CRITICAL MAGNETIC FIELD OF ANISOTROPIC TWO-BAND MAGNETIC SUPERCONDUCTORS

A DISSERTATION
BY
ARPAPONG CHANGJAN

Presented in Partial Fulfillment of the Requirements for the
Doctor of Philosophy Degree in Physics
at Srinakharinwirot University
July 2011

CRITICAL MAGNETIC FIELD OF ANISOTROPIC
TWO-BAND MAGNETIC SUPERCONDUCTORS

A DISSERTATION
BY
ARPAPONG CHANGJAN

Presented in Partial Fulfillment of the Requirements for the
Doctor of Philosophy Degree in Physics
at Srinakharinwirot University
July 2011

CRITICAL MAGNETIC FIELD OF ANISOTROPIC
TWO-BAND MAGNETIC SUPERCONDUCTORS

AN ABSTRACT
BY
ARPAPONG CHANGJAN

Presented in Partial Fulfillment of the Requirements for the
Doctor of Philosophy Degree in Physics
at Srinakharinwirot University
July 2011

Arpapong Changjan. (2011). *Critical magnetic field of anisotropic two-band magnetic superconductors*. Dissertation, Ph.D. (Physics). Bangkok: Graduate School, Srinakharinwirot University. Advisor Committee: Assoc.Prof. Dr. Pongkaew Udomsamuthirun , Dr. Supitch Khemmani and Dr.Anusit Thongnum

In this dissertation, we study the upper critical field, the lower critical field and the critical magnetic field ratio of anisotropic magnetic superconductors by Ginzburg-Landau theory both in one-band and two-band models. Firstly, the 1st and 2nd Ginzburg–Landau equations of anisotropic magnetic superconductors are derived. Secondly, the upper critical field, the lower critical field and the critical magnetic field ratio are calculated. Thirdly, in one-band model, the effect of the Ginzburg–Landau parameter (κ_0), magnetic susceptibility (χ) and magnetic-to-anisotropic parameter ratio (θ) on the critical field ratio are studied. We find that the value of critical field ratio increases with increasing κ_0 and θ, and decreases with increasing χ. The highest and the lowest value of the critical magnetic field ratio are found respectively in the diamagnetic and the ferromagnetic superconductors. In two-band model, the very high value of zero-temperature upper critical field can be found in the negative differential susceptibility region. Finally, the temperature-dependent upper critical field is investigated and applied to Fe-based superconductors. The temperature-dependent is investigated in 4 cases; linear GL theory, non-linear GL theory, the Zhu et al. assumption and the Shanenko et al. assumption. The numerical calculations of the upper critical magnetic field are compared to the experimental data of Fe-based superconductors. The best fit provide values of zero temperatures upper critical magnetic fields in the range 42-71 Tesla with the non-linear GL theory case. Moreover, the upper critical magnetic fields show the linear behavior near critical temperature and non-linear behavior near zero temperature.

Keywords: Ginzburg-Landau theory, upper critical field, lower critical field, two-band superconductors, Fe-based superconductors

สนามแม่เหล็กวิกฤตของตัวนำยวดยิ่งชนิดเป็นแม่เหล็กแบบสองแถบพลังงานที่ขึ้นกับทิศทาง

บทคัดย่อ

ของ

อาภาพงศ์ ชั่งจันทร์

เสนอต่อบัณฑิตวิทยาลัย มหาวิทยาลัยศรีนครินทรวิโรฒ เพื่อเป็นส่วนหนึ่งของการศึกษา

ตามหลักสูตรปริญญาปรัชญาดุษฎีบัณฑิต สาขาวิชาฟิสิกส์

กรกฎาคม 2554

อาภาพงศ์ ชั่งจันทร์. (2554). *สนามแม่เหล็กวิกฤตของตัวนำยวดยิ่งชนิดเป็นแม่เหล็กแบบสองแถบพลังงานที่ขึ้นกับทิศทาง*. ปริญญานิพนธ์ ปร.ด. (ฟิสิกส์). กรุงเทพฯ: บัณฑิตวิทยาลัย มหาวิทยาลัยศรีนครินทรวิโรฒ. คณะกรรมการควบคุม: รองศาสตราจารย์ ดร.พงษ์แก้ว อุดมสมุทรหิรัญ, อาจารย์ ดร.สุพิชญ แขมมณี, อาจารย์ ดร.อนุศิษฏ์ ทองนำ.

งานวิจัยนี้มีความมุ่งหมายเพื่อศึกษาค่าสนามแม่เหล็กวิกฤตบน สนามแม่เหล็กวิกฤตล่าง และค่าอัตราส่วนสนามแม่เหล็กวิกฤตของตัวนำยวดยิ่งชนิดเป็นแม่เหล็กที่มีแถบพลังงานขึ้นกับทิศทางตามทฤษฎีกินซ์เบิร์ก-แลนดาวแบบแถบพลังงานเดียวและสองแถบพลังงาน การคำนวณเริ่มจากการหาสมการกินซ์เบิร์ก-แลนดาวที่หนึ่งและที่สอง จากนั้นนำสมการที่ได้มาคำนวณหาค่าสนามแม่เหล็กวิกฤตบน สนามแม่เหล็กวิกฤตล่าง และค่าอัตราส่วนสนามแม่เหล็กวิกฤต ในกรณีแถบพลังงานเดียวได้พิจารณาผลกระทบเนื่องจากค่ากินซ์เบิร์ก-แลนดาวพารามิเตอร์ (κ_0) ค่าสภาพความซาบซึมได้ (χ) และค่าอัตราส่วนความไม่สมมาตรของสนามแม่เหล็ก (θ) ที่มีผลต่อค่าอัตราส่วนสนามแม่เหล็กวิกฤต จากการศึกษาพบว่าค่าอัตราส่วนสนามแม่เหล็กวิกฤตจะมีค่าเพิ่มขึ้นเมื่อค่า κ_0 และ θ เพิ่มขึ้น และมีค่าลดลงเมื่อค่า χ เพิ่มขึ้น ค่าสูงสุดและต่ำสุดของอัตราส่วนสนามแม่เหล็กวิกฤตสามารถพบได้ในตัวนำยวดยิ่งชนิดเป็นสารแม่เหล็กไดอาและตัวนำยวดยิ่งชนิดเป็นสารแม่เหล็กเฟอร์โรตามลำดับ สำหรับกรณีสองแถบพลังงาน ค่าสนามแม่เหล็กวิกฤตบนที่อุณหภูมิศูนย์เคลวินซึ่งมีค่าสูงมากสามารถพบได้ในบริเวณที่ค่าอนุพันธ์ของค่าสภาพความซาบซึมได้มีค่าเป็นลบ จากนั้นนำผลการคำนวณค่าสนามแม่เหล็กวิกฤตบนที่ขึ้นกับอุณหภูมิมาอธิบายตัวนำยวดยิ่งชนิดมีเหล็กเป็นองค์ประกอบโดยใช้ลักษณะการขึ้นกับอุณหภูมิแบ่งเป็น 4 กรณี คือ กรณีทฤษฎีกินซ์เบิร์ก-แลนดาวแบบเชิงเส้น กรณีทฤษฎีกินซ์เบิร์ก-แลนดาวแบบไม่เป็นเชิงเส้น กรณีของซูและคณะ และกรณีของชาเนนโกและคณะ จากนั้นได้นำผลการคำนวณเชิงตัวเลขของค่าสนามแม่เหล็กวิกฤตบนมาเปรียบเทียบกับผลการทดลองของตัวนำยวดยิ่งชนิดมีเหล็กเป็นองค์ประกอบ พบว่าค่าสนามแม่เหล็กวิกฤตบนที่มีความสอดคล้องกับผลการทดลองมากที่สุด ให้ค่าสนามแม่เหล็กวิกฤตบนที่อุณหภูมิศูนย์เคลวินมีค่าในช่วง 42-71 เทสลา ภายใต้กรณีทฤษฎีกินซ์เบิร์ก-แลนดาวแบบไม่เป็นเชิงเส้นค่าสนามแม่เหล็กวิกฤตบนได้แสดงความเป็นเชิงเส้นในบริเวณใกล้ๆค่าอุณหภูมิวิกฤต และแสดงความไม่เป็นเชิงเส้นในบริเวณใกล้ๆค่าอุณหภูมิศูนย์เคลวิน

คำสำคัญ: ทฤษฎีกินซ์เบิร์ก-แลนดาว, สนามแม่เหล็กวิกฤตบน, สนามแม่เหล็กวิกฤตล่าง, ตัวนำยวดยิ่งแบบสองแถบพลังงาน, ตัวนำยวดยิ่งชนิดมีเหล็กเป็นองค์ประกอบ

The dissertation titled
“Critical magnetic field of anisotropic two-band magnetic superconductors”
by
Arpapong Changjan

has been approved by the Graduate School as partial fulfillment of the requirements for the Doctor of Philosophy Degree in Physics of Srinakharinwirot University.

....…………………………………… Dean of Graduate School
(Assoc. Prof. Dr. Somchai Santiwatanakul)

July /.........../ 2011

Dissertation Committee

.... Major-advisor
(Assoc. Prof. Dr.Pongkaew Udomsamuthirun)

.... Co-advisor
(Dr. Supitch Khemmani)

.... Co-advisor
(Dr. Anusit Thongnum)

Oral Defense Committee

.... Chair
(Assoc.Prof. Dr. Nason Phonpoke)

.... Committee
(Prof. Dr. Suthat Yoksan)

.... Committee
(Assoc.Prof. Dr.Pongkaew Udomsamuthirun)

.... Committee
(Dr. Supitch Khemmani)

.... Committee
(Dr. Anusit Thongnum)

Acknowledgements

This thesis could not successfully completed without the kindness of advisor's team. First and foremost to my major advisor, Assoc. Prof. Dr. Pongkaew Udomsamuthirun, who gave good advice and be guidance of this thesis since start until successful. My co-advisor, Dr. Supitch Khemmani and Dr. Anusit Thongnum for suggestions, checked and corrected the fault of this thesis, and the special thanks for Prof. Dr. Suthat Yoksan and Assoc. Prof. Dr. Nason Phonpoke for all of comments and good suggestions.

I would like to special thank for all of staff at Excellent center of Prasarnmit Physics research unit in Department of Physics, Faculty of Science, Srinakharinwirot University for their helpful in providing facilities and materials for my thesis. The financial support by the graduate school of Srinakharinwirot University, Phathumwan Institute of Technology and Thailand Center of Excellence in Physics (ThEP), Commission Higher on Education are acknowledged.

Finally, my graduation would not be achieved without best wish from my parents, my sisters and my wife who help me for everything and always gives me greatest love and willpower. And the last gratefully special thanks to my friends for their help and encouragements.

Arpapong Changjan

TABLE OF CONTENTS

TABLE OF CONTENTS (continue)

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1
INTRODUCTION

1 Historical background

In 1908, Onnes (Buckel, 1991:1) found the way to liquefy helium and to reach temperatures as low as 4K. In 1911, he discovered that below the temperature of the order of 4K, Hg looses the resistivity. This discovery leads to realize a new state of solid matter. When the critical field $H_c(T)$ (or above) was applied, the normal proper--ties of the metal were recovered. Furthermore, a critical current $J_c(T)$ (or above) could produce the same effect. He called the new phenomenon, superconductivity.

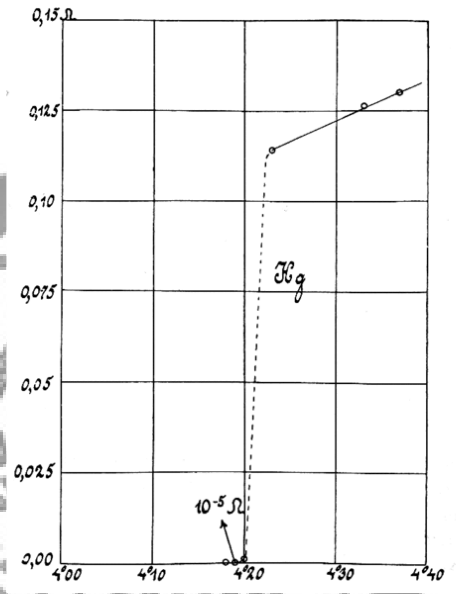


Figure 1 The resistance of mercury sample at the low temperatures. :Kittel.(1991:89).

Onnes recognized the importance of his discovery to the scientific community as well as its commercial potential. An electrical conductor with no resistance could carry current to any distance with no losses. In one of Onnes experiments he started a current flowing through a loop of lead wire cooled to 4 K. A year later the current was still flowing without significant current loss. Onnes found that the superconductor exhibited what he called persistent currents, electric currents that continued to flow without an electric potential driving them. Onnes had discovered superconductivity, and was awarded the Nobel Prize in 1913.

