the three reinforcement contingency groups were found to occur over the entire 520 trials. These findings are consistent with those reported by Buss and Buss (1956) and Wallace (1964) that different reinforcement contingencies differentially affected performance in concept learning. They are also consistent with the findings of Siegel (1959) that learning was differentially affected. The findings in this study, however, are not consistent with those of Bourne, et al (1967) that different reinforcement contingencies had no differential effects on performance. Inasmuch as there are few studies in this area of concept learning and reinforcement contingencies, it is impossible to say whether the present findings support or contradict a recognized body of research results.

There is a perplexing aspect to the performance and learning differences among the three reinforcement contingency groups. While the Feedback only group began with the worst performance, it ended with the best, and, while the Reward group began with the best performance, it ended with the worst (see Figure 5). Assuming that the Reward (plus feedback) employed in this study should have facilitated performance more than feedback alone, then the results conflict with what would be expected. The same may be said in the case of the Punishment group. Although the Punishment group began with better performance than the Feedback only group, it ended with worse performance. This result contradicts the findings of Buss and Buss (1956) and Wallace (1964) that punishment (of the incorrect response) facilitated performance.

The findings that both performance and learning in the Feedback only group were superior to performance and learning in the Reward and Punishment groups is difficult to rationalize. Nevertheless, two possible explanations are apparent. First, the fact that the _S_s in the Feedback only group were told that they would receive five baht for participating, whether or not they "solved the problem", may have created a general motivation to do well (perhaps in repayment of E's beneficience). In that the problem was a difficult one to solve, this motivation may have been an important factor. Second, the effects of various reinforcement contingencies on concept learning (and learning in general) may not be the same for Thai children as for Western children.[1] Even urban Thai children (as those employed in this study) have many cultural differences from their Western peers. This second explanation may be difficult to discount until there is further research into the area of learning in Thai children and the effects of various reinforcement contingencies on such learning.

Hypothesis 3 (Effects of Stimulus Complexity)

The two stimulus complexity groups were found to differ significantly in their performance, but not their learning. Inspection of Figure 6 shows the learning curves to be nearly parallel, with the High stimulus complexity group making a smaller proportion of errors in every block of trials. The fact that the High stimulus complexity group showed better performance (mean number of errors = 73.43) than the Low stimulus complexity group (mean = 95.83) is not in accord with the findings of other investigators (e.g. Bulgarella and Archer, 1962, and Schvaneveldt, 1966). Generally, it has been found that increases

---

[1] The possible influence of cultural differences on concept learning should be considered in interpreting all findings in this study.

in stimulus complexity lead to decreases in performance.

In this study, stimulus complexity was increased by the illumination of one of the lights in group 3 of the stimulus presentation. The position of the light illuminated corresponded to the number of lights illuminated in group 1. In that S was instructed to attend only to the lights in groups 1 and 2 and compare them for "equality" or "non-equality", the light illuminated in group 3 provided information superfluous (i.e. unnecessary) to concept attainment. Nevertheless, the group presented with superfluous light illuminations performed better. This result may be explained by the fact that the light illuminations in groups 1 and 3 were perfectly redundant (i.e. perfectly correlated). Bourne and Haygood (1959, 1961) found that increasing the number of perfectly redundant relevant dimensions in a concept identification problem increased performance. The lights in group 3 definitely qualify as a perfectly redundant relevant dimension. Bourne and Haygood (1959) noted that, in an attribute identification type of problem (in which S must discover the relevant attributes of the concept, as in the present study), the probability of S selecting a relevant attribute is logically related to the proportion of variable attributes which are relevant. Therefore, increasing this proportion (by the illumination of group 3 lights) increases the probability of a correct response. This explanation would appear to adequately explain the result obtained in this investigation. However, one discrepancy emerges. Bourne and Haygood (1959) reported that Ss notice, at least, some of the perfectly redundant relevant dimensions (attributes) and can often name several usable dimensions after concept attainment. All Ss in the present study received a short post-experiment interview, requiring them to explain how they solved or attacked the problem. Of the Ss in the High stimulus complexity group (group 3 lights employed), all explained in terms of relations between group 1 and group 2 lights. Not one of the 54 Ss mentioned the lights in group 3. This does not negate the explanation proposed by Bourne and Haygood. Indeed, their explanation remains the most plausible. Yet it does suggest that perfectly redundant relevant dimensions, although facilitative of performance, need not be reported as so by S.

Hypothesis 4 (Effects of Interaction of Reinforcement Contingencies and Stimulus Complexity)

Contingencies of reinforcement and stimulus complexity[1] were found to interact in their effect on learning, but not on performance. Hence, the contention that the reason some researchers have found differences in performance resulting from differences in reinforcement contingencies, while others have not is due to differences in concept complexity was not supported. On the other hand, the contention that the reason some researchers have found differences in learning resulting from differences in reinforcement contingencies, while others have not is due to differ-

[1] Henceforth, to be interpreted, also, as the presence and absence of a perfectly redundant relevant stimulus dimension.

ences in concept complexity was supported.

The finding that reinforcement contingencies and stimulus complexity interacted in their effects on learning (or that blocks of trials and reinforcement contingencies and stimulus complexity interacted in their effects on error responses) required further analysis to be useful. This further analysis showed that differences in reinforcement contingencies led to differences in learning (Blocks X Reinforcement contingencies interaction) at the High stimulus complexity level only (see Table VIII). It was also found that these differences could be attributed to differences between each of the three pairs of reinforcement contingencies (i.e. differences between Feedback only and Reward, Feedback only and Punishment, and Reward and Punishment). Figure 7 shows that, while the Feedback only group showed a higher proportion of errors initially, it showed a lower proportion of errors (than the other two groups) at the end. Indeed, the Feedback only group (at the High stimulus complexity level) was the only group in which all 18 Ss learned to criterion within the 520 trials. In the Reward and Punishment groups, only 13 and 12, respectively, of the 18 Ss learned to criterion. The finding that the Feedback only group showed a higher rate of error decrease should not be interpreted as applying to each individual S. It is most likely the result of more and more Feedback only Ss learning to criterion and ceasing to make errors altogether, while several Reward and Punishment Ss continued to make errors on all blocks of trials.

When the data were analyzed at each individual level of reinforce-contingencies, it was found that differences in learning (i.e. Blocks X Stimulus complexity interaction) resulting from differences in stimulus complexity occurred at the Feedback only and Punishment levels, but not at the Reward level. It can be seen in Figure 8 that, for the Feedback only group, while both the High and Low stimulus complexity groups had similar proportions of errors initially, the High stimulus complexity group's errors decreased more rapidly. It can be seen in Figure 9 that, for the Punishment group, High stimulus complexity was also more facilitative of learning (although this facilitation took a different form).

In summary, when the Feedback only and Punishment groups were presented with the High and Low stimulus complexity conditions, the High stimulus complexity condition was more facilitative of learning. This result is consistent with the Bourne and Haygood (1959, 1961) findings that learning improved with the inclusion of perfectly redundant relevant dimensions. The failure of the High stimulus complexity condition to facilitate learning in the Reward group, however, is not consistent.