In Onnes's time superconductors were simple metals like mercury, lead, bismuth, etc. These elements become superconductors only at the very low temperatures of liquid helium. During the 75 years that followed, great strides were made in the understanding of how superconductors worked. Over that time, various alloys were found that were superconductors at somewhat higher temperatures. Unfortunately, none of these alloy superconductors worked at temperatures much more than 23 K. Thus, liquid helium remained the only convenient refrigerant that could be employed with these superconductors.

Then in 1986, Alex Muller and Georg Bednorz (Bednorz and Muller, 1986 :189) researchers at an IBM laboratory in Switzerland, discovered that ceramics from a class of materials called perovskites were superconductors at a temperature of about 35 K. This event sparked great excitement in the world of physics, and earned the Swiss scientists a Nobel Prize in 1987. As a result of this breakthrough, scientists began to examine the various perovskite materials very carefully. In February of 1987, a perovskite ceramic material was found to be a superconductor at 90 K. This was very significant because now it became possible to use liquid nitrogen as the refrigerant. Since these material superconduct at a significantly higher temperature, they are called high temperature superconductors.

There are several advantages in using liquid nitrogen instead of liquid helium. Firstly, the 77 K temperature of liquid nitrogen is far easier to attain and maintain than the chilly 4.2 K of liquid helium. Liquid nitrogen also has a much greater capacity to keep things colder than liquid helium does. Most importantly, nitrogen constitutes 78% of the air we breathe, and thus unlike liquid helium, for which there are only a few limited sources. Moreover, it is relatively much cheaper.

2 Basics properties

2.1 Heat capacity

At zero temperatures the heat capacity of superconductors is zero and when the temperature was increases the heat capacity is increasing in an exponential form. The heat capacity is decreased immediately at critical temperatures that the superconductor is in the normal state.

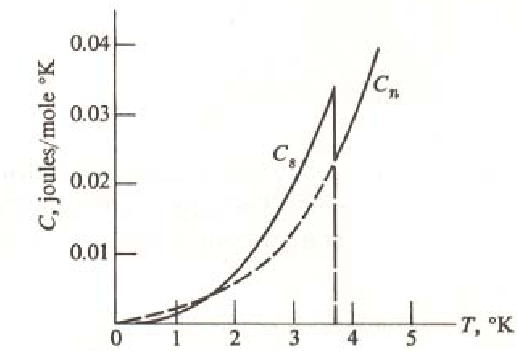


Figure 2 The relationship between the heat capacity and temperature are shown. Where C_s is the heat capacity in superconducting state and C_n is the heat capacity in normal state.:Omar.(1975:503).

The existence of discontinuity of the heat capacity at critical temperature called the specific heat jump, as shown in figure 2.

2.2 Isotope effect

The isotope effect is a phenomenon that critical temperature varies with isotopes mass. In mercury, critical temperature has changed from 4.158 K to 4.168 K while atomic mass changes from 199.5 to 203.4. From experiment of each isotopes series, we have a relationship

$$M^{\alpha}T_c = const. \qquad (1.1)$$

Where M is the isotope mass for the same element,
α is the isotope coefficient.

2.3 Superconductor in magnetic field

By 1933 Walther Meissner and R.Ochsenfeld(Omar.1975:500) discovered that superconductors are more properties than a perfect conductor, they also have an interesting magnetic property of excluding a magnetic field. A superconductor will not allow a magnetic field to penetrate its interior. This effect called the Meissner effect, causes a phenomenon that is a very popular demonstration of superconductivity. The Meissner effect will occur only if the magnetic field is relatively small. If the magnetic fields are higher than critical magnetic field, the metal loses its supercon -ductivity.

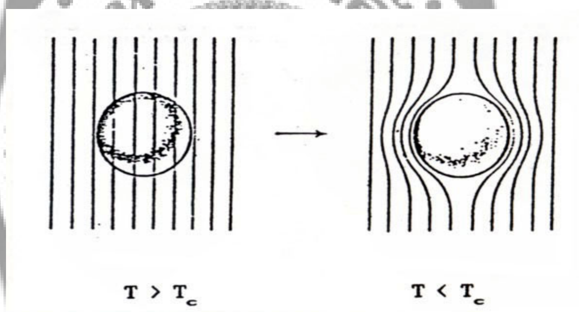


Figure 3 Meissner Effect: Kittel.(1998:344).

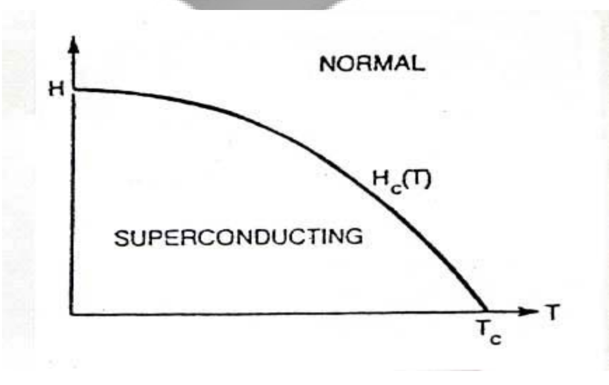
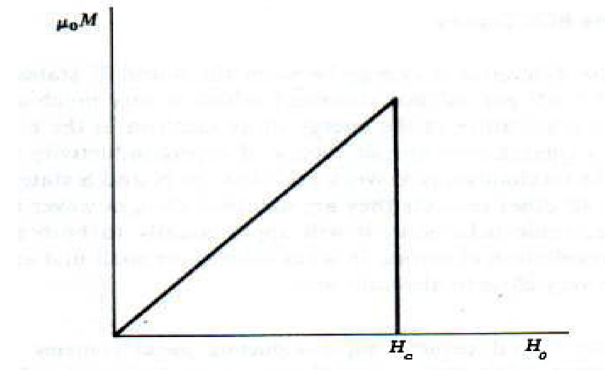


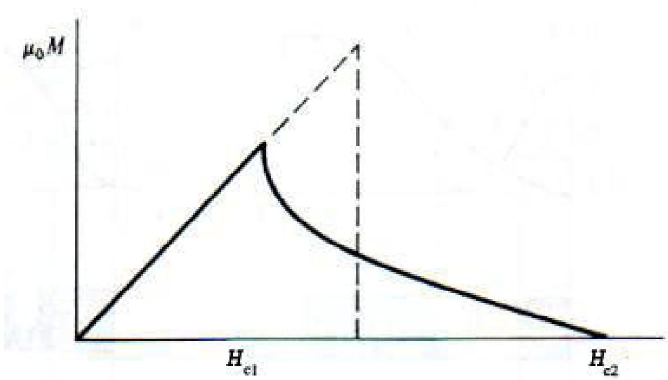
Figure 4 The relationship between critical magnetic field and temperature.:Ketterson ; & Song.(1998:25).

2.4 The classification of superconductors

Superconductors can be classified according to their magnetism properties as two types, called Type I superconductor and Type II superconductor.



(a)



(b)

Figure 5 (a) Type I superconductor and (b) Type II superconductor.:Ketterson; & Song.(1999:25).

When an external magnetic field is increasing until the critical magnetic field (H_c), Type I superconductors changes to a normal conductor immediately. For Type II superconductors, they are mostly metal or non metal transitions with high resistivity at normal phase. They have two critical field as lower critical field (H_{c1}) and upper critical field (H_{c2}). During lower and upper critical field $(H_{c1} < H < H_{c2})$, the sample is in a mixed condition with non-uniform flux penetration. H_{c1} and H_{c2} are called respectively the lower and upper critical fields.

2.5 Iron-based superconductors

In February 2008, Hideo Hosono and co-workers reported the discovery of 26 K superconductivity in fluorine-doped LaOFeAs (Kamihara et.al.,2008:3296-3297) marking the beginning of worldwide efforts to investigate this new family of superconductors. Historically, the typically antagonistic relationship between superconductivity and magnetism has led researchers to avoid using magnetic elements-ferromagnetic in particular - as potential building blocks of new superconducting materials. Because elemental iron is strongly magnetic, the discovery of Fe-based superconductors with high critical temperature was completely unexpected. This has opened a new avenue of research driven by the fact that our fundamental understanding of the origins of superconductivity needs significant improvement.

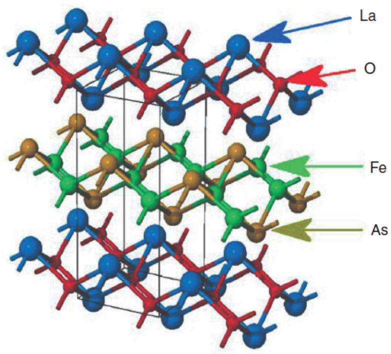


Figure 6 Crystal structure of the first iron-based superconductor.:Nakamura et.al.(2008:153).

2.6 Two-band superconductors

In solid-state physics, the electronic band structure (or simply band structure) of a solid describes those ranges of energy an electron is "forbidden" or "allowed". Band structure is derived from the diffraction of the quantum mechanical electron waves in a periodic crystal lattice with a specific crystal system and Bravais lattice. The band structure of a material determines several characteristics, in particular, the material's electronic and optical properties.

In superconductor, a band structure (energy band, energy gap or band gap) is associated with binding energy of a cooper pair. To obtain the superconducting condition, electrons near the Fermi surface form cooper pair. The energy requires to do this is the binding energy of the pairing, or phonon interaction. This effect, the binding energy manifests itself as an energy gap in the energy spectrum, centered at the Fermi level (see Fig.7).

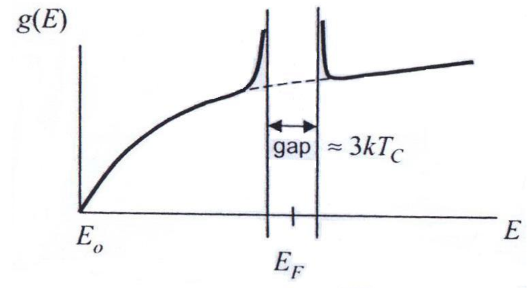


Figure 7 Energy gap in the energy spectrum.:Fischer-Cripps.(2008:191).

The existence of two kinds of superconducting gaps in MgB_2 has been suggested by several experiments (Bouquet, F. et al., 2002:257001, Tsuda, S. et al., 2001:177006, Chen, X. K. et al.,2001:157002). In theoretical point to view, the two-band Ginzburg-Landau theory can be applied to two-band superconductor. Firstly, we consider the free energy (Doh, H.et al. 1999:5350-5353, Udomsamuthirun et.al., 2006:62-66)

$$F_{sc} = \int d^3r\left(F_1 + F_{12} + F_2 + \frac{H^2}{8\pi}\right), \tag{1.2}$$

where $F_i = \dfrac{\hbar^2}{4m_i}\left|\left(\vec{\nabla} - \dfrac{2\pi i\vec{A}}{\phi_0}\right)\psi_i\right|^2 + \alpha_i(T)\psi_i^2 + \dfrac{\beta_i\psi_i^4}{2}$,

$$F_{12} = \varepsilon\left(\psi_1^*\psi_2 + c.c.\right) + \varepsilon_1\left\{\left(\vec{\nabla} + \frac{2\pi i\vec{A}}{\phi_0}\right)\psi_1^*\left(\vec{\nabla} - \frac{2\pi i\vec{A}}{\phi_0}\right)\psi_2 + c.c.\right\}.$$

Here F_i is the free energy density of separate band ($i = 1,2$), F_{12} is the free energy density of interaction between bands and H is external magnetic fields.

3 The purpose of the research

To study the lower critical magnetic field, the upper critical magnetic field and the critical magnetic field ratio of anisotropic two-band magnetic superconductors by Ginzburg-Landau theory.

4 The importance of the research

1. To describe the critical magnetic fields of magnetic superconductors by Ginzburg-Landau theory.

2. To predict the zero-temperature upper critical field of Fe-based superconductors.

5 Scope of the research

1. To calculate the lower critical magnetic field, the upper critical magnetic field and the critical magnetic field ratio of anisotropic two-band magnetic superconductors by Ginzburg-Landau theory.

2. To compare our numerical calculation to the experimental results.

.

CHAPTER 2
LITERATURE REVIEW

Current successful theories in describing the superconductivity state are the BCS theory (Bardeen, Cooper and Schrieffer. 1957: 1175 - 1204) and the Ginzburg-Landau theory (Abrikosov;Paulter.1991:1-47). Each theory is different from the concept however, they can describe superconductivity. There are advantages and disadvantages to each, i.e., BCS theory can explain superconductivity of the material that has low critical temperatures while the substance having a high critical temperature, the result is not consistent with the experiments. Ginzburg-Landau theory is able to describe superconductors with high critical temperature, but the result consistent within the vicinity of critical temperature only. In addition, the concept of starting either of these theories is different. The concept of the BCS theory starts from the micro-level systems (microscopic) while the Ginzburg-Landau theory begins to describe the system in the macro-level (macroscopic). These will be discussed in further detail.