<u>Hypothesis 5 (Predictions of Statistics by the Bower and Trabasso Model)</u>

Employing the data of the 76 <u>Ss</u> (of the total of 108 <u>Ss</u>) who learned to criterion, the Bower and Trabasso (1964) model of concept learning successfully predicted the following statistics:

- Total number of errors (TE)
- $P(E_{n+1} | C_n)$
- Runs of errors (including total, from 1-5, and over 5)

The Bower and Trabasso model failed to predict the following statistics:

- Standard deviation of the trial of last error (SD_{TLE})
- Standard deviation of the total number of errors (SD_{TE})
- Number of errors before the first success
- $P(E_{n+1} | E_n)$

The failure of the model to accurately predict four of the statistics is not indicative of the model's success in previous investigations (e.g. Bower and Trabasso, 1964) and requires explanation. The concept learning situation employed in the present study differed from those previously analyzed by the Bower and Trabasso model in two important respects: (1) the concept was mathematical, requiring S (theoretically, at least) to respond to the relation between number and the binary representation of a number, and (2) the concept was very difficult (as evidenced by the fact that 32 of 108 Ss did not learn to criterion within 520 trials and those who did required an average of 205 trials). Restle (1971, pp. 25-56) has noted that simple concept learning models, such as the Bower and Trabasso model, are not necessarily appropriate for the analysis of other than simple concept learning. Since there is no strong evidence that mathematical concept learning is basically different from the other types of concept learning studied (including the evidence resulting from the test of Hypothesis 1), the difficulty of the concept appears to be a plausible explanation of the model's failure in predicting statistics.[1]

Hypothesis 6a (Stationarity of Response Probability)

The hypothesis that the probabilities $P(E)$, $P(E_{n+1} \mid E_n)$, and $P(E_{n+1} \mid C_n)$ are stationary prior to the trial of last error was clearly not supported by the data. This result has serious implications for both the Bower and Trabasso model and an all-or-none theory of learning in the type of learning situation employed in this investigation. Laming (1973, p. 246) considers the stationarity of response probability assumption to be critical to acceptance of the Bower and Trabasso model and an all-or-none theory of learning. Reference to Figures 10 and 11 shows that all three of the above response probabilities not only did not remain stationary, but decreased over Vincent quartiles. Such a finding is especially important, because it also lends support to an incremental theory of learning.

Hypothesis 6b (Independence of Response Probability)

The hypothesis that error responses are independent of previous responses, or $P(E_{n+1} \mid E_n) = P(E_{n+1} \mid C_n)$, prior to the trial of last error was not supported. Thus, another assumption basic to the Bower

[1] Additional explanation of this failure is implicit in the lack of evidence in support of two assumptions (Hypotheses 6a and 6b) necessary to the support of an all-or-none theory of learning.

and Trabasso model failed to receive support. However, the failure of this assumption is not as serious as the failure of the stationarity assumption, since the independence of response probabilites is an assumption of a linear model and an incremental theory of learning as well (Laming, 1973, p. 246).

Hypothesis 6c (Invariance of θ)

The hypothesis that the learning parameter, θ, is invariant over trials was somewhat supported. Figure 12 shows that the fit of the theoretical distribution of total errors to the observed distribution was fairly good, though not so good as that achieved by Bower and Trabasso (1964). This result not only provides evidence in support of the Bower and Trabasso model and an all-or-none theory of learning, but a linear model and an incremental theory of learning (which assumes a constant value of θ) as well.

In summary (of Hypotheses 5, 6a, 6b, and 6c), the Bower and Trabasso model of concept learning was not supported by the data in two of its three major assumptions (stationarity and independence). Neither was it able to predict statistics of the learning process with as much success as previous studies. These results suggest the inapplicability of both the Bower and Trabasso model and an all-or-none theory of learning in the type of concept learning situation employed in this investigation.

Evidence of Probability Learning

The general failure of the Bower and Trabasso model of concept learning (and a Markov chain model of learning) in a concept learning situation which differed from previously studied situations in two respects (i.e. a mathematical concept and a more difficult concept) demands more than logical guesses in explanation. As a result, the data were re-inspected for evidence of the operation of contributing factors. Some evidence was found for one such factor - probability learning.

The following is an example of probability learning. Suppose S is faced with a situation in which he must predict which of two uncertain events will occur. For instance, will a die come up with the number 6, or some other number? If S has no information other than the frequency of occurrence of a 6 or some other number in the past upon which to base his predictions, he will come to predict each of the two events according to their probabilities of occurrence in the past. For an unbiased die, he will predict a 6 on one-sixth of the total trials and some other number on five-sixths of the total trials.

In the present investigation, the number of positive instances (18) of the concept was greater than the number of negative instances (8) in each 26 trial sequence. S's responses were limited to "equal" (indicating that S considered the stimulus presentation to be a positive instance, or group 1 = group 2) and "not equal" (indicating that S considered the stimulus presentation to be a negative instance, or group 1 $\neq$ group 2). Without any attempt to actually "learn" the concept, then, S could improve his probability of a correct response from the chance level simply by probability learning,[1] i.e. by matching the probabilities of his responses to the probabilities of occurrence of the positive (an "equal" stimulus presentation) and negative (a "not equal" stimulus presentation) instances of the concept. For example, S might press the "equal" switch on 18 of every 26 trials, regardless of the individual stimulus presentations. By adopting this strategy, S could increase his probability of a correct response from .500 to .574, since, by chance alone,

$$\begin{aligned} P(\text{correct}) &= P(=S) \times P(=R) + P(\neq S) \times P(\neq R) \\ &= (18/26) \times (18/26) + (8/26) \times (8/26) \\ &= .479 + .095 \\ &= .574 \end{aligned}$$

where:

$= S$	is an "equal" stimulus presentation, i.e. a positive instance of the concept
$= R$	is an "equal" response
$\neq S$	is a "not equal" stimulus presentation, i.e. a negative instance of the concept
$\neq R$	is a "not equal" response

Should S adopt the strategy of pressing the "equal" switch even more often than its probability of occurrence, he could further increase

[1] Since feedback was given on every trial, S could determine from the correctness of his responses whether a stimulus presentation was a positive or negative instance of the concept.

his probability of a correct response without actually "learning" the concept.[1] For example, S could maximize his probability of a correct response by chance by pressing the "equal" switch on every trial, with

$$P(\text{correct}) = (18/26) \times (26/26) + (8/26) \times (0/26)$$
$$= .692$$

The question here is whether all or some of the Ss in the present concept learning situation adopted a probability learning strategy at some time. To begin, it was assumed that, if probability learning occurred, it would have occurred during the early and middle parts of the response sequences, since all 76 Ss eventually learned to criterion, and the actual concept learning must have occurred no later than the latter part of the sequence. Therefore, it was deemed most appropriate to inspect the response sequences from the early to the middle trials (first part) and from the middle to the latter trials (last part). As a result, the individual S response sequences were divided into thirds in the manner of Vincent quartiles.

If Ss were using a probability learning strategy in the first part and a concept learning strategy in the last part, two results would be expected to occur. First, the probability of a "not equal" response, $P(\neq R)$, should decrease during the first part and, then, remain

---

[1] Responding at probabilities higher than the probabilities of occurrence of events is also considered to be probability learning, inasmuch as such behavior is characteristic of probability learning situations involving added reinforcements (see Siegel, 1959).

stationary or increase somewhat during the last part (if S had initially decreased $P(\neq R)$ to less than the probability of occurrence of a negative instance, $P(\neq S)$, he would eventually have to increase it again in order to learn to criterion). This would indicate that S employed probability learning in decreasing $P(\neq R)$ in the first part (probably to a value approximately equal to $P(\neq S)$, or 8/26 = .307), then ceased employing probability learning in the last part, since $P(\neq R)$ ceased decreasing. At the same time, the probability of an error, P(E), would be expected to decrease in both parts. Its decrease in the first part could be attributed to both concept learning and probability learning. Its decrease in the last part could be attributed only to concept learning, since $P(\neq R)$ ceased decreasing.

Second, if only probability learning were operating, then, while $P(\neq R)$ decreased, the probability of an error on a "not equal" stimulus presentation would increase, because

$$P(E \text{ on } \neq S) = P(\neq S) \times P(= R) \qquad \text{where: } P(\neq S) = k$$

In summary, if probability learning were occurring, two results would be expected:

(1) $P(\neq R)$ should decrease in the first part, then cease decreasing in the last part, while P(E) should decrease during both parts, and

(2) $P(E \text{ on } \neq S)$ should increase as long as $P(\neq R)$ decreases.