1 BCS theory

The theory that success in describing the general properties of low temperature superconductors is the theory of the Bardeen, Cooper and Schrieffer (BCS theory). The key mechanism described by this theory that enables the normal conductor to superconductors are the capture pairs of electrons called the Cooper Pairs. When electrons move into the lattice, it interacts with the crystal lattice structure and then creates crystal lattice deforming as shown in figure 8. This interaction is an electron - lattice - electron interaction, which occurs when electrons move through the group of ions with positive charge on the lattice. Electrons will attract positive ions in the surrounding areas and deform the lattice around it. Density of the positive ions increases and then attracts other electrons. As a result, it seems that two electrons effectively attract each other. Pairing of electron made it loss some energies. It causes an energy gap at Fermi surface in superconductor. The superconductors that obey the BCS theory are called s-wave superconductors.

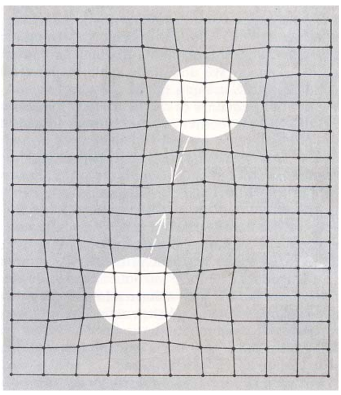


Figure 8 Deformation of the lattice when electrons move into the lattice and interact with the lattice.:Buckel.(1991:31).

2 Ginzburg – Landau theory (GL theory)

Within the vicinity of the critical temperature (T_c), the order parameter is assumed to be small and the free energy density can be expanded as

$$f_s = f_n + a|\psi|^2 + \frac{1}{2}b|\psi|^4 + \ldots \ , \tag{2.1}$$

where f_s is the free energy density for superconducting state,
f_n is the free energy density for normal state,
a and b are arbitrary functions.

The free energy is defined by $F_s = \int d\vec{r} f_s(\vec{r})$ and we can perform a variation calculation with respect to ψ^* as

$$\delta F_s = \int d\vec{r}\, \frac{\partial f_s}{\partial \psi^*} \delta\psi^* \ . \tag{2.2}$$

From eq.(2.1) $\delta F_s = 0$ implies $\frac{\partial f_s}{\partial \psi^*} = 0 = a\psi + b|\psi|^2\psi$, we have two solutions

$$\psi = 0 \qquad \text{for normal state,} \tag{2.3}$$

and

$$|\psi|^2 = -\frac{a}{b} \qquad \text{for superconducting state.} \tag{2.4}$$

We must have $|\psi|^2 > 0$ and $f_s < f_n$. So, we conclude that $a < 0$ and $b > 0$.
For the superconducting state solution, the free energy density is

$$f_s = f_n - \frac{a^2}{2b} . \tag{2.5}$$

In order to include spatial variations in ψ, GL theory included a term $\frac{1}{2m^*}\left|-i\hbar\vec{\nabla}\psi\right|^2$ (kinetic energy) in the free energy density. Here m^* is the effective mass of the superconducting particles (actually $m^* = 2m$ since these particles are cooper pairs).

For a superconductor in an external magnetic field, some flux may penetrate the sample. We can write the internal magnetic field $(\vec{h}(\vec{r}))$ in term of an electro-magnetic vector potential $\vec{A}(\vec{r})$ as $\vec{h} = \vec{\nabla}\times\vec{A}$. In general, for a particle of charge q in a magnetic field the Hamiltonian is $\hat{H} = \frac{1}{2m}\left|-i\hbar\vec{\nabla} + \frac{q}{c}\vec{A}\right|^2$. So, GL generalized the term including spatial variations in ψ to read

$$\frac{1}{2m^*}\left|\left(-i\hbar\vec{\nabla} + \frac{e^*}{c}\vec{A}\right)\psi\right|^2 . \tag{2.6}$$

Here, e^* is the effective charge.
The free energy density is now

$$f_s = f_n + a|\psi|^2 + \frac{1}{2}b|\psi|^4 + \frac{1}{2m^*}\left|\left(-i\hbar\vec{\nabla} + \frac{e^*}{c}\vec{A}\right)\psi\right|^2 + \frac{h^2}{8\pi} , \tag{2.7}$$

where the last term is the energy density of the magnetic field.

In an experimental situation, we have an external magnetic field $\left(\vec{H}\right)$ which can be controlled while the internal field $\left(\vec{h}\right)$ cannot be controlled. So it is convenient to make a Legendre transformation to a new free energy density given by

$$g_s = f_s - \frac{\vec{h}\cdot\vec{H}}{4\pi} \ , \tag{2.8}$$

where g_s is the free energy density of superconducting state in terms of an external magnetic field.

2.1 The 1st Ginzburg-Landau equation

From eq.(2.7) and eq.(2.8), we get the free energy density

$$g_s = f_n + a|\psi|^2 + \frac{1}{2}b|\psi|^4 + \psi^* \frac{1}{2m^*}\left(-i\hbar\vec{\nabla} + \frac{e^*}{c}\vec{A}\right)^2 \psi + \frac{h^2}{8\pi} - \frac{\vec{h}\cdot\vec{H}}{4\pi}. \tag{2.9}$$

The free energy is

$$G_s = \int d\vec{r}\, g_s \ . \tag{2.10}$$

The variation of G_s with respect to ψ^* is

$$\delta G_s = \int d\vec{r}\, \frac{\partial g_s}{\partial \psi^*}\delta\psi^* \ .$$

From $\dfrac{\partial g_s}{\partial \psi^*} = 0$ and the divergence theorem is

$$\int_v d\vec{r}\,\vec{\nabla}\cdot\vec{P} = \oint_s \vec{P}\cdot d\vec{s} \ ,$$

where s is the closed surface enclosing the volume v, we have

$$\begin{aligned}
&\frac{1}{2m^*}\int \left|\left(-i\hbar\vec{\nabla} + \frac{e^*}{c}\vec{A}\right)\psi\right|^2 d\vec{r} \\
&= \frac{1}{2m^*}\int d\vec{r}\left[-\hbar^2\psi^*\nabla^2\psi - i\hbar\frac{e^*}{c}\psi^*\vec{\nabla}\cdot\vec{A}\psi - i\hbar\frac{e^*}{c}\vec{A}\psi^*\cdot\vec{\nabla}\psi + \frac{e^{*2}}{c^2}A^2|\psi|^2\right] \\
&\quad + \frac{1}{2m^*}\oint_s\left[\hbar^2\psi^*\vec{\nabla}\psi + \frac{i\hbar e^*}{c}\psi^*\vec{A}\psi\right]\cdot d\vec{s} \ .
\end{aligned} \tag{2.11}$$

From eq.(2.11), we can be rewritten as

$$\begin{aligned}
&\frac{1}{2m^*}\int \left|\left(-i\hbar\vec{\nabla} + \frac{e^*}{c}\vec{A}\right)\psi\right|^2 d\vec{r} \\
&= \frac{1}{2m^*}\int_v d\vec{r}\left[\psi^*\left(-i\hbar\vec{\nabla} + \frac{e^*}{c}\vec{A}\right)^2\psi\right] + \frac{i\hbar}{2m^*}\oint_s\left[\psi^*(-i\hbar\vec{\nabla} + \frac{e^*}{c}\vec{A})\psi\right]\cdot d\vec{s} \ .
\end{aligned}$$

Using the surface condition as

$$\hat{n}\cdot\left(-i\hbar\vec{\nabla} + \frac{e^*}{c}\vec{A}\right)\psi = 0, \tag{2.12}$$

for all points on the surface of the superconductor with normal vector $\hat{n}$, we then get the 1st Ginzburg-Landau equation

$$\frac{1}{2m^*}\left(-i\hbar\vec{\nabla}+\frac{e^*}{c}\vec{A}\right)^2\psi+a\psi+b|\psi|^2\psi=0\,. \tag{2.13}$$

2.2 The 2nd Ginzburg-Landau equation

Consider the variation of eq.(2.9) with respect to $\vec{A}$. Since the free energy density is a function of $\vec{A}$ and its spatial derivatives $\left(\vec{h}=\vec{\nabla}\times\vec{A}\right)$, we need to consider

$$\int d\vec{r}\left[\frac{\partial g_s}{\partial A_i}\delta A_i+\sum_i\frac{\partial g_s}{\partial\left(\frac{\partial A_i}{\partial i}\right)}\frac{\partial\left(\delta A_i\right)}{\partial i}\right]=0\,\cdot \tag{2.14}$$

We have three equations since $i=1,2,3$ (or x,y,z). The second term, after integrated by parts, is

$$\sum_i\frac{\partial g_s}{\partial\left(\frac{\partial A_i}{\partial i}\right)}\cdot\delta A_i\Bigg|_s-\int d\vec{r}\sum_i\frac{\partial}{\partial i}\left[\frac{\partial g_s}{\partial\left(\frac{\partial A_i}{\partial i}\right)}\right]\delta A_i\,.$$

The part of g_s which depends on the derivatives of $\vec{A}$ is $\frac{h^2}{8\pi}-\frac{\vec{h}\cdot\vec{H}}{4\pi}$.

Clearly the surface term vanishes if $\vec{h}=\vec{H}$ on the surface. We now have

$$\frac{1}{2m^*}\left(i\hbar\frac{e^*}{c}\psi\vec{\nabla}\psi^*-i\hbar\frac{e^*}{c}\psi^*\vec{\nabla}\psi+2\frac{e^{*2}}{c^2}\vec{A}|\psi|^2\right)=-\frac{1}{4\pi}\vec{\nabla}\times\left(\vec{h}-\vec{H}\right)$$

$$-\frac{1}{4\pi}\vec{\nabla}\times\left(\vec{h}-\vec{H}\right)=\frac{e^*\hbar}{2m^*i}\left(\psi^*\nabla\psi-\psi\vec{\nabla}\psi^*\right)+\frac{e^{*2}}{m^*c}\vec{A}|\psi|^2\,. \tag{2.15}$$

We will consider the external magnetic field to be uniform that we have $\vec{\nabla}\times\vec{H}=0$.

From Maxwell's equation the current density is given by $\vec{j}=\frac{c}{4\pi}\vec{\nabla}\times\vec{h}$.

Now, we get

$$\vec{j}=-\frac{e^*\hbar}{2m^*i}\left(\psi^*\nabla\psi-\psi\vec{\nabla}\psi^*\right)-\frac{e^{*2}}{m^*c}\vec{A}|\psi|^2\,. \tag{2.16}$$

This is the 2nd Ginzburg-Landau equation.

3 Type I Superconductors

In general, high temperatures and high magnetic fields destroy the superconductivity. If the temperature is less than critical temperature and superconductors are called type I superconductors the following relationship between the internal and external fields is as figure9.

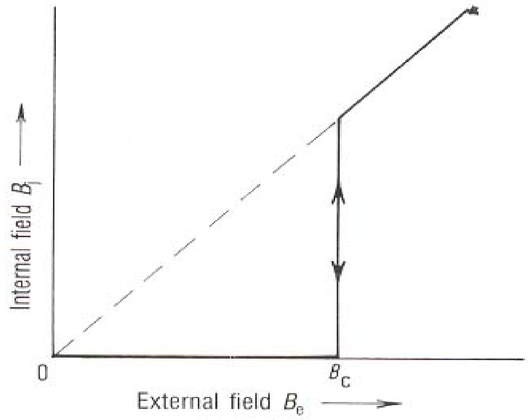


Figure 9 The relationship between the internal and external fields.:Buckel.(1991: 115).

For external fields higher than critical fields, we have the normal state which is assumed to be nonmagnetic $(\vec{B}_i = \vec{B}_e)$. For the external fields less than critical fields, we have the Meissner effect $(\vec{B}_i = 0)$. To determine the critical magnetic field from GL theory, we write order parameter in the form of

$$\psi = |\psi| e^{i\phi}, \tag{2.17}$$

where ϕ is the phase of order parameter. We get

$$\psi^* \vec{\nabla}\psi - \psi\vec{\nabla}\psi^* = 2i|\psi|^2 \vec{\nabla}\phi . \tag{2.18}$$

Substituting eq.(2.18) into eq.(2.16), the 2nd Ginzburg-Landau equation is

$$\vec{j} = -\frac{e^{*2}}{m^* c}\left(\frac{\hbar c}{e^*}\vec{\nabla}\phi + \vec{A}\right)|\psi|^2 . \tag{2.19}$$

For a uniform external field $\vec{H}$, we have $\vec{h} = \vec{H}$ that give

$$\left(\frac{\hbar c}{e^*}\vec{\nabla}\phi + \vec{A}\right)|\psi|^2 = 0.$$

Since $\vec{\nabla} \times \vec{\nabla}\phi = 0$ and $\vec{\nabla} \times \vec{A} = \vec{h} = \vec{H} \neq 0$, we must have $\psi = 0$ (normal state).

On the other hand, if $\vec{h} = 0$ then we can have $\psi \neq 0$ (superconducting state).

Thus, the free energy density can be written as

$$g_s(H) = g_n(H) + a|\psi|^2 + \frac{1}{2}b|\psi|^4 + \frac{1}{2m^*}\left|\left(-i\hbar\vec{\nabla} + \frac{e^*}{c}\vec{A}\right)\psi\right|^2 , \tag{2.20}$$

where $g_n(H)$ is the free energy density for the normal phase which

$$g_n(H) = f_n + \frac{h^2}{8\pi} - \frac{\vec{h}\cdot\vec{H}}{4\pi} .$$

In the normal phase $(\vec{h} = \vec{H})$, $g_n(H) = g_n(0) - \frac{H^2}{8\pi}$ and in the superconducting phase $(\vec{h} = 0)$, $g_s(H) = g_s(0)$. Gibbs free energy is constant in the superconducting state that give $g_s(0) = g_s(H_c) = g_n(H_c) = g_n(0) - \frac{H_c^2}{8\pi}$. And for an infinite sample with a uniform GL order parameter; $g_s(0) = g_n(0) - \frac{a^2}{2b}$, we must have $g_s(H_c) = g_n(H_c)$ for $H = H_c$. Then, we get

$$\frac{H_c^2}{8\pi} = -\frac{a^2}{2b},$$

or

$$H_c^2 = -\frac{4\pi a^2}{b}. \tag{2.21}$$

4 Type II Superconductors

For Type II Superconductors, the degree of flux penetration is according to the following diagram in figure 10.

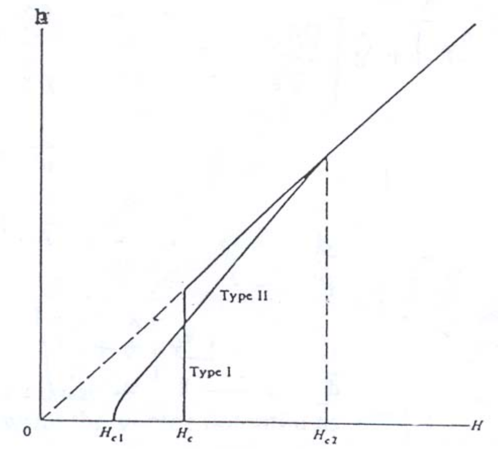


Figure 10 Compare relationship between internal and external field of type I and type II Superconductor.: Abrikosov; Paulter.(1991:18)

There are two critical magnetic field in type II superconductors, H_{c1} and H_{c2}. We show the detail calculating process of these critical fields from GL theory in the following topics.

For $H > H_{c2}$, the sample is normal $(\vec{h} = \vec{H})$, whilst for $H < H_{c1}$, the samples are complete Meissner effect $(\vec{h} = 0)$. In the intermediate state $H_{c1} < H < H_{c2}$, the sample is in a mixed condition with non uniform flux penetration. H_{c1} and H_{c2} are called the lower and upper critical fields, respectively.

4.1 Upper critical field

By considering the constant external fields in z-axis, we get

$$\vec{\nabla}\cdot\vec{A}=0,$$

$$\vec{\nabla}\times\vec{A}=H\hat{z}.$$

For external fields just below H_{C2}, the internal fields will be slightly different from external fields. We can write the vector potential as

$$\vec{A}=\vec{A}_0+\delta\vec{A},$$

so that

$$\vec{h}=\vec{H}+\vec{\nabla}\times\delta\vec{A}.$$

Since $\vec{H}$ is uniform,

$$\vec{\nabla}\times\vec{h}=\vec{\nabla}\times\vec{\nabla}\times\delta\vec{A}.$$

By using the 2nd Ginzburg-Landau equation, we get

$$\vec{\nabla}\times\vec{\nabla}\times\delta\vec{A}=-\frac{4\pi e^{*2}}{m^*c^2}\left(\frac{\hbar c}{e^*}\vec{\nabla}\phi+\vec{A}\right)|\psi|^2. \tag{2.22}$$

In the 1st Ginzburg-Landau equation with $H<H_{c2}$, the GL order parameter will be very small. If we neglect all the term of order $|\psi|^3$ which is very small, then we can write

$$\frac{1}{2m^*}\left(-i\hbar\vec{\nabla}+\frac{e^*}{c}\vec{A}\right)^2\psi+a\psi=0. \tag{2.23}$$

This is the linearized GL equation. We can compare this equation to the Schrodinger equation with a particle of mass m^* and charge $-e^*$ moving in a magnetic field. If we choose $\vec{H}=H\hat{z}$, then the eigen values of Hamiltonian of $\frac{1}{2m^*}\left(-i\hbar\vec{\nabla}+\frac{e^{*2}}{c}\vec{A}_0\right)^2$ is in the form of

$$E=(n+\frac{1}{2})\hbar\omega, \tag{2.24}$$

where $\omega=\frac{e^*H}{m^*c}$, is the energy levels corresponding to motion perpendicular to the magnetic field. Thus, we have

$$|a|=\left(n+\frac{1}{2}\right)\frac{\hbar e^*H}{m^*c},$$

and

$$H=\frac{2m^*c}{(2n+1)\hbar e^*}|a|.$$

In order to determine H_{c2}, we must take the largest possible value of H. This means that we must choose $n=0$ which give the upper critical magnetic field as

$$H_{c2}=\frac{2m^*c}{\hbar e^*}|a|. \tag{2.25}$$

4.2 Lower critical field

Consider H just above H_{c1}, the flux penetration were uniform then $\psi = 0$ everywhere. We consider a non uniform local flux penetration and keeping as much as possible that we choose a flux line

$$\vec{h} = h(\vec{r})\hat{z}.$$

The function $h(\vec{r})$ has its maximum value at $r = 0$ and tends to zero for large r, as shown in figure11.

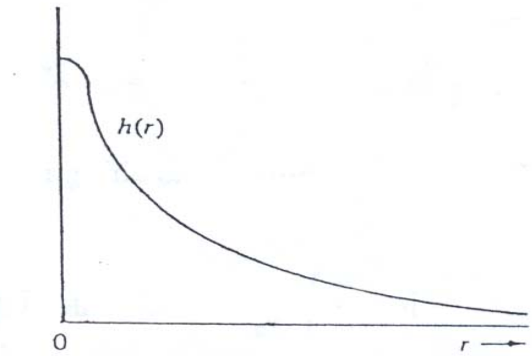


Figure 11 The relationship between $h(\vec{r})$ and r .: Abrikosov; Paulter.(1991:20)

A suitable choice of vector potential is

$$\vec{A} = A(r)\hat{\theta}.$$

The internal field is

$$\vec{h} = \vec{\nabla} \times \vec{A} = \hat{z}\frac{1}{r}\frac{d}{dr}(rA(r)),$$

and

$$h(r) = \frac{1}{r}\frac{d}{dr}(rA(r)).$$

Let

$$A(r) = \sum_{n=1}^{\infty} a_n r^n .$$

Then $h(r) = \sum_{n=1}^{\infty} a_n (n+1) r^{n-1}$ is finite as required.

So $$A(r) = \frac{1}{r}\int_0^r r'h(r')dr .$$

For small value of r $$A(r) = \frac{1}{2}h(0)r .$$

Far from the flux line $(r \to \infty)$ we have $\vec{h} = 0, |\psi| = \left(\frac{a}{b}\right)^{\frac{1}{2}}$ and $\vec{A} = -\frac{\hbar c}{e^*}\vec{\nabla}\phi$.

Integrating this expression around the circle c with radius $r \to \infty$, we have

$$\oint_c \vec{A} \cdot d\vec{l} = -\frac{\hbar c}{e^*} \oint_c \vec{\nabla}\phi \cdot d\vec{l} .$$

The left hand side is transformed via stokes theorem

$$\oint_c \vec{A} \cdot d\vec{l} = \int_s \vec{\nabla} \times \vec{A} \cdot d\vec{s} = \int_s \vec{h} \cdot d\vec{s} ,$$

where s is a surface bounded by c. This quantity is the total flux (Φ) passing through the surface as

$$\Phi = \int_s \vec{h} \cdot d\vec{s} = -\frac{\hbar c}{e^*} \oint_c \vec{\nabla}\phi \cdot d\vec{l} .$$

The GL order parameter must be single valued. This means that the phase (ϕ) must change by integral multiples of 2π in making a complete circuit, $\oint_c \vec{\nabla}\phi \cdot d\vec{l} = 2\pi n$.

Note that the integral cannot be transformed by the stokes's theorem to give zero in every case.

Now $$A(r)\hat{\theta} = -\frac{\hbar c}{e^*}\vec{\nabla}\phi .$$

So, we must choose ϕ to depend only on θ. The most physically sensible choice is $\phi = -\theta$.

Then $$\oint_c \vec{\nabla}\phi \cdot d\vec{l} = -\int_0^{2\pi} d\theta = -2\pi ,$$

and $$\Phi = 2\pi \frac{\hbar c}{e^*} . \tag{2.26}$$

Here Φ is the minimum flux that can pass through the superconductor and is quantized.
The value of $A(r)$ for $r \to \infty$ is

$$A(r) = \frac{\hbar c}{e^*}\frac{1}{r} \to 0 .$$

The supercurrent is given by

$$\vec{j} = \frac{c}{4\pi}\vec{\nabla} \times \vec{h} = -\frac{c}{4\pi}\frac{dh(r)}{dr}\hat{\theta} = \frac{e^*}{m^* c}\left(\frac{\hbar c}{e^*}\frac{1}{r} - \vec{A}\right)|\psi|^2 \hat{\theta} , \tag{2.27}$$

which $\vec{j}$ is in the direction of $\hat{\theta}$. This means that the flux line corresponds to a vortex of supercurrent.
We set the following expression of the order parameter as

$$\psi(r) = \left(\frac{|a|}{b}\right)^{\frac{1}{2}} f(r) e^{-i\theta} .$$

For $r \to \infty$, we get $f(r) \to 1$.
From the 1st Ginzburg-Landau equation, if $\vec{A} = A(r)\hat{\theta}$ and $\vec{\nabla} \cdot \vec{A} = 0$, then

$$\frac{\hbar^2}{2m^*}\left(-\nabla^2 - \frac{2ie^*}{\hbar c}\frac{A(r)}{r}\frac{\partial}{\partial\theta} + \frac{e^{*2}}{\hbar^2 c^2}A(r)^2\right)\psi + a\psi + b|\psi|^2\psi = 0 . \tag{2.28}$$

The magnetic flux is symmetry in plane, then

$$\nabla^2\psi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2}. \quad (2.29)$$

Substituting eq.(2.29) into eq.(2.28), we get

$$-\xi^2\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{df(r)}{dr}\right)-\left(\frac{1}{r}-\frac{e^*}{\hbar c}A(r)\right)^2 f(r)\right]-f(r)+f(r)^3=0, \quad (2.30)$$

where $\xi^2=\frac{\hbar^2}{2m^*|a|}$ is the coherence length.

The equation for the supercurrent is

$$\vec{j}=-\frac{c}{4\pi}\frac{dh(r)}{dr}\hat{\theta}=\frac{e^{*2}}{m^*c}\left(\frac{\hbar c}{e^*}\frac{1}{r}-A(r)\right)\frac{|a|}{b}f(r)^2\hat{\theta},$$

where $\lambda=\left(\frac{m^*c^2b}{4\pi e^{*2}|a|}\right)^{\frac{1}{2}}$ is the penetration depth,

and

$$\frac{dh(r)}{dr}=-\frac{1}{\lambda^2}\left(\frac{\hbar c}{e^*}\frac{1}{r}-A(r)\right)f(r)^2. \quad (2.31)$$

According to the complicated form of eq.(2.30) and eq.(2.31). The equations for $f(r)$ and $h(r)$ can be generally solved by numerical methods. For small r and $f(r)\cong r$, the solution for $f(r)$ and $h(r)$ generally resemble in figure12.