Figure 13 shows the results of dividing the S response sequences into equal thirds (in the manner of Vincent quartiles) and calculating

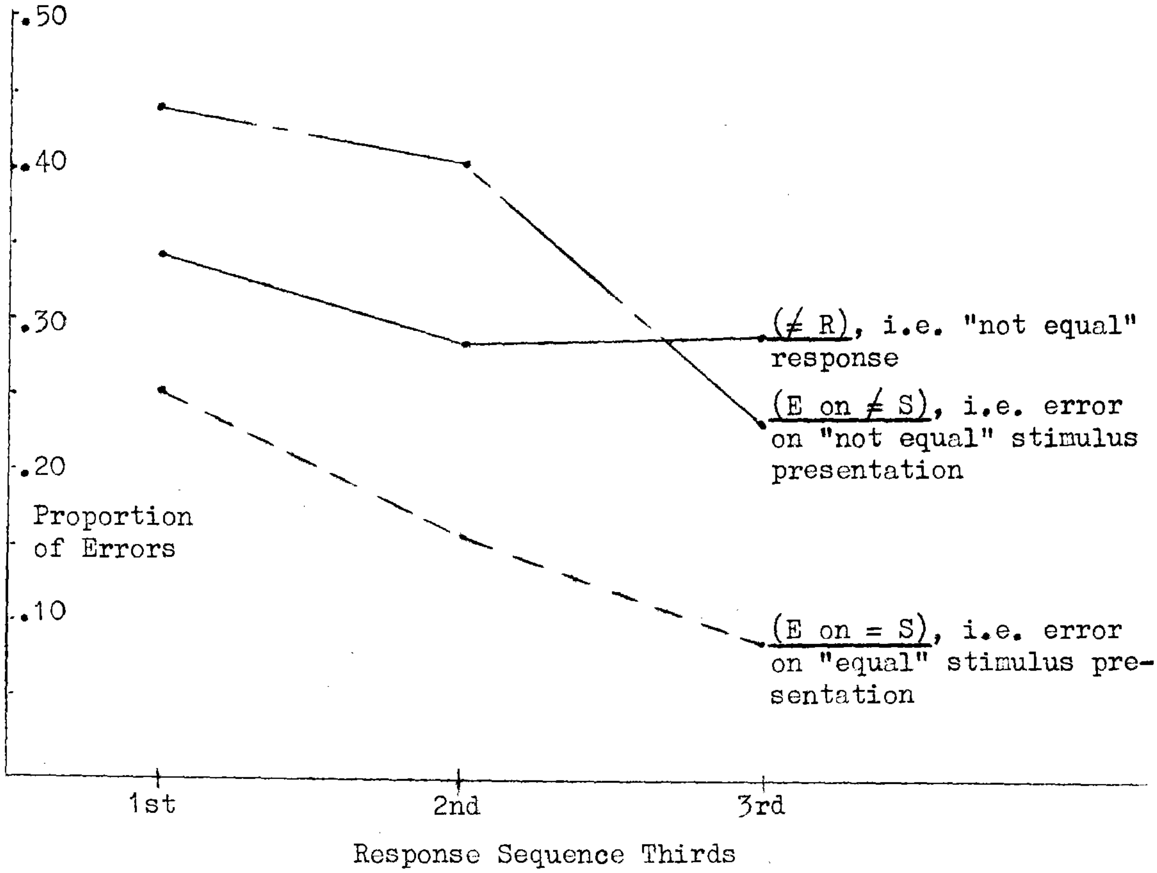


Figure 13. Proportions of responses for the 76 Ss who learned to criterion.

the following proportions: (E on ≐ S), (E on ≠ S), and (≠ R), averaged for all Ss. It can be seen that the proportion of (≠ R) decreased during the first part (to a value approximately equal to P(≠ S), .292 ≈ .307), then remained relatively stationary during the last part. At the same time, the proportion of errors, including both P(E on = S) and P(E on ≠ S), decreased during both parts of the response sequence. In that these results are in accordance with the a priori expectations previously noted, there is some evidence that probability learning was occurring.

The proportion of (E on ≠ S) did not change exactly as expected. However, this proportion did decrease more slowly than the proportion of (E on = S) in the first part, then more rapidly in the last part. Furthermore, the proportion of (E on ≠ S) decreased at about the same rate as the proportion of (≠ R) in the first part, then decreased sharply in the last part, while the proportion of (≠ R) remained relatively stationary. These results are not completely in accordance with the a priori expectations previously noted. However, the attenuated decrease of (E on ≠ S) in the first part and its sharp decrease in the last part suggests that probability learning may have prevented (E on ≠ S) from decreasing as rapidly as it would have if only concept learning were operating.

The evidence in Figure 13 is suggestive, but not conclusive. As a result, a search was conducted for further evidence. The previous results indicated that, if probability learning was occurring, it was probably not occurring alone, but in combination with concept learning. Moreover, some Ss could be expected to employ probability learning more than others. If these Ss can be identified and evidence in support of their probability learning produced, then there will be a firmer basis for concluding that probability learning was employed by, at least, some Ss.

The following criteria were used to identify "probability learners": (1) a decrease in the proportion of ($\neq$ R) during the first part and an increase during the last part, and (2) an increase in the proportion of (E on $\neq$ S) during the first part and a decrease during the last part. Nineteen "probability learners" were identified by these criteria. The remaining 57 Ss were termed "non-probability learners" for comparison.

Figures 14 and 15 show the mean response proportions for the "probability learners" and the "non-probability learners", respectively. The differences in their responses are obvious. During the first part, the "probability learners" sharply increased their proportion of (E on $\neq$ S), while decreasing their proportion of ($\neq$ R). This is strong evidence of probability learning. During the last part, the "probability learners" sharply decreased their proportion of (E on $\neq$ S), while increasing their proportion of ($\neq$ R). This is evidence of the cessation of probability learning (and an increase in concept learning). On the other hand, the "non-probability learners" showed a decrease in the proportion of (E on $\neq$ S) during both parts. They also showed a decrease in the proportion of ($\neq$ R) during the first part and no change during

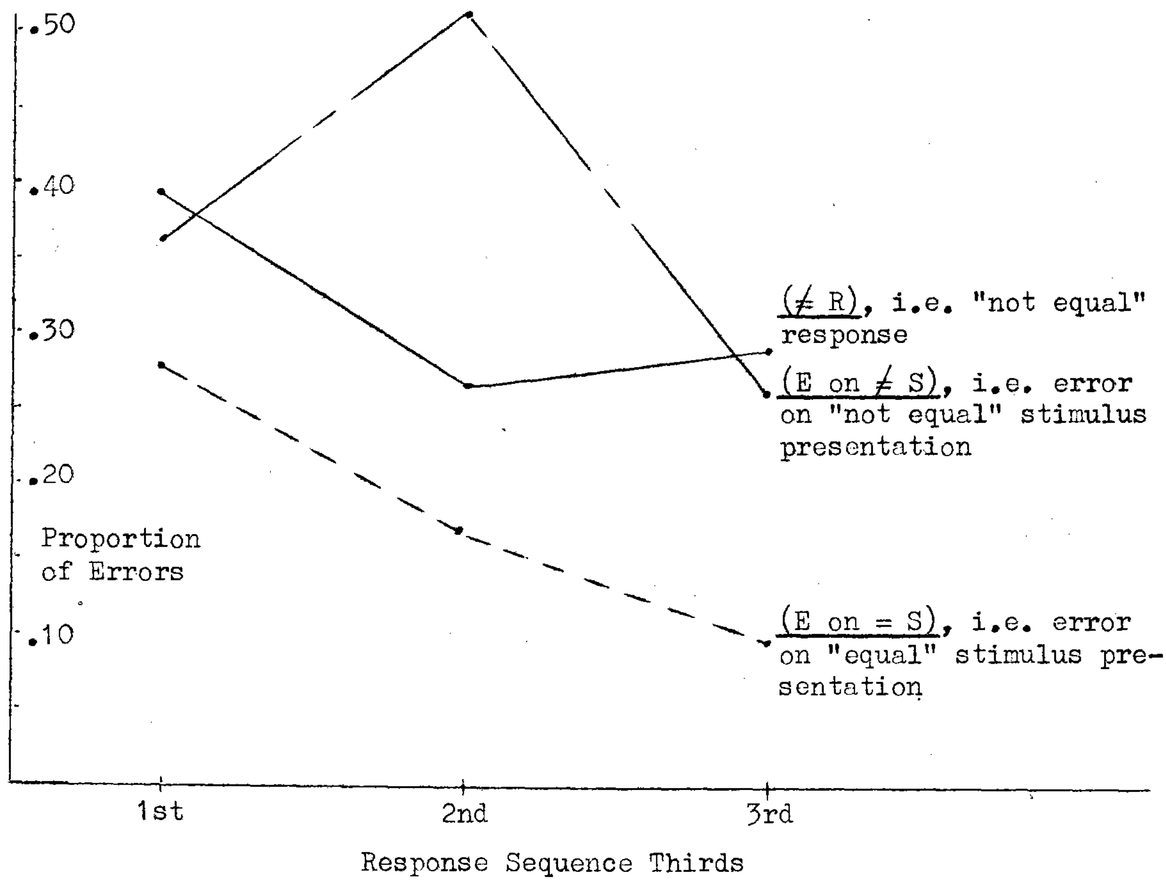


Figure 14. Proportions of responses for the "probability learners".

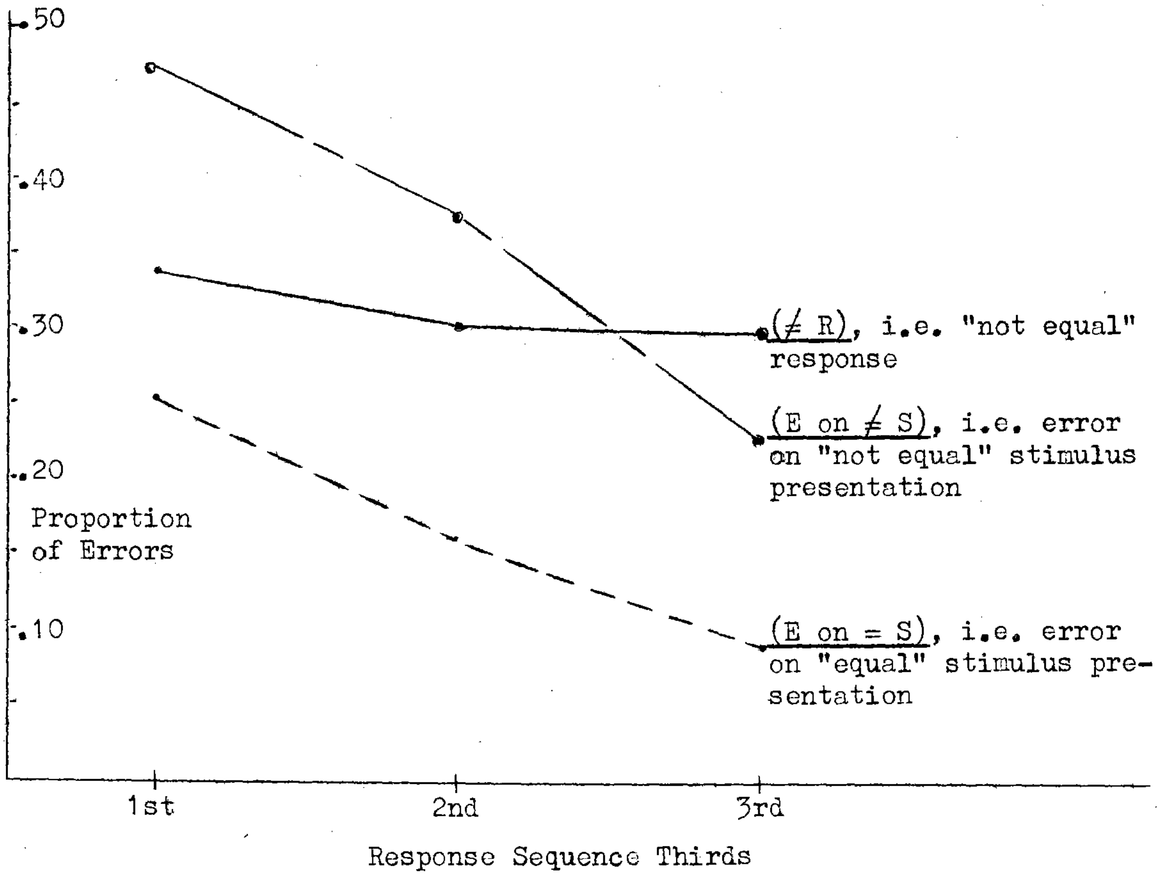


Figure 15. Proportions of responses for the "non-probability learners".

the last part.

In summary, evidence that probability learning was occurring in, at least, some Ss was found. Inasmuch as probability learning is a phenomenon well represented by a linear model of learning and supportive of an incremental theory of learning (Hilgard and Bower, 1966, pp. 348-351), part of the failure of the Bower and Trabasso model to account for learning in the present investigation might be attributed to the occurrence of probability learning in, at least, some Ss.

With regard to the behavior of "probability learners" and "non-probability learners", one other finding is of interest. The overall performance of the two groups did not differ significantly. The mean number of total errors for the "probability learners" was 55.63, while for the "non-probability learners" the mean was 52.39 .[1] The mean trial of last error for the "probability learners" was 203.58, while for the "non-probability learners" it was 206.12 .[2] It appears that, although apparent probability learning was somewhat disruptive of concept learning during the first part, i.e. an increase in P(E on $\neq$ S), it did not hinder overall performance. "Probability learners" also improved their performance on every block of trials. These results suggest the possibility that the frequent finding that a predominance of positive instances is generally facilitative of concept learning (e.g. Bruner, et

[1] A t test of the difference between means showed $t = .68$ ($p > .05$).

[2] A t test of the difference between means showed $t = .15$ ($p > .05$).

al, 1956) is, in part, due to the improvement in performance of, at least, some Ss resulting from probability learning.

Implications for Education in Thailand

The results of this investigation are suggestive of several things, including the following:

(1) Concept learning in Thai children may follow a "negatively decelerated" exponential form as it normally does in Western children, implying some (cross-cultural) generality of the learning process and, perhaps, allowing the application of Western research results in this area to Thai education.

(2) The effects of various reinforcement contingencies on the behavior of Thai children may not be the same as their effects on the behavior of Western children, requiring caution in the application of Western research results in this area to Thai education.

(3) The inclusion of perfectly redundant relevant stimulus dimensions in concept learning situations may facilitate performance in Thai children as it does in Western children, perhaps allowing the application of Western research results in this area to Thai education.

(4) The learning of a difficult and mathematical concept may not conform to a Markov chain model of learning and an all-or-none theory of learning.

Nevertheless, it would be both unwarranted and unwise to attempt any immediate application of the results of this investigation to educational practice in Thailand. As noted above, these results are merely suggestive. The intelligent use of research results demands that these results be replicated in both the original context and a wider context, and that they be interpreted in light of the totality of research in the same and related areas. Unfortunately, in Thailand, research in the area of interest of this investigation, as well as related areas, is lacking. Therefore, it is hoped that results obtained will serve two purposes: (1) to point out the need for further research in the area of learning in Thai children, and (2) to indicate worthwhile directions for future research aimed at application to Thai education.

Chapter V

SUMMARY AND CONCLUSIONS

One purpose of the study was to investigate behavior in a mathematical concept learning situation and the effects of successive blocks of trials, reinforcement contingencies, levels of stimulus complexity, and interactions among these variables on this behavior. Another purpose of the study was to employ a Markov chain model of learning (specifically, the Bower and Trabasso, 1964, model of concept learning) in the data analysis in order to: (1) achieve a more thorough understanding of the learning process through the testing of some of the model's assumptions, and (2) test the ability of the model to predict certain statistics defining the learning process.

108 ninth grade Thai students were required to learn a binary number concept under one of six treatment conditions, involving three different reinforcement contingencies and two levels of stimulus complexity. Nine females and nine males were randomly assigned to each of the six treatments. The task required S to press a switch labeled "equal" when a number of lights (from one to three) equaled a binary representation (in lights) of that number and to press a switch labeled "not equal" otherwise. All Ss were individually run for a total of 520 trials or until they reached a criterion of 26 consecutive correct responses. The criterion measure employed was the number of error responses in each block of trials.

Reinforcement contingencies were administered as follows:

Feedback only - S was told that he would be given five baht for participating, whether or not he solved the problem. Positive or negative feedback was given on every trial.

Reward (plus feedback) - S was told that each correct response would add one point to his initial score of 50 points and that, with a high enough score, he could win a monetary prize. Positive or negative feedback was given on every trial.

Punishment (plus feedback) - S was told that each incorrect response would subtract one point from his initial score of 150 points and that, with a high enough score, he could win a monetary prize. Positive or negative feedback was given on every trial.