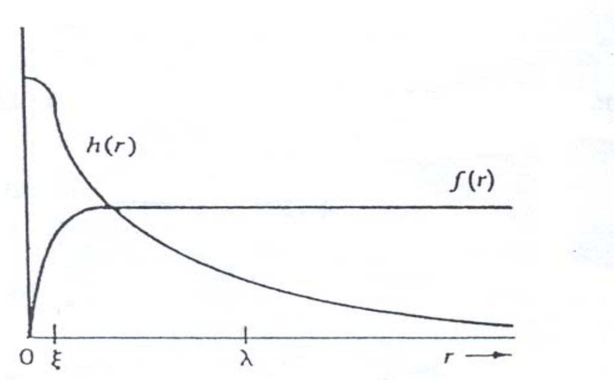


Figure 12 The solution for $f(r)$ and $h(r)$: Abrikosov; Paulter.(1991:22).

The Ginzburg – Landau parameter (κ) is defined as

$$\kappa=\frac{\lambda}{\xi}=\frac{m^*c}{\hbar e^*}\left(\frac{b}{2\pi}\right)^{\frac{1}{2}}. \quad (2.32)$$

Considering the case of $\kappa>>1$, we have $\lambda>>\xi$ and the function $f(r)$ is essentially equal to unity except for a core region of radius ξ.

From eq. (2.31) and $rh(r) = \frac{d}{dr}(rA(r))$, we get

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dh(r)}{dr}\right) = \frac{1}{\lambda^2}h(r)f^2(r) - \frac{1}{\lambda^2}\left(\frac{\hbar c}{e^*}\frac{1}{r} - A(r)\right)\frac{df^2(r)}{dr}.$$

Outside of the core region $f(r) \cong 1$, we get

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dh(r)}{dr}\right) = \frac{1}{\lambda^2}h(r). \tag{2.33}$$

The solutions of this equation are dependent on the modified Bessel function and can be approximated by

$$I_0\left(\frac{r}{\lambda}\right) \quad \text{and} \quad K_0\left(\frac{r}{\lambda}\right).$$

For the large value of r, the modified Bessel function is

$$I_0\left(\frac{r}{\lambda}\right) \propto e^{\frac{r}{\lambda}}\left(\frac{\lambda}{r}\right)^{\frac{1}{2}}$$

and

$$K_0\left(\frac{r}{\lambda}\right) \propto e^{-\frac{r}{\lambda}}\left(\frac{\lambda}{r}\right)^{\frac{1}{2}}.$$

So we must choose $h(r) \propto K_0\left(\frac{r}{\lambda}\right)$ that

$$h(r) = c'K_0\left(\frac{r}{\lambda}\right),$$

where c' is a constant normalization factor.

From eq. (2.26) the total flux passing through the superconductor is

$$\Phi = 2\pi\frac{\hbar c}{e^*} = \int_s \vec{h}\cdot d\vec{s} = 2\pi\int_0^\infty rdrh(r).$$

If $h(r) = c'K_0\left(\frac{r}{\lambda}\right)$ is taken to be true for all r, we can neglect the effect of the vortex core. Let

$$\frac{\hbar c}{e^*} = \int_0^\infty rdrc'K_0\left(\frac{r}{\lambda}\right) = c'\lambda^2\int_0^\infty xdxK_0(x) = c'\lambda^2$$

with $c' = \frac{\hbar c}{e^*}\frac{1}{\lambda^2}$.

We can find

$$h(r) = \frac{\hbar c}{e^*}\frac{1}{\lambda^2}K_0\left(\frac{r}{\lambda}\right). \tag{2.34}$$

If $\psi = \left(\frac{|a|}{b}\right)^{\frac{1}{2}}e^{-i\theta}$ and $\vec{A} = A(r)\hat{\theta}$, the free energy density is

$$g_s = f_n + \frac{h^2}{8\pi} - \frac{\vec{h}\cdot\vec{H}}{4\pi} - \frac{|a|^2}{b} + \frac{1}{2}\frac{|a|^2}{b} + \frac{\hbar^2}{2m^*}\frac{|a|}{b}\left(\frac{1}{r} - \frac{e^*}{\hbar c}A(r)\right)^2 .$$

Setting $f(r)=1$, eq.(2.31) can be written as

$$\frac{\hbar c}{e^*}\frac{1}{r} - A(r) = -\lambda^2 \frac{dh(r)}{dr}.$$

For $\vec{H} = H\hat{z}$, we get

$$g_s(H) = f_n - \frac{1}{2}\frac{|a|^2}{b} + \frac{1}{8\pi}\left(h(r)^2 + \lambda^2\left(\frac{dh(r)}{dr}\right)^2\right) - \frac{h(r)H}{4\pi}.$$

The integration over all volume are recruited for free energy and required

$$\varepsilon_1 = \frac{1}{4}\int_0^\infty rdr\left(h(r)^2 + \lambda^2\left(\frac{dh(r)}{dr}\right)^2\right) = \frac{H_{c1}}{2}\int_0^\infty rdrh(r) = \frac{H_{c1}}{2}\frac{\hbar c}{e^*}.$$

The lower critical field is given by

$$H_{c1} = 2\frac{e^*}{\hbar c}\varepsilon_1 . \tag{2.35}$$

We find

$$\varepsilon_1 = \frac{1}{4}\int_0^\infty rdr\left(h(r)^2 + \lambda^2\left(\frac{dh(r)}{dr}\right)^2\right)$$

and $$\int_0^\infty rdr\left(\frac{dh(r)}{dr}\right)^2 = rh(r)\frac{dh(r)}{dr}\bigg|_0^\infty - \int_0^\infty drh(r)\frac{d}{dr}\left(r\frac{dh(r)}{dr}\right).$$

From eq.(2.33), $\displaystyle\int_0^\infty rdr\left(\frac{dh(r)}{dr}\right)^2 = rh(r)\frac{dh(r)}{dr}\bigg|_0^\infty - \frac{1}{\lambda^2}\int_0^\infty rdrh(r)^2$.

We get

$$\varepsilon_1 = \frac{\lambda^2}{4}rh(r)\frac{dh(r)}{dr}\bigg|_0^\infty . \tag{2.36}$$

By considering $r \to \infty$ and $h(r) \propto e^{-\frac{r}{\lambda}}\left(\frac{\lambda}{r}\right)^{\frac{1}{2}}$, we get

$$\varepsilon_1 = -\frac{\lambda^2}{4}\lim_{r\to\infty} rh(r)\frac{dh(r)}{dr}.$$

For small r, $K_0\left(\frac{r}{\lambda}\right) = -\ln\frac{r}{2\lambda} - \gamma$ where $\gamma \approx 0.5772$, we have $\dfrac{dh(r)}{dr} = -\dfrac{\hbar c}{e^*}\dfrac{1}{\lambda^2}\dfrac{1}{r}$

or $r\dfrac{dh(r)}{dr} = -\dfrac{\hbar c}{e^*}\dfrac{1}{\lambda^2}$ that is

$$\varepsilon_1 = \frac{1}{4}\frac{\hbar c}{e^*}\lim_{r\to 0} h(r).$$

Since $h(r) \propto K_0\left(\frac{r}{\lambda}\right)$ diverges for $r \to 0$, we take $\varepsilon_1 = \frac{1}{4}\frac{\hbar c}{e^*}h(\xi)$ that is we cut off the divergence at the core radius.

We have

$$h(\xi) = \frac{1}{\lambda^2}\frac{\hbar c}{e^*}K_0\left(\frac{\xi}{\lambda}\right) \approx \frac{\hbar c}{e^*}\frac{1}{\lambda^2}\ln\kappa .$$

Since $\kappa >> 1$, we get

$$\varepsilon_1 \approx \frac{1}{4}\frac{\hbar^2 c^2}{e^{*2}}\frac{1}{\lambda^2}\ln\kappa .$$

Thus, the lower critical field is given by

$$H_{c1} \approx \frac{1}{2}\frac{\hbar c}{e^*}\frac{1}{\lambda^2}\ln\kappa . \tag{2.37}$$

5 Relevant research of two-band anisotropic magnetic superconductors

In June1998, Hampshire (1998: 1-11) proposed a free energy model for magnetic superconductors by including the spatial variation and the nonlinear magnetic response of the magnetic ions in-field as

$$f_s(H,T) = f_N + \alpha|\psi|^2 + \frac{1}{2}\beta|\psi|^4 + \frac{1}{2m}\left|\left(-i\hbar\vec{\nabla} - 2e\vec{A}\right)\psi\right|^2 + \int H_s dB , \tag{2.38}$$

where

$$H_s = \frac{B}{\mu_0} - M_{ions} , \; M = M_{sc} + M_{ions} \text{ and } M_{ions} = \chi H_s .$$

For small change in B-field, a series in B was introduced

$$\gamma_0 + \gamma_1 B + \gamma_2 \frac{B^2}{2\mu_0} = \int \left(B - \mu_0 M_{ions}\right)\frac{dB}{\mu_0} - \left(B - \mu_0 M\right)M_{sc} - \left(B - \mu_0 M\right)M_{ions} .$$

The Gibbs's free energy of magnetic superconductor became

$$g_s(B,T) = f_N + \alpha|\psi|^2 + \frac{1}{2}\beta|\psi|^4 + \frac{1}{2m}\left|\left(-i\hbar\vec{\nabla} - 2e\vec{A}\right)\psi\right|^2 + \gamma_0 + \gamma_1 + \gamma_2\frac{B^2}{2\mu_0} .$$

In December 1999, Doh et al.(1999: 5305 - 5353) proposed a free energy model for multi-band superconductors for a group of compounds $Ho_{1-x}Dy_xNi_2B_2C$ and showed irregularities of upper critical field by Ginzburg-Landau Theory. The results were also consistent with experiments.

In January 2001, Akimisu and Nagamatsu (2001: 63) reported that magnesium diboride could show the properties as superconductors below 40 K. After providing this information, there were a strong interest in the scientific community because the magnesium diboride is a s-wave two-band superconductors with high critical temperature.

In December 2001, Hass & Maki (2001:020502) proposed a model showing anisotropic superconductor with ellipsoid shaped of Fermi surface as

$$\Delta(k) = \Delta(0)\left(\frac{1 + a'z^2}{1 + a'}\right) \tag{2.39}$$

where $z = \cos\theta$,

θ is polar angle,

a' is anisotropic parameter between c-axis and ab-plane.

They used this model to describe the thermodynamic properties of anisotropic superconductors.

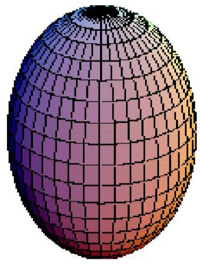


Figure 13 The energy band for anisotropic s-wave superconductors as Hass & Maki model in momentum space.:Hass; & Maki(2001: 020502)

In January 2002, Askerzade, Gencer and Güçlü (Askerzade, Gencer and Güçlü.2002: L13-L16) studied the temperature dependence of upper critical magnetic field in magnesium diboride superconductors by two band Ginzburg- Landau theory. The equation of the upper critical magnetic field that could explain the experimental results in vicinity of the critical temperature is shown in figure14.

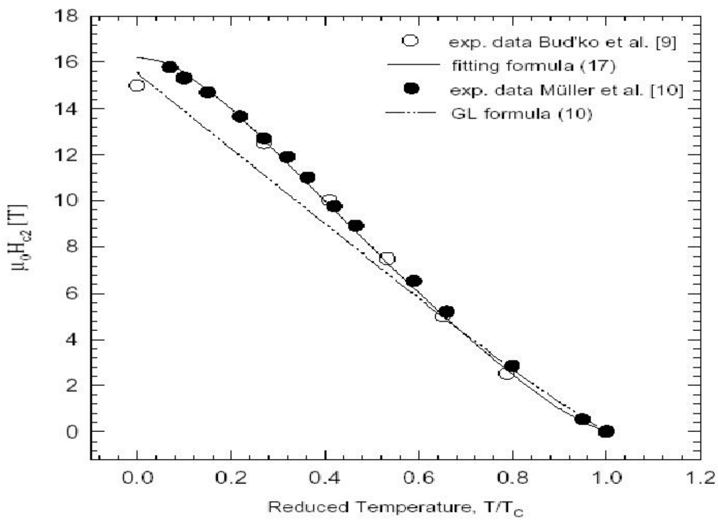


Figure 14 The upper critical magnetic field of two band superconductor that comparison between experimental and theoretical results are shown.
: Askerzade, Gencer and Güçlü(2002: L13-L16)

They consider the two-band free energy density model that proposed by Doh et al.(1999: 5350 - 5353).

$$F_{sc}[\psi_1,\psi_2]=\int d^3r\left(f_1+f_{12}+f_2+\frac{H^2}{8\pi}\right) \qquad (2.40)$$

where

$$f_i = \frac{\hbar^2}{4m_i}\left|\left(\nabla - \frac{2\pi i\vec{A}}{\phi_0}\right)\psi_i\right|^2 + \alpha_i(T)\psi_i^2 + \frac{\beta_i\psi_i^4}{2}, \tag{2.41}$$

$$f_{12} = \varepsilon\left(\psi_1^*\psi_2 + \psi_1\psi_2^*\right) + \varepsilon_1\left\{\left(\nabla + \frac{2\pi i\vec{A}}{\phi_0}\right)\psi_1^*\left(\nabla - \frac{2\pi i\vec{A}}{\phi_0}\right)\psi_2 + \left(\vec{\nabla} - \frac{2\pi\vec{A}}{\phi_0}\right)\psi_1\left(\vec{\nabla} + \frac{2\pi i\vec{A}}{\phi_0}\right)\psi_2^*\right\}. \tag{2.42}$$

Here, f_i is the free energy density of the separate bands, f_{12} is the interaction term between bands, m_i denotes the effective mass of carriers, ψ_i is the order parameter and $|\psi_i|^2$ is proportional to the density of carriers, the coefficient α_i and β_i are the temperature-dependence and temperature-independence parameter. The coefficient ε and ε_1 are the interband mixing of two order parameters and their gradients. The first term in eq.(2.41) accounts for the kinetic energy of the carriers and the lasted term in eq.(2.40) accounts for the energy stored in the local magnetic fields.
By taking the variation of free energy with respect to ψ_i^*

$$\frac{\delta F}{\delta\psi_1^*} = 0, \frac{\delta F}{\delta\psi_2^*} = 0,$$

we get

$$-\frac{\hbar^2}{4m_1}\left(\frac{d^2}{dx^2} - \frac{x^2}{l_s^2}\right)\psi_1 + \alpha_1(T)\psi_1 + \varepsilon\psi_2 + \varepsilon_1\left(\frac{d^2}{dx^2} - \frac{x^2}{l_s^2}\right)\psi_2 = 0$$

and

$$-\frac{\hbar^2}{4m_2}\left(\frac{d^2}{dx^2} - \frac{x^2}{l_s^2}\right)\psi_2 + \alpha_2(T)\psi_2 + \varepsilon\psi_{12} + \varepsilon_1\left(\frac{d^2}{dx^2} - \frac{x^2}{l_s^2}\right)\psi_1 = 0.$$

That gave the upper critical magnetic field as

$$h_{c2} = \left(\theta - c_0 + \left(A\theta^2 - B\theta + c_0^2\right)^{\frac{1}{2}}\right)a_0^{-1} \tag{2.43}$$

with $A = \dfrac{(x-1)^2}{(x+1)^2} + A_1\eta^2, A_1 = 64a_1a_2\dfrac{x^2}{(x+1)^2}, x = \dfrac{\gamma_1 m_1}{\gamma_2 m_2},$

$$B = \frac{2(x-1)(a_1x - a_2)}{(x+1)^2} + (a_1 + a_2)A_1\eta^2 + 2B_1\eta,$$

$$c_0 = \frac{(a_1x + a_2)}{(x+1)} + B_1\eta,\ \theta = 1 - \frac{T}{T_c},$$

$$a_0 = 1 - \frac{16x\eta^2\left(\frac{\varepsilon}{T_c}\right)^2}{\gamma_1\gamma_2},\ a_i = 1 - \frac{T_{ci}}{T_c},\ \eta = \frac{T_c m_2\varepsilon_1\gamma_2}{\hbar^2\varepsilon}.$$

Later in April of the same year, Askerzade,Gencer Güçlü and Kiliç (2002: L17-L20) proposed the relationship between temperature and the lower critical magnetic field in magnesium diboride superconductors by two-band Ginzburg-Landau theory. They used the same free energy (eq.(2.40)) to calculate lower critical magnetic field. After variation of free energy with respect to vector potentials $(\vec{A})$, they found

$$\frac{\nabla\times\nabla\times\vec{A}}{4\pi}=\frac{2\pi}{\phi_0}\left[\frac{\hbar^2}{4m_1}n_1(T)\left(\frac{d\phi_1}{dr}-\frac{2\pi}{\phi_0}\vec{A}\right)+\frac{\hbar^2}{4m_2}n_2(T)\left(\frac{d\phi_2}{dr}-\frac{2\pi}{\phi_0}\vec{A}\right)\right.$$

$$\left.+\varepsilon_1(n_1(T)n_{21}(T))^{\frac{1}{2}}\cos(\phi_1-\phi_2)\left[\left(\frac{d\phi_1}{dr}-\frac{2\pi}{\phi_0}\vec{A}\right)+\left(\frac{d\phi_2}{dr}-\frac{2\pi}{\phi_0}\vec{A}\right)\right]\right]$$

so that the upper critical magnetic field was

$$h_{c1}=B(T)\ln\kappa(T) \tag{2.44}$$

with

$$B(T)=-\frac{2}{x+D^{-1}}\left(\varepsilon^2+\chi(\tau-\tau_{c1})^2+2\chi\eta\varepsilon^2(\tau-\tau_{c1})\left[\frac{\theta^2+(2-\tau_{c1}-\tau_{c2})\theta}{\varepsilon^3D(\tau-\tau_{c2})+(\tau-\tau_{c1})^3}\right]\right).$$

Here

$$D=\frac{\beta_1\gamma_2^2}{\beta_2\gamma_1^2}\ ,\ \tau_{c1,c2}=\frac{T_{c1,c2}}{T_c}\ ,\ \chi=\frac{\gamma_1 m_1}{\gamma_2 m_2},\ \eta=\frac{\varepsilon_1 T_c\gamma_2 m_2}{\varepsilon\hbar^2}\ ,\ \tau=\frac{T}{T_c},\ \ \theta=\tau-1.$$

In January 2006, Udomsamuthirun, Changjan, Kumvongsa and Yoksan (2006:62-66) studied the upper critical field (H_{c2}) of two-band superconductors by two-band Ginzburg–Landau approach. The analytical formula including anisotropic of order parameter and anisotropic of effective-mass were

$$H_{c2}=-\frac{\phi_0}{2\pi\xi_{12}^{'2}}\left(\frac{\xi_{12}^{'2}}{\xi_1^2}+\frac{\xi_{12}^{'2}}{\xi_2^2}+2\sqrt{\Omega\kappa}\right)$$
$$\times\left[\frac{1}{1-\Omega\kappa^2}-\frac{\left(\frac{\xi_{12}^{'4}}{\xi_1^2\xi_2^2}-1\right)}{\left(\frac{\xi_{12}^{'2}}{\xi_1^2}+\frac{\xi_{12}^{'2}}{\xi_2^2}+2\sqrt{\Omega\kappa}\right)^2}+\frac{(1-\Omega\kappa^2)\left(\frac{\xi_{12}^{'4}}{\xi_1^2\xi_2^2}-1\right)^2}{\left(\frac{\xi_{12}^{'2}}{\xi_1^2}+\frac{\xi_{12}^{'2}}{\xi_2^2}+2\sqrt{\Omega\kappa}\right)^4}\right], \tag{2.45}$$

where $\Omega=\frac{\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle^2}{\left\langle f_1(\hat{k})\right\rangle^2\left\langle f_2(\hat{k})\right\rangle^2}$, $\xi_{12}^{'}=\sqrt{\frac{\hbar^2}{2m\varepsilon_1\sqrt{\Omega}}}=\frac{\xi_{12}}{\Omega^{1/4}}$, $\langle...\rangle$ denotes an average over Fermi surface, $f_1(\hat{k})$ and $f_2(\hat{k})$ are the anisotropic function of the first and second band. The numerical calculation of upper critical field in ab-plane and c-axis were comparing to the experimental data as shown in figure15.

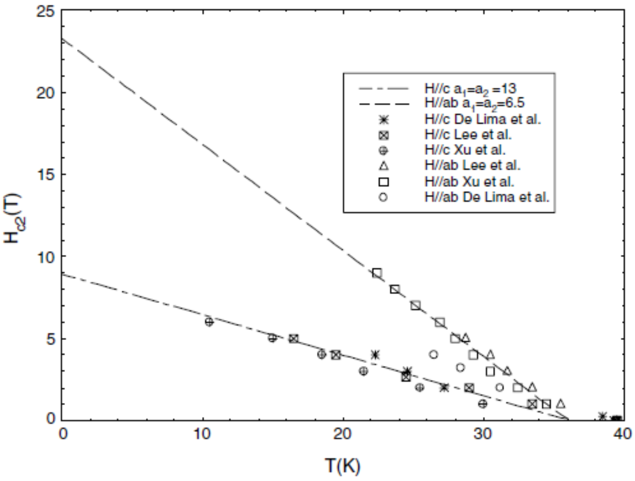


Figure 15 The relationship of the upper critical magnetic field of single crystal MgB_2 and the critical temperature, comparison between experimental and theoretical results.: Udomsamuthirun et al.(2006:62-66)

In March 2008, Kamihara et al. (2008:3296-3297) reported the discovery of 26K superconductivity in fluorine-doped LaOFeAs. This discovery aroused great interest in the scientific community because it was the first time that noncuprate superconductor showed high critical temperature with very high upper critical magnetic field. This material showed a very high in upper critical magnetic field exceeded the capability of common high field measurement system.

In April 2008, Ren et al. (2008:2215-2216) reported the superconductivity in iron-based oxyarsenide $Sm[O_{1-x}F_x]FeAs$, with the onset resistivity transition temperature at 55 K and Meissner transition at 54.6 K. This compound has the same crystal structure as LaOFeAs with shrunk crystal lattices and becomes the superconductor with the highest critical temperature among all materials besides copper oxides.

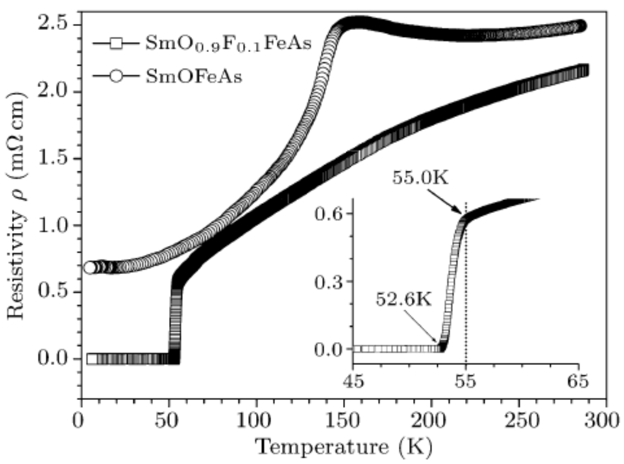


Figure 16 The temperature dependence of resistivity for the undoped SmOFeAs and the $Sm[O_{0.9}F_{0.1}]FeAs$ superconductor.:Ren et al. (2008:2215-2216)

In July 2008, Zhu et al.(2008:105001) synthesized the new iron-based superconductor $LaFeAsO_{0.9}F_{0.1-\delta}$. The resistive transition curves under different magnetic fields were measured, leading to the determination of the upper critical field $H_{c2}(T)$ of this new superconductor. The value of H_{c2} at zero temperatures was estimated to be about 50 T roughly.

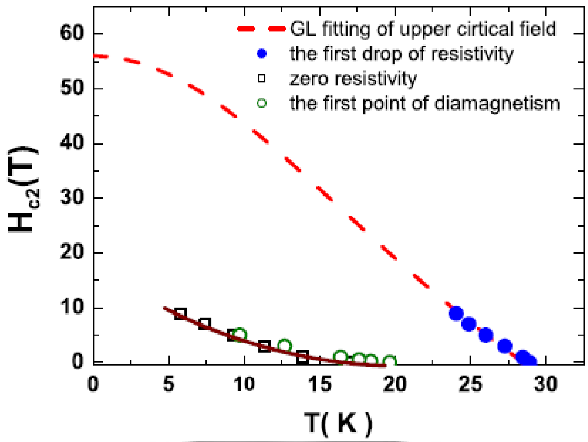


Figure 17 Phase diagram derived from the resistive transition curves. :Zhu et al. (2008:105001)

CHAPTER 3
RESEARCH METHODOLOGY

In this research, we study the lower critical field, the upper critical field, and the critical magnetic field ratio of two-band anisotropic magnetic superconductors.