Stimulus complexity was varied as follows:

High stimulus complexity - one perfectly redundant and relevant dimension was added to the normal stimulus presentation explained above.

Low stimulus complexity - no dimension was added to the stimulus presentation.

The hypotheses under investigation and the results obtained are as follows:

1. If Ss are presented with a task, then the probability of making an error on trial n, p_n, over successive trials is a "negatively

decelerated" exponential function of the form

$$p_n = ae^{-bn} + c \qquad \text{where: } n = \text{trial number}$$
$$e = \text{base of natural logarithms}$$
$$a, b, c = \text{parameters}$$

This hypothesis was somewhat supported (i.e. not rejected).

2. If Ss are presented with a task under different contingencies of reinforcement, then the Ss under different contingencies show different performance and different learning.

The performance part of this hypothesis was not supported for the experiment as a whole. However, it was supported for the last half of the experiment. The learning part of this hypothesis was supported. Contrary to the results of previous studies, the Feedback only group showed both better performance and learning than the Reward and Punishment groups. Two possible explanations of this result are: (1) the effect of a general motivation to do well resulting from the instructions to the Feedback only Ss that they would receive five baht for participating, and (2) differences in the effects of reinforcement contingencies on the behavior of Thai students and their Western peers.

3. If Ss are presented with a task under different levels of stimulus complexity, then the Ss under different levels show different performance and different learning.

The performance part of this hypothesis was supported. Contrary to what would be expected, the High stimulus complexity group performed better than the Low. This result was attributed to the fact that the added stimulus dimension was perfectly redundant and relevant to concept attainment (a finding also reported by other researchers). Of interest is the fact that, in the post-experiment interviews, not one of the 54 High stimulus complexity Ss reported using this added dimension in learning the concept. The learning part of this hypothesis was not supported.

4. If Ss are presented with a task under both different contingencies of reinforcement and levels of stimulus complexity, then contingencies of reinforcement and levels of stimulus complexity interact in their effects on performance and learning.

The performance part of this hypothesis was not supported. The learning part of this hypothesis was supported. Further analysis of the data showed the following:

(1) There were "differences in learning" among the different reinforcement contingency groups at the High stimulus complexity level, but not at the Low stimulus complexity level.

(2) There were "differences in learning" between the two stimulus complexity groups at the Feedback only and Punishment levels, but not at the Reward level.

5. If Ss are presented with a task, then the Bower and Trabasso (1964) model of concept learning predicts the following statistics:

- Standard deviation of the trial of last error (SD_{TLE})

- Total number of errors (TE)

- Standard deviation of the total number of errors (SD_{TE})

- Number of errors before the first success

- $P(E_{n+1} \mid E_n)$ where: n = trial number

  E = error response

- $P(E_{n+1} \mid C_n)$ C = correct response

- Runs of errors (including total, from 1-5, and over 5)

In that only the predictions of TE, $P(E_{n+1} \mid C_n)$, and Runs of errors were accurate, this hypothesis was not supported. The ability of the Bower and Trabasso model to predict statistics in the present mathematical concept learning situation was considerably lower than that found in previous easier and non-mathematical concept learning situations.

6a. If Ss are presented with a task, then, prior to the trial of last error, the following probabilities are constant:

$$P(E)$$

$$P(E_{n+1} \mid E_n)$$

$$P(E_{n+1} \mid C_n)$$

This hypothesis (of stationarity of response probability) was not supported. In fact, all three of the above response probabilities decreased over Vincent quartiles, lending support to a linear model (rather than a Markov model) and an incremental (rather than an all-or-none) theory of learning in the present situation.

6b. If Ss are presented with a task, then, prior to the trial of last error,

$$P(E_{n+1} \mid E_n) = P(E_{n+1} \mid C_n)$$

This hypothesis (of response independence) was not supported. This result, like the result of 6a, implies the inapplicability of an assumption basic to an all-or-none theory of learning and a Markov chain model of learning in the present situation.

6c. If Ss are presented with a task, then the learning parameter θ (of the Bower and Trabasso model) has a constant value over trials.

This hypothesis was somewhat supported (i.e. not rejected). Although the invariance of θ is an assumption of a Markov chain model, it is also an assumption of a linear model.

The overall performance of the Bower and Trabasso (1964) model of concept learning, including predictions of statistics and tests of basic assumptions, was markedly inferior to that reported in previous studies. In fact, more support was found for a linear model than a Markov chain model of learning. This result was partially explained by the evidence of probability learning by, at least, some Ss. Probability learning is a phenomenon well represented by a linear model.

บทคัดย่อ และข้อสรุป

การศึกษาครั้งนี้มีความมุ่งหมายที่จะศึกษาถึงพฤติกรรมในการเรียนรู้มโนภาพ (Concept) ทางคณิตศาสตร์ และผลของตัวแปรต่าง ๆ ได้แก่ จำนวนครั้งในการตอบติดต่อกัน การเสริมแรงในรูปแบบต่าง ๆ ระดับความซับซ้อนของตัวเร้า ตลอดจน interaction ระหว่างตัวแปรเหล่านี้ ที่มีต่อพฤติกรรมในการเรียนรู้นั้น และความมุ่งหมายอีกประการหนึ่งคือ ต้องการใช้ Markov chain model (model ของ Bower และ Trabasso, 1964) ในการวิเคราะห์ข้อมูล เพื่อเพิ่มความเข้าใจเกี่ยวกับกระบวนการเรียนรู้ โดยการทดสอบข้อตกลงพื้นฐานบางอย่างของ model และเพื่อทดสอบความสามารถของ model ในการทำนายค่าสถิติต่าง ๆ ที่เกี่ยวกับกระบวนการเรียนรู้

กลุ่มตัวอย่างในการทดลองครั้งนี้ ได้แก่นักเรียนชั้นมัธยมศึกษาปีที่ ๒ จำนวน ๑๐๘ คน ซึ่งแบ่งออกเป็น ๖ กลุ่มให้เรียนรู้มโนภาพเกี่ยวกับจำนวนเลขฐานสอง (binary number) โดยได้รับการปฏิบัติที่แตกต่างกันตามรูปแบบของการเสริมแรงและความซับซ้อนของตัวเร้า ในแต่ละกลุ่มประกอบด้วยนักเรียนชายและหญิงฝ่ายละ ๙ คน การจัดเข้ากลุ่มได้ใช้วิธีสุ่มจากกลุ่มตัวอย่าง

ชิ้นงานเฉพาะ (task) ที่นักเรียนแต่ละคนต้องปฏิบัติคือ เมื่อดวงไฟจำนวนหนึ่ง (ตั้งแต่หนึ่งถึงสามดวง) เท่ากับตัวแทนเลขฐานสองของจำนวนนั้น (ซึ่งเป็นดวงไฟอีกหนึ่งชุด) ต้องกดสวิทช์ตอบว่า "เท่า" และในกรณีตรงกันข้ามต้องกดสวิทช์ตอบว่า "ไม่เท่า" นักเรียนจะต้องปฏิบัติทั้งสิ้น ๕๒๐ ครั้ง โดยแบ่งออกเป็น ๑๐ ชุด (blocks of trials) ชุดละ ๕๒ ครั้ง หรือปฏิบัติจนกระทั่งตอบถูกถึง ๒๖ ครั้ง ติดต่อกัน พฤติกรรมที่เป็นเกณฑ์ในการวัดคือจำนวนครั้งที่ตอบผิดในแต่ละชุดของการปฏิบัติ

รูปแบบต่าง ๆ ของการเสริมแรงมีดังนี้ คือ

(๑) <u>การให้รู้ผลการตอบ</u> (feedback) นักเรียนได้รับคำชี้แจงว่าจะได้รับเงินจำนวน ๕ บาท จากการปฏิบัติภารกิจ ไม่ว่าจะแก้ปัญหาได้หรือไม่ และในขณะทดลองนักเรียนจะรู้ผล