1 Critical magnetic field of one-band anisotropic magnetic superconductors

1.1 Upper critical field

According to the Ginzburg and Landau theory, the Helmholtz free energy density of non-magnetic superconductors is of the form

$$f_s(H,T) = f_N + \alpha|\psi|^2 + \frac{1}{2}\beta|\psi|^4 + \frac{1}{2m}\left|\left(-i\hbar\vec{\nabla} - 2e\vec{A}\right)\psi\right|^2 + \int H_s dB \tag{3.1}$$

Here, f_N and f_s are respectively the Helmholtz free energy density in the normal state and superconducting state,

B is the net field in the superconductivity region,

ψ is the order parameter and $\left(|\psi|^2\right)$ is proportional to the density of carriers,

m denotes the effective mass of the carriers, the coefficient α depends linearly on the temperature, while coefficient β is independent of temperature.

The fourth term accounts for the kinetic energy of the carriers and the lasted term accounts for the energy stored in the local magnetic fields.

Hampshire(Hampshire.1998:1) had proposed the Gibbs's free energy of magnetic superconductor by assuming that $g_s(B,T) = f_N(B,T) - \mu_0 HM$, $H_s = \frac{B}{\mu_0} - M_{ions}$, $M = M_{sc} + M_{ions}$ and $M_{ions} = \chi H_s$. Therefore Gibbs free energy density of magnetic superconductors can be written as

$$g_s(B,T) = f_N + \alpha|\psi|^2 + \frac{1}{2}\beta|\psi|^4 + \frac{1}{2m}\left|\left(-i\hbar\vec{\nabla} - 2e\vec{A}\right)\psi\right|^2 + \int (B - \mu_0 M_{ions})\frac{dB}{\mu_0} - (B - \mu_0 M)M \tag{3.2}$$

For small change in B-field, a series in B was introduced

$$\gamma_0 + \gamma_1 B + \gamma_2 \frac{B^2}{2\mu_0} = \int (B - \mu_0 M_{ions})\frac{dB}{\mu_0} - (B - \mu_0 M)M_{sc} - (B - \mu_0 M)M_{ions}.$$

Here γ_0, γ_1 and γ_2 are coefficient parameters. We can get the γ_2 by differentiating above equation twice and taking M_{sc} to be small that

$$\gamma_2 = 1 - \mu_0 \frac{dM_{ions}}{dB} - \mu_0 \frac{d^2}{dB^2}\left[(B - \mu_0 M_{ions})M_{ions}\right],$$

and $\gamma_2 = 1$ for non-magnetic superconductors. The coefficient parameter γ_1 depends on the gradient of field and $\gamma_1 = 0$ for the uniform applied field. Finally, we can write the Gibbs's free energy of magnetic superconductor as

$$g_s(B,T) = f_N + \alpha|\psi|^2 + \frac{1}{2}\beta|\psi|^4 + \frac{1}{2m}\left|\left(-i\hbar\vec{\nabla} - 2e\vec{A}\right)\psi\right|^2 + \gamma_0 + \gamma_1 + \gamma_2 \frac{B^2}{2\mu_0} \tag{3.3}$$

When the magnetic superconductors place in magnetic field, there are the field produced by ions $(\mu_0 M_{ions})$, the applied field$(\mu_0 H)$ and the field produced by the carriers $(\mu_0 M_{sc})$ which provide the relationship between fields as

$$B = \mu_0 H + \mu_0 M_{sc} + \mu_0 M_{ions} \tag{3.4}$$

$$M_{ions} = \chi H_{c_2}(T) + \chi'(H - M_{sc} - H_{c_2}) \tag{3.5}$$

So that

$$B = \mu_0(\chi - \chi')H_{c_2} + \mu_0(1+\chi')(H + M_{sc}) \tag{3.6}$$

where the differential susceptibility (χ') is $\left(\frac{\partial M_{ions}}{\partial H}\right)_{T,H=H_{c_2}}$ and susceptibility (χ) is $\left(\frac{M_{ions}}{H}\right)_{T,H=H_{c_2}}$.

By comparing Ginzburg-Landau theory to BCS theory (Fetter & Walecka, 1995:471), we can get

$$\psi \sim \Delta$$

where Δ is the gap function from BCS theory,
ψ is the order parameter from GL theory.
The anisotropic gap function can be written as

$$\Delta(\hat{k},T) \sim f(\hat{k})\Delta(T)$$

where $f(\hat{k})$ is the anisotropic function.
Similarity, we can write anisotropic order parameter

$$\psi(\hat{k},T) = f(\hat{k})\psi(T). \tag{3.7}$$

The anisotropy gap function was proposed by Haas and Maki(Haas and Maki. 2001:020502): $f(\theta) = \frac{1+a\cos^2\theta}{1+a}$ and by Posazhennikova et al.(Posazhennikova et al.2003:577): $f(\theta) = \frac{1}{\sqrt{1+a\cos^2\theta}}$, here θ is the polar angle and a is anisotropy parameter. In case of the symmetry order parameter $f(\hat{k}) = 1$.

Minimising g_s of eq.(3.3) with respect to ψ^* and $\vec{A}$, here we use anisotropic order parameter in eq.(3.7), they lead to the 1st and 2nd Ginzburg-Landau equations, which now include the effect of anisotropic function and magnetic ions as

$$\alpha\psi\left\langle f^2(\hat{k})\right\rangle + \frac{1}{2}\beta|\psi|^2\psi\left\langle f^4(\hat{k})\right\rangle + \frac{1}{2m}\left(-i\hbar\vec{\nabla} - 2e\vec{A}\right)^2\psi\left\langle f^2(\hat{k})\right\rangle = 0 \tag{3.8}$$

$$\left(\frac{\gamma_1}{B} + \frac{\gamma_2}{\mu_0}\right)\mu_0\vec{\nabla}\times(1+\chi')M_{sc} = \left[-\frac{ie\hbar}{m}\left(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right) - \frac{4e^2\vec{A}|\psi|^2}{m}\right]\left\langle f^2(\hat{k})\right\rangle. \tag{3.9}$$

Where $\vec{\nabla}\times M_{sc}=J_{sc}$ is the supercurrent density and $<...>$ is an averaged over Fermi surface. By eq.(3.8) and eq.(3.9) can be reduced to the isotropic magnetic superconductors by setting $\langle ...\rangle$ to 1 and $\gamma_1=0$.

An expression for upper critical field can be found from the linearized 1st Ginzburg-Landau equation

$$\alpha|\psi|^2+\frac{1}{2m}\left(-i\hbar\vec{\nabla}-2e\vec{A}\right)^2\psi=0. \tag{3.10}$$

Where the vector potential is taken as

$$\vec{A}=\left(0,\left[\mu_0(\chi-\chi')H_{c_2}+\mu_0(1+\chi')(H+M_{sc})\right]x,0\right).$$

For a particle of mass m and charge e moving in a magnetic field $\vec{B}=\vec{\nabla}\times\vec{A}$, near the upper critical field (H_{c2}) then $H\rightarrow H_{c2}$ and $M_{sc}\rightarrow 0$, we get

$$\vec{A}=\mu_0 H_{c2}(1+\chi)x\hat{j} \quad\text{and}\quad \vec{B}=\mu_0 H_{c2}(1+\chi)\hat{k}. \tag{3.11}$$

From

$$\hat{H}\varphi=E\varphi,$$

where $\hat{H}=\frac{1}{2m}\left(-i\hbar\vec{\nabla}-2e\vec{A}\right)^2$, the eigenvalue of this Hamiltonian are $E=\left(n+\frac{1}{2}\right)\hbar\omega$ with $\omega=\frac{eB}{m}$.

For the case of linearized 1st Ginzburg-Landau equation (eq.(3.11))

$$|\alpha|=\left(n+\frac{1}{2}\right)\hbar\omega=\left(n+\frac{1}{2}\right)\frac{\hbar e\mu_0 H_{c2}(1+\chi)}{m}$$

or

$$\mu_0 H_{c2}=\frac{2|\alpha|m}{(2n+1)\hbar e(1+\chi)}.$$

To determine the upper critical field, we must take the largest possible value of magnetic field ($n=0$)

$$\mu_0 H_{c2}=\frac{2|\alpha|m}{\hbar e(1+\chi)}. \tag{3.12}$$

From coherence length $\xi^2(T)=\frac{\hbar^2}{2m|\alpha|}$ and magnetic flux quantum $\phi=\frac{2\pi\hbar}{e}$,

we get

$$\mu_0 H_{c2}=\frac{\phi_0}{2\pi\xi^2(1+\chi)}=\frac{\mu_0 H_{c2}(\chi=0,T)}{(1+\chi)} \tag{3.13}$$

where $\mu_0 H_{c2}(\chi=0,T)$ is upper critical magnetic field of non-magnetic superconductor.

1.2 Lower critical field

Consider the magnetic field just above lower critical field (B_{c1}). If the flux penetration is uniform, then we would have $\psi = 0$ everywhere. So, we consider a non-uniform flux penetration keeping as much symmetry as possible. For lower critical field's calculation, we choose a flux line in cylindrical coordinate $\vec{B} = B(r)\hat{z}$, where $B(r)$ has its maximum value at the core and tends to zero at large radial. A vector potential in cylindrical coordinate is chosen as

$$\vec{A} = \left[\mu_0(\chi - \chi')H_{c_2}r + \mu_0(1+\chi')(H + M_{sc})r\right]\hat{\theta} = A(r)\hat{\theta}. \tag{3.14}$$

From the solution for the wave function $\psi(T)$ provided by

$$\psi(T) = |\psi(T)|e^{i\phi}, \tag{3.15}$$

where ϕ is the phase of order parameter and by using the relevant Maxwell's equation, $\vec{j} = \frac{1}{\mu_0}\vec{\nabla}\times\vec{B}$ (for the magnetostatic case, $\frac{\partial\vec{D}}{\partial t} = 0$) and 2nd Ginzburg-Landau equation in uniform field, the vector potential and magnetic flux quantum are $\vec{A} = \frac{\hbar}{2e}\vec{\nabla}\phi$ and $\Phi = \frac{2\pi\hbar}{e}$, the equation for the vector potential takes the form

$$\vec{\nabla}\times\vec{\nabla}\times\vec{B} = -\frac{\mu_0}{\gamma_{2(1+\chi')}}\cdot\frac{4e^2|\alpha|}{m\beta}\left(\vec{\nabla}\times\vec{A}\right)\left\langle f^2(\hat{k})\right\rangle = -\frac{\mu_0}{\gamma_{2(1+\chi')}}\cdot\frac{4e^2|\alpha|\vec{B}}{m\beta}\left\langle f^2(\hat{k})\right\rangle. \tag{3.16}$$

Let our sample has a surface perpendicular to the x-axis and the external magnetic field is $\vec{B} = B_0\hat{z}$, so the internal magnetic field should be the form $\vec{b} = b(x)\hat{z}$. Then, the London equation is of the form $\frac{d^2b(x)}{dx^2} - \frac{1}{\lambda^2}b(x) = 0$. Here λ is the London penetration depth of anisotropic magnetic superconductors

$$\lambda = \left(\frac{\gamma_2(1+\chi')m\beta}{\mu_0\left(4e^2\right)\left\langle f^2(\hat{k})\right\rangle|\alpha|}\right)^{\frac{1}{2}}. \tag{3.17}$$

The lower critical field can be obtained as

$$B_{c1} = \frac{\hbar}{4e\lambda^2}\ln\kappa. \tag{3.18}$$

Where κ is Ginzburg-Landau parameter, $\kappa = \frac{\lambda}{\xi}$. The lower critical field eq.(3.18) is the same form of Abrikosov(Abrikosov.1957:1174) but it has the difference in κ that depend on λ (eq.(3.17)).

1.3 Critical magnetic field ratio

We introduce a dimensionless parameter, the critical magnetic field ratio (η) as

$$\eta = \frac{B_{c2}}{B_{c1}} \tag{3.19}$$

Substitution eq.(3.13) and eq.(3.18) in eq.(3.19), we get

$$\eta = \frac{4\theta\kappa_0^2}{(1+\chi)\ln\theta\kappa_0^2}. \tag{3.20}$$

In case $\kappa >> 1$, the approximation critical magnetic field ratio is

$$\eta \approx \frac{2\theta\kappa_0^2}{(1+\chi)\left(\sqrt{\theta}\,\kappa_0 - 1\right)}. \tag{3.21}$$

Here, κ_0 is the isotropic non-magnetic Ginzburg-Landau parameter, $\kappa_0 = \dfrac{\lambda_0}{\xi_0}$ and $\theta = \dfrac{\gamma_2(1+\chi')}{\left\langle f^2(\hat{k})\right\rangle}$ is the magnetic-to-anisotropic parameter ratio. For the pure super-conductors that is the isotropic non-magnetic superconductors, eq.(3.20) can be reduced to $\eta = \dfrac{2\kappa_0{}^2}{\ln\kappa_0}$ that agreed with the Ginzburg-Landau relation.