การตอบของตนทุกครั้งว่าถูกหรือผิด

(๒) การให้รางวัลและการรู้ผลการตอบ นักเรียนได้รับคำชี้แจงว่า ในตอนแรกจะได้รับคะแนนฟรี ๕๐ คะแนน เมื่อตอบถูกแต่ละครั้งจะได้รับเพิ่ม ๑ คะแนน ถ้าผู้ใดได้รับคะแนนรวมมากพอควรจะได้รับรางวัลเป็นเงินจำนวนหนึ่ง และในแต่ละครั้งที่ตอบนักเรียนจะรู้ผลการตอบของตนด้วย

(๓) การลงโทษ และการรู้ผลการตอบ นักเรียนได้รับคำชี้แจงว่าในตอนแรกจะได้รับคะแนนฟรี ๑๕๐ คะแนน เมื่อตอบผิดแต่ละครั้งจะถูกหักออก ๑ คะแนน เมื่อสิ้นสุดการปฏิบัติผู้ใดมีคะแนนมากพอควรจะได้รับรางวัลเป็นเงินจำนวนหนึ่ง และในแต่ละครั้งที่ตอบนักเรียนจะรู้ผลการตอบของตนด้วย

ในด้านระดับความซับซ้อนของตัวเร้ามีดังนี้ คือ

(๑) ความซับซ้อนมาก หมายถึงการเพิ่มตัวเร้าอีกมิติหนึ่ง (ดวงไฟอีกหนึ่งชุด) นอกเหนือจากตัวเร้าที่กล่าวถึงแล้ว มิติที่เพิ่มขึ้นนี้จะซ้ำแบบมิติอื่น และเกี่ยวข้องกับการแก้ปัญหาด้วย

(๒) ความซับซ้อนน้อย หมายถึงการไม่มีมิติเพิ่ม

สมมติฐานที่ทดสอบและผลของการทดสอบมีดังนี้

(๑) เมื่อนักเรียนปฏิบัติชิ้นงานเฉพาะ ความน่าจะเป็นที่จะตอบผิดในครั้งที่ n (p_n) เป็น "negatively decelerated" exponential function กล่าวคือ

$$p_n = ae^{-bn} + c$$

เมื่อ n คือ ลำดับที่ในการปฏิบัติ

e คือ ฐานของ natural logarithms

a, b, c คือ parameters

สมมติฐานนี้ได้รับการสนับสนุนบ้าง กล่าวคือไม่ได้รับการปฏิเสธ

(๒) ถ้านักเรียนปฏิบัติชิ้นงานเฉพาะโดยได้รับการเสริมแรงในรูปแบบที่ต่างกัน นักเรียนที่ได้รับการเสริมแรงต่างกันแสดงการกระทำและการเรียนรู้ต่างกัน

ส่วนของสมมติฐานที่กล่าวถึงความแตกต่างกันของการกระทำไม่ได้รับการสนับสนุนสำหรับการปฏิบัติชิ้นงานเฉพาะทั้งหมด แต่ได้รับการสนับสนุนสำหรับครึ่งหลังของการปฏิบัติ และส่วนของสมมติฐานที่กล่าวถึงความแตกต่างกันของการเรียนรู้ได้รับการสนับสนุน แต่ปรากฏว่าผลการทดลองครั้งนี้ไม่ตรงกับผลการวิจัยของผู้อื่น กล่าวคือ กลุ่มที่รู้ผลการตอบอย่างเดียวมีการกระทำและการเรียนรู้ดีกว่ากลุ่มที่ได้รับรางวัล และการลงโทษ การที่เป็นเช่นนี้ผู้วิจัยมีความเห็นว่าอาจจะเนื่องมาจาก

ก. การที่กลุ่มที่รู้ผลการตอบอย่างเดียวได้รับคำชี้แจงว่าจะได้รับเงินจำนวน ๕ บาท ไม่ว่าจะแก้ปัญหาได้หรือไม่ ทำให้เกิดแรงจูงใจทั่วไปในการแก้ปัญหา และ

ข. รูปแบบต่าง ๆ ของค่าเสริมแรงอาจส่งผลต่อพฤติกรรมที่แตกต่างกันระหว่างนักเรียนไทยและนักเรียนในประเทศตะวันตก

(๓) ถ้านักเรียนปฏิบัติชิ้นงานเฉพาะโดยได้รับตัวเร้าที่มีระดับความซับซ้อนแตกต่างกัน นักเรียนที่ได้รับตัวเร้าที่มีระดับความซับซ้อนต่างกัน แสดงการกระทำและการเรียนรู้ต่างกัน

ส่วนของสมมติฐานที่กล่าวถึงความแตกต่างของการกระทำได้รับการสนับสนุน แต่ปรากฏว่าผลของการทดลองนี้ไม่ตรงกับผลที่คาดหวังกันโดยทั่วไป กล่าวคือกลุ่มที่ได้รับตัวเร้าซึ่งมีความซับซ้อนมาก แสดงการกระทำดีกว่ากลุ่มที่ได้รับตัวเร้าที่มีความซับซ้อนน้อย ซึ่งผู้วิจัยมีความเห็นว่าอาจเนื่องมาจาก มิติของตัวเร้าที่เพิ่มขึ้นนั้นเป็นมิติที่ซ้ำแบบมิติอื่น และเกี่ยวข้องกับการแก้ปัญหา ซึ่งสอดคล้องกับผลการวิจัยของผู้อื่น แต่อย่างไรก็ตามมีข้อที่น่าพิจารณาก็คือ จากการสัมภาษณ์นักเรียนในกลุ่มที่รับตัวเร้าที่มีความซับซ้อนมากทั้ง ๕๔ คน ไม่มีนักเรียนคนใดพูดถึงการใช้มิติที่เพิ่มขึ้นในการแก้ปัญหา

(๔) ถ้านักเรียนปฏิบัติชิ้นงานเฉพาะโดยได้รับการเสริมแรงในรูปแบบที่ต่างกัน และตัวเร้าที่มีระดับความซับซ้อนต่างกัน รูปแบบของการเสริมแรงและความซับซ้อนของตัวเร้า interact ในการส่งผลต่อการกระทำและการเรียนรู้

ส่วนของสมมติฐานที่กล่าวถึงความแตกต่างกันของการกระทำนั้นไม่ได้รับการสนับสนุน แต่ส่วนของสมมติฐานที่กล่าวถึงความแตกต่างกันของการเรียนรู้ได้รับการสนับสนุน จากการวิเคราะห์รายละเอียดได้พบว่า

ก. มีความแตกต่างกันในการเรียนรู้ระหว่างกลุ่มที่ได้รับการเสริมแรงในรูปแบบที่แตกต่างกัน เฉพาะในกลุ่มนักเรียนที่ใช้ตัวเร้าที่มีความซับซ้อนมาก

ข. มีความแตกต่างกันในการเรียนรู้ระหว่างกลุ่มที่ใช้ตัวเร้าที่มีระดับความซับซ้อนแตกต่างกัน เฉพาะในกลุ่มที่รู้ผลการตอบอย่างเดียว และกลุ่มที่ได้รับการลงโทษ

(๕) เมื่อนักเรียนปฏิบัติชิ้นงานเฉพาะ model ของ Bower และ Trabasso (1964) สามารถทำนายค่าสถิติดังต่อไปนี้ได้ คือ

ก. ความเบี่ยงเบนมาตรฐานของครั้งการปฏิบัติ (trial) ที่ตอบผิดเป็นครั้งสุดท้าย (SD_{TLE})

ข. ยอดรวมของจำนวนครั้งที่ตอบผิด (TE)

ค. ความเบี่ยงเบนมาตรฐานของยอดรวมของจำนวนครั้งที่ตอบผิด (SD_{TE})

ง. จำนวนครั้งที่ตอบผิดก่อนที่จะตอบถูกครั้งแรก

จ. $P(E_{n+1} | E_n)$ เมื่อ n คือลำดับที่ของครั้งการปฏิบัติ และ E คือคำตอบผิด

ฉ. $P(E_{n+1} | C_n)$ เมื่อ C คือคำตอบถูก

ช. Runs of errors (ทั้งหมด ตั้งแต่หนึ่งถึงห้า และมากกว่าห้า)

ผลปรากฏว่า model ของ Bower และ Trabasso ทำนายถูกเฉพาะ TE, $P(E_{n+1} | C_n)$ และ Runs of errors เท่านั้น สมมติฐานนี้จึงไม่ได้รับการสนับสนุน และผลการทำนายของ model นี้ในการทดลองการเรียนรู้มโนภาพทางคณิตศาสตร์ ด้อยกว่าผลการทำนายในการวิจัยอื่นที่ใช้มโนภาพที่ง่ายกว่า และไม่ใช่มโนภาพทางคณิตศาสตร์

(๖) ก. เมื่อนักเรียนปฏิบัติชิ้นงานเฉพาะ ก่อนการตอบผิดครั้งสุดท้าย ความน่าจะเป็นต่อไปนี้มีค่าคงที่ คือ $P(E)$, $P(E_{n+1} | E_n)$ และ $P(E_{n+1} | C_n)$ *

สมมติฐานนี้ไม่ได้รับการสนับสนุน เพราะความน่าจะเป็นที่กล่าวข้างต้นได้ลดค่าลงในทุก ๆ Vincent quartile ตามลำดับขั้น ซึ่งเป็นหลักฐานสนับสนุน linear model (แทน Markov model) และทฤษฎีการเรียนรู้แบบ incremental (แทนแบบ all - or-none)

ข. เมื่อนักเรียนปฏิบัติชิ้นงานเฉพาะ ก่อนการตอบผิดครั้งสุดท้าย

$P(E_{n+1} | E_n) = P(E_{n+1} | C_n)$

สมมติฐานนี้ไม่ได้รับการสนับสนุน ซึ่งผลของการทดสอบสมมติฐานทั้ง (๖) ก. และ ข. ชี้ให้เห็นถึงความไม่เหมาะสมของข้อตกลงพื้นฐานสองข้อของ Markov model และทฤษฎีการเรียนรู้แบบ all - or - none.

ค. เมื่อนักเรียนปฏิบัติชิ้นงานเฉพาะ parameter แห่งการเรียนรู้ (θ ของ model) มีค่าคงที่

สมมติฐานนี้ได้รับการสนับสนุนบ้าง กล่าวคือไม่ได้รับการปฏิเสธ

การใช้ model ของ Bower และ Trabasso (Markov chain model) ในการวิจัยครั้งนี้ โดยการทดสอบข้อตกลงพื้นฐานและความสามารถในการทำนายค่าสถิติของ model ได้ผลด้อยกว่าผลที่เคยพบในการวิจัยที่ผ่านมาแล้ว ผลการวิจัยนี้พบหลักฐานสนับสนุน linear model มากกว่า Markov model ซึ่งอาจเนื่องมาจาก probability learning ที่พบในการตอบของนักเรียนบางคน และ probability learning นั้นเป็นปรากฏการณ์ที่เกิดขึ้นตาม linear model.

REFERENCES

Azuma, H. and Cronbach, L. Cue response correlations in the attainment of a scalar concept. Amer. J. Psychol., 1966, 79, 38-49.

Bourne, L. and Haygood, R. The role of stimulus redundancy in the identification of concepts. J. exp. Psychol., 1959, 58, 232-238.

Bourne, L., Dodd, D., and Justesen, D. Concept identification: the effects of varying length and informational components of the intertrial interval. J. exp. Psychol., 1965, 69, 624-629.

Bourne, L., Guy, D., and Wadsworth, N. Verbal reinforcement combinations and the relative frequency of informative feedback in a card-sorting task. J. exp. Psychol., 1967, 73, 220-226.

Bourne, L., Ekstrand, B., and Dominowski, R. The Psychology of Thinking. Englewood Cliffs, New Jersey: Prentice-Hall, 1971.

Bower, G. and Theios, J. A learning model for discrete performance levels. In R. Atkinson (Ed.), Studies in Mathematical Psychology. Stanford: Stanford University Press, 1964.

Bower, G. and Trabasso, T. Concept identification. In R. Atkinson (Ed.), Studies in Mathematical Psychology. Stanford: Stanford University Press, 1964.

Bruner, J., Goodnow, J., and Austin, G. A Study of Thinking. New York: Wiley & Sons, 1956.

Bulgarella, R. and Archer, E. Concept identification of auditory stimuli as a function of amount of relevant and irrelevant information. J. exp. Psychol., 1962, 63, 254-257.

Buss, A. and Buss, E. The effect of verbal reinforcement combinations on concept learning. J. exp. Psychol., 1956, 52, 283-287.

Carroll, J. Language and Thought. Englewood Cliffs, New Jersey: Prentice-Hall, 1964.

Chumbley, J. A duoprocess theory of concept learning. J. math. Psychol., 1972, 9, 17-35.

Estes, W. An experimental study of punishment. Psychol. Monogr., 1944, 57, No. 23.

Falmagne, R. Construction of a hypothesis model for concept identification. J. math. Psychol., 1970, 7, 60-96.

Feller, W. An Introduction to Probability Theory and Its Applications, Vol. I (3rd ed.). New York: Wiley & Sons, 1968.

Greeno, J. and Steiner, T. Markovian processes with identifiable states: general considerations and applications to all-or-none learning. Psychometrika, 1964, 29, 309-393.

Haygood, R. and Stevenson, M. Effects of number of irrelevant dimensions in non-conjunctive concept learning. J. exp. Psychol., 1967, 74, 302-304.

Hilgard, E. and Bower, G. Theories of Learning (3rd ed.). New York: Appleton-Century-Crofts, 1966.

Hunt, E. Concept Learning: An Information Processing Problem. New York: Wiley & Sons, 1962.

Kintsch, W. Learning, Memory, and Conceptual Processes. New York: Wiley & Sons, 1970.

Kirk, R. Experimental Design: Procedures for the Behavioral Sciences. Belmont, Calif.: Brooks/Cole Publishing Co., 1968.

Laming, D. Mathematical Psychology. New York: Academic Press, 1973.

Lewis, D. Quantitative Methods in Psychology. New York: McGraw-Hill, 1960.

Marx, M. (Ed.) Learning: Interactions. New York: Macmillan, 1970.

Millward, R. and Wickens, T. Concept-identification models. In D. Krantz, et al (Eds.), Learning, Memory, and Thinking. San Francisco: W. H. Freeman & Co., 1974.

Restle, F. The selection of strategies in cue learning. Psychol. Rev., 1962, 69, 329-343.

Restle, F. Mathematical Models in Psychology: An Introduction. Middlesex: Penguin Books, 1971.

Schvaneveldt, R. Concept identification as a function of probability of positive instances and number of relevant dimensions. J. exp. Psychol., 1966, 72, 649-660.

Siegel, S. Decision-making behavior in a two-choice uncertain-outcome situation. J. exp. Psychol., 1959, 57, 37-42.

Thorndike, E. A proof of the law of effect. Science, 1933, 77, 173-175.

Thurstone, L. Influence of motion pictures on children's attitudes. J. soc. Psychol., 1931, 2, 291-305.

Trabasso, T. and Bower, G. Component learning in a four-category concept problem. J. math. Psychol., 1964, 1, 143-169.

Uhl, C. Learning of interval concepts I: Effects of differences in stimulus weights. J. exp. Psychol., 1963, 66, 264-273.

Uhl, C. Effects of multiple stimulus validity and criterion dispersion in learning of interval concepts. J. exp. Psychol., 1966, 72, 519-525.

Walker, C. and Bourne, L. Concept identification as a function of amounts of relevant and irrelevant information. Amer. J. Psychol., 1961, 74, 410-417.

Wallace, J. Concept dominance, type of feedback, and intensity of feedback as related to concept attainment. J. educ. Psychol., 1964, 55, 159-166.

A P P E N D I X   A

Since "learning" in the Bower and Trabasso model requires that S first makes an error, then transits to the learned state (with probability equal to θ), the probability that the last error occurs on trial k is

$$P(TLE = k) = (1-\pi)(\theta)\left[1-(1-\pi)(\theta)\right]^{k-1}$$ (see Millward, 1971)

and

$$E(TLE) = \sum_{k-1}^{\infty} (k)(1-\pi)(\theta)\left[1-(1-\pi)(\theta)\right]^{k-1}$$

$$= \frac{1}{(1-\pi)(\theta)}$$

Therefore, the best estimate of θ is

(1) $$\hat{\theta} = \frac{1}{(1-\pi)E(TLE)} = .01888$$ where: $E(TLE) = \bar{X}_{TLE}$

The value of θ can also be estimated from the total number of errors (TE). Since errors are recurrent events (see Feller, 1968, p. 303) and S can learn only be transiting from an error state (E), the probability that the total number of errors equals j is

$$P(TE = j) = \theta(1-\theta)^{j-1}$$ (see Millward, 1971)

and

$$E(TE) = \sum_{j-1}^{\infty} (j)(\theta)(1-\theta)^{j-1}$$

$$= \frac{1}{\theta}$$

ESTIMATION OF PARAMETERS AND PREDICTIONS OF STATISTICS

Bower and Trabasso (1964) Model of Concept Learning

For references purposes the transition probability matrix of the Bower and Trabasso (1964) model of concept learning is presented below.

		trial n+1 L	trial n+1 C	trial n+1 E	INITIAL PROBABILITIES
	L	1	0	0	0
trial n	C	0	π	$1-\pi$	π
	E	θ	$\pi(1-\theta)$	$(1-\pi)(1-\theta)$	$1-\pi$

Estimation of Paramter Values

The probabilities of making a correct response (π) and an error response ($1-\pi$) before learning are assumed by the model to be constant. Therefore, as noted by Restle (1971, pp. 136-137), their estimation is a simple matter and the best estimates of their values are the mean correct and error response proportions up to the trial of last error (TLE), or

$$\hat{\pi} = \frac{\text{total correct responses}}{\text{total responses up to TLE}} = .74112$$

$$(\widehat{1-\pi}) = \frac{\text{total error responses}}{\text{total responses up to TLE}} = .25888$$

where: $\wedge =$ estimate of

Therefore, another best estimate of Θ is

$$(2) \qquad \hat{\Theta} = \frac{1}{E(TE)} \qquad \text{where: } E(TE) = \bar{X}_{TE}$$

The value of Θ, then, can be estimated from either equation (1) or equation (2). It should be noted that

$$\hat{\Theta} = \frac{1}{(1-\pi)E(TLE)} = \frac{1}{E(TE)}$$

and

$$E(TE) = (1-\pi)E(TLE)$$

Thus, as noted in the Results section, if Θ is estimated from either E(TE) or E(TLE), then it is very likely that the prediction of TLE or TE will be accurate.

Predictions of Statistics

Equations for the predictions of statistics are as follows:

$$(3) \qquad SD_{TLE} = \frac{1-\Theta(1-\pi)}{[\Theta(1-\pi)]^2}$$ (see Millward, 1971)

$$(4) \qquad TE = E(TE) = \frac{1}{\Theta}$$ (see Millward, 1971)

$$(5) \qquad SD_{TE} = \frac{1-\Theta}{\Theta^2}$$ (see Millward, 1971)

(6) Number of errors before the first success $= \dfrac{(1-\pi)}{1-(1-\pi)(1-\Theta)}$

(see Kintsch, 1970, p. 74)

(7) $P(E_{n+1} \mid E_n) = (1-\pi)(1-\theta)$ (from the Bower and Trabasso model)

(8) $P(E_{n+1} \mid C_n) = (1-\pi)$ (from the Bower and Trabasso model)

Runs of errors:

Let R_i stand for the event "a sequence of i consecutive error responses preceded and terminated by one correct response", i.e a run of i errors. For example,

R_1 = C E C

R_2 = C E E C

R_3 = C E E E C

Since R_i is a non-periodic persistent event, its mean recurrence time (in this case time is equivalent to trials), μ_{R_i}, is

$$\mu_{R_i} = \frac{1}{p_{RR}^{(1)}}$$

where: $p_{RR}^{(1)}$ is the probability of remaining in state R or, alternatively, transiting from R to R in one step (see Feller, 1968, p. 443); and n (the number of trials), and, therefore, i, are finite

and the expected value of the number of runs of errors of length i, R_i, in n trials, N_{R_i}, is

$$E(N_{R_i}) \approx \frac{n}{\mu_{R_i}} = (n)(p_{R_i R_i}^{(1)})$$ (see Feller, 1968, p. 321)

As per the definition of R_i above, the recurrence of R_i is always preceded by a correct response. Therefore,

$$p^{(1)}_{R_iR_i} = (p_{C|C})(p_{E|C})(p_{E|E}) \ldots (p_{E|E})(p_{C|E})$$

$$\quad C \quad E_1 \quad E_2 \quad \ldots \quad E_i \quad C$$

and

$$E(N_{R_i}) \approx n\left[(p_{C|C})(p_{E|C})(p_{E|E}) \ldots (p_{E|E})(p_{C|E})\right]$$

$$\approx n\left[(\pi)(1-\pi)(1-\theta)(1-\pi) \ldots (1-\theta)(1-\pi)(1-\theta)(\pi)\right]$$

(9) $$E(N_{R_i}) \approx n\pi^2\left[(1-\theta)(1-\pi)\right]^i$$

Therefore,

(10) $$E(N_{R_{total}}) \approx \sum_{i=1}^{n-2} E(N_{R_i})$$

A P P E N D I X B

INSTRUCTIONS

Following is the text of the instructions given to Ss in the Reward group. The instructions given to the Feedback only and Punishment groups differ where marked with an asterisk (*).

Instructions[1] Today you will have the opportunity to solve a simple problem. If you receive a high enough score, you will win a prize of 20 or 30 baht.* The problem you will have today is about a new system for representing numbers. This is a very simple system, but you may have never seen it before. When you understand the principle involved, you will be able to solve the problem quickly. To begin, you will receive 50 free points.* Thereafter, each time you answer correctly, you will receive one more point.*

The rules for problem solution are as follows:

In front of you is a brown box. On the front of this box are tiny red lights arranged in rows and divided into the following groups: the first group consists of the top three rows, the second group consists of the fourth row, and the third group consists of the bottom row. Each time that these lights illuminate, from one to three of the lights in group 1 will illuminate. You should notice how many illuminate. At the same time, one or two lights in group 2 will

---

[1] These instructions are translated from the original Thai.

also illuminate. These lights in group 2 represent a number, which is not necessarily equal to the number of lights illuminated, but conforms to the principle determining the new number representation system. The numbers represented will be one, two, and three. For example, only one light may illuminate in group 2, but that light might represent the number two.

Your task will be to decide whether the lights in group 2 are equal or not equal to the lights in group 1. You needn't observe the lights in group 3.

The method for responding is as follows. When you think the number represented in group 2 equals the number of lights in group 1, you should press the switch labeled "equal". When you think that group 1 and group 2 are not equal, you should press the switch labeled "not equal". For example, three lights illuminate in group 1 and two lights illuminate in group 2, and you think that the lights in group 2 represent the number three. Therefore, you should press the switch labeled "equal". In pressing a switch, you should press it for only a short time. And you should press only one switch at a time. There is no time limit in this problem, so you can use as much time as you want before deciding which switch to press.

After you press a switch, all of the little lights will go out. If you have answered correctly, the blue light at the top of the box will illuminate for a short time. If you have answered incorrectly, the yellow light at the bottom of the box will illuminate. Besides that, if you have answered correctly, the counter at the top of the box will add one point.* You should remember that, if you answer correctly, your chance of winning a prize will increase, but if you answer incorrectly your chance of winning a prize will not be affected.

After either the blue or yellow light illuminates and goes out, new set of the little lights will illuminate, but it will not be the same as the previous set. You will have to decide again whether the lights in group 2 are equal or not equal to the lights in group 1 and then press a switch.

You will have to answer many, many times today. You should continue answering until the teacher in charge tells you to stop. After you have solved the problem, you should not tell your friends about it, because everyone will have a different problem to solve. If you tell your friends, they may misunderstand and be unable to solve their problems. If there is anything that you are not sure about, you should ask the teacher in charge.

The asterisked portions of the above instructions differed (in essence) for the other two groups as follows:

Punishment group The Ss in this group were instructed that they would begin with 150 points and have one point subtracted for each incorrect answer. There were no other differences.

Feedback only group In place of the first asterisked sentence Ss were instructed that they would receive five baht for participating, whether or not they solved the problem. All other asterisked sentences were deleted.