2 Critical magnetic field of two-band anisotropic magnetic superconductors

2.1 Upper critical field

The free energy of an anisotropic two-band magnetic superconductors can be obtained as

$$F_{sc}[\psi_1,\psi_2] = \int d^3r f_{sc} = \int d^3r\left(f_1 + f_2 + f_{12} + \gamma_0 + \gamma_1 B + \gamma_2\frac{B^2}{2\mu_0}\right) \tag{3.22}$$

with

$$f_{i(i=1,2)} = \frac{1}{2m_i}\left|\left(-i\hbar\vec{\nabla} - 2e\vec{A}\right)\psi_i\right|^2\left\langle f_i^2(\hat{k})\right\rangle + \alpha_i|\psi_i|^2\left\langle f_i^2(\hat{k})\right\rangle + \frac{1}{2}\beta_i|\psi_i|^4\left\langle f_i^4(\hat{k})\right\rangle \tag{3.23}$$

$$f_{12} = \varepsilon\left(\psi_1^*\psi_2 + c.c.\right)\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle + \varepsilon_1\left\{\left(i\hbar\vec{\nabla} - 2e\vec{A}\right)\psi_1^*\left(-i\hbar\vec{\nabla} - 2e\vec{A}\right)\psi_2 + c.c.\right\}\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle. \tag{3.24}$$

Minimizing F_{sc} of Eq.(7) with respect to ψ_i^* ($i = 1,2$) and $\vec{A}$, the 1st and 2nd GL equations including the anisotropic function and magnetic ions can be found as

$$\frac{\left\langle f_1^2(\hat{k})\right\rangle}{2m_1}\left(-i\hbar\vec{\nabla}-2e\vec{A}\right)^2\psi_1+\alpha_1\left\langle f_1^2(\hat{k})\right\rangle\psi_1+\beta_1\left\langle f_1^4(\hat{k})\right\rangle\left(\psi_1^*\psi_1\right)\psi_1 \\ +\varepsilon\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle\psi_2-\varepsilon_1\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle\left(-i\hbar\vec{\nabla}-2e\vec{A}\right)^2\psi_2=0 \tag{3.25}$$

$$\frac{\left\langle f_2^2(\hat{k})\right\rangle}{2m_2}\left(-i\hbar\vec{\nabla}-2e\vec{A}\right)^2\psi_2+\alpha_2\left\langle f_2^2(\hat{k})\right\rangle\psi_2+\beta_2\left\langle f_2^4(\hat{k})\right\rangle\left(\psi_2^*\psi_2\right)\psi_2 \\ +\varepsilon\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle\psi_1-\varepsilon_1\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle\left(-i\hbar\vec{\nabla}-2e\vec{A}\right)^2\psi_1=0 \tag{3.26}$$

and

$$-\left(\frac{\gamma_1}{B}+\frac{\gamma_2}{\mu_0}\right)\mu_0\vec{\nabla}\times(1+\chi')M_{sc}=\frac{\left\langle f_1^2(\hat{k})\right\rangle}{2m_1}\left[2ie\hbar\left(\psi_1^*\vec{\nabla}\psi_1-\psi_1\vec{\nabla}\psi_1^*\right)+8e^2\vec{A}|\psi_1|^2\right] \\ +\frac{\left\langle f_2^2(\hat{k})\right\rangle}{2m_2}\left[2ie\hbar\left(\psi_2^*\vec{\nabla}\psi_2-\psi_2\vec{\nabla}\psi_2^*\right)+8e^2\vec{A}|\psi_2|^2\right] \\ +\varepsilon_1\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle\left[\begin{array}{l}2ie\hbar\left(\psi_1^*\vec{\nabla}\psi_2-\psi_2\vec{\nabla}\psi_1^*\right)+8e^2\vec{A}\psi_1^*\psi_2\\+2ie\hbar\left(\psi_2^*\vec{\nabla}\psi_1-\psi_1\vec{\nabla}\psi_2^*\right)+8e^2\vec{A}\psi_2^*\psi_1\end{array}\right] \tag{3.27}$$

where $\vec{\nabla}\times M_{sc}=J_{sc}$ is the supercurrent density and $<\ldots>$ denotes an average over Fermi surface. An expression for upper critical field can be found from the linearize 1st Ginzburg-Landau equation

$$\frac{\left\langle f_1^2(\hat{k})\right\rangle}{2m_1}\left(-i\hbar\vec{\nabla}-2e\vec{A}\right)^2\psi_1+\alpha_1\left\langle f_1^2(\hat{k})\right\rangle\psi_1+\varepsilon\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle\psi_2-\varepsilon_1\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle\left(-i\hbar\vec{\nabla}-2e\vec{A}\right)^2\psi_2=0$$

$$\frac{\left\langle f_2^2(\hat{k})\right\rangle}{2m_2}\left(-i\hbar\vec{\nabla}-2e\vec{A}\right)^2\psi_2+\alpha_2\left\langle f_2^2(\hat{k})\right\rangle\psi_2+\varepsilon\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle\psi_1-\varepsilon_1\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle\left(-i\hbar\vec{\nabla}-2e\vec{A}\right)^2\psi_1=0,$$

where the vector potential is taken as $\vec{A}=\left[\mu_0(\chi-\chi')H_{c_2}x+\mu_0(1+\chi')(H+M_{sc})x\right]\hat{j}$. Substitute $\vec{A}$ in the above equation, we get

$$-\frac{\left\langle f_1^2(\hat{k})\right\rangle\hbar^2}{2m_1}\left(\frac{d^2}{dx^2}-\frac{x^2}{l_s^2}\right)\psi_1+\alpha_1(T)\left\langle f_1^2(\hat{k})\right\rangle\psi_1+\varepsilon\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle\psi_2+\hbar^2\varepsilon_1\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle\left(\frac{d^2}{dx^2}-\frac{x^2}{l_s^2}\right)\psi_2=0 \tag{3.28}$$

$$-\frac{\left\langle f_2^2(\hat{k})\right\rangle\hbar^2}{2m_2}\left(\frac{d^2}{dx^2}-\frac{x^2}{l_s^2}\right)\psi_2+\alpha_2(T)\left\langle f_2^2(\hat{k})\right\rangle\psi_2+\varepsilon\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle\psi_1+\hbar^2\varepsilon_1\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle\left(\frac{d^2}{dx^2}-\frac{x^2}{l_s^2}\right)\psi_1=0 \tag{3.29}$$

where $l_s^2 = \dfrac{\hbar^2}{4e^2\left(\mu_0(\chi-\chi')H_{c_2}x + \mu_0(1+\chi')(H+M_{sc})x\right)}$.

Substitute eq.(3.28) and eq.(3.29) with the eigen function $\psi \propto e^{-\frac{ax^2}{2}}$ corresponding to a lowest energy state, i.e. $\psi_1 = \lambda_1 e^{-\frac{ax^2}{2}}$ and $\psi_2 = \lambda_2 e^{-\frac{bx^2}{2}}$, we get

$$-\frac{\left\langle f_1^2(\hat{k})\right\rangle\hbar^2}{2m_1}\left(-a\left[1-ax^2\right]-\frac{x^2}{l_s^2}\right)\psi_1 + \alpha_1(T)\left\langle f_1^2(\hat{k})\right\rangle\psi_1 + \varepsilon\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle\psi_2$$
$$+\hbar^2\varepsilon_1\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle\left(-b\left[1-bx^2\right]-\frac{x^2}{l_s^2}\right)\psi_2 = 0, \qquad (3.30)$$

$$-\frac{\left\langle f_2^2(\hat{k})\right\rangle\hbar^2}{2m_2}\left(-b\left[1-bx^2\right]-\frac{x^2}{l_s^2}\right)\psi_2 + \alpha_2(T)\left\langle f_2^2(\hat{k})\right\rangle\psi_2 + \varepsilon\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle\psi_1$$
$$+\hbar^2\varepsilon_1\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle\left(-a\left[1-ax^2\right]-\frac{x^2}{l_s^2}\right)\psi_1 = 0. \qquad (3.31)$$

These two equations can be rewritten in the single matrix form as

$$\begin{bmatrix} -\frac{\left\langle f_1^2(\hat{k})\right\rangle\hbar^2}{2m_1}\left(-a(1-ax^2)-\frac{x^2}{l_s^2}\right)+\alpha_1(T)\left\langle f_1^2(\hat{k})\right\rangle & \left(\varepsilon+\hbar^2\varepsilon_1\left(-b(1-bx^2)-\frac{x^2}{l_s^2}\right)\right)\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle \\ \left(\varepsilon+\hbar^2\varepsilon_1\left(-a(1-ax^2)-\frac{x^2}{l_s^2}\right)\right)\left\langle f_1(\hat{k})f_2(\hat{k})\right\rangle & -\frac{\left\langle f_2^2(\hat{k})\right\rangle\hbar^2}{2m_2}\left(-b(1-bx^2)-\frac{x^2}{l_s^2}\right)+\alpha_2(T)\left\langle f_2^2(\hat{k})\right\rangle \end{bmatrix}$$
$$\times\begin{bmatrix}\psi_1\\ \psi_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}. \qquad (3.32)$$

By solving secular equation, we get

$$
\left[\frac{\left\langle f_1^2\left(\hat{k}\right)\right\rangle \hbar^2 a}{2m_1}+\alpha_1(T)\left\langle f_1^2\left(\hat{k}\right)\right\rangle + x^2\left(\frac{\left\langle f_1^2\left(\hat{k}\right)\right\rangle \hbar^2}{2m_1 l_s^2}-\frac{\left\langle f_1^2\left(\hat{k}\right)\right\rangle \hbar^2 a^2}{2m_1}\right)\right]\times
$$

$$
\left[\frac{\left\langle f_2^2\left(\hat{k}\right)\right\rangle \hbar^2 b}{2m_2}+\alpha_2(T)\left\langle f_2^2\left(\hat{k}\right)\right\rangle + x^2\left(\frac{\left\langle f_2^2\left(\hat{k}\right)\right\rangle \hbar^2}{2m_2 l_s^2}-\frac{\left\langle f_2^2\left(\hat{k}\right)\right\rangle \hbar^2 b}{2m_2}\right)\right]
$$

$$
-\left[\varepsilon\left\langle f_1\left(\hat{k}\right) f_2\left(\hat{k}\right)\right\rangle-\varepsilon_1 a\hbar^2\left\langle f_1\left(\hat{k}\right) f_2\left(\hat{k}\right)\right\rangle - x^2\left(\frac{\hbar^2\varepsilon_1\left\langle f_1\left(\hat{k}\right) f_2\left(\hat{k}\right)\right\rangle}{l_s^2}-\hbar^2\varepsilon_1 a^2\left\langle f_1\left(\hat{k}\right) f_2\left(\hat{k}\right)\right\rangle\right)\right]\times
$$

$$
\left[\varepsilon\left\langle f_1\left(\hat{k}\right) f_2\left(\hat{k}\right)\right\rangle-\varepsilon_1 b\hbar^2\left\langle f_1\left(\hat{k}\right) f_2\left(\hat{k}\right)\right\rangle - x^2\left(\frac{\hbar^2\varepsilon_1\left\langle f_1\left(\hat{k}\right) f_2\left(\hat{k}\right)\right\rangle}{l_s^2}-\hbar^2\varepsilon_1 b^2\left\langle f_1\left(\hat{k}\right) f_2\left(\hat{k}\right)\right\rangle\right)\right]=0.
$$

(3.33)

Let $\quad \dfrac{\left\langle f_1^2\left(\hat{k}\right)\right\rangle \hbar^2}{2m_1 l_s^2}-\dfrac{\left\langle f_1^2\left(\hat{k}\right)\right\rangle \hbar^2 a^2}{2m_1}=0$, we get $\quad a^2=\dfrac{1}{l_s^2}$

and $\quad \dfrac{\left\langle f_2^2\left(\hat{k}\right)\right\rangle \hbar^2}{2m_2 l_s^2}-\dfrac{\left\langle f_2^2\left(\hat{k}\right)\right\rangle \hbar^2 b^2}{2m_2}=0$, we get $\quad b^2=\dfrac{1}{l_s^2}$.

Substitute in eq.(3.33), we find that

$$
\left[\frac{\left\langle f_1^2\left(\hat{k}\right)\right\rangle \hbar^2 a}{2m_1}+\alpha_1(T)\left\langle f_1^2\left(\hat{k}\right)\right\rangle\right]\left[\frac{\left\langle f_2^2\left(\hat{k}\right)\right\rangle \hbar^2 b}{2m_2}+\alpha_2(T)\left\langle f_2^2\left(\hat{k}\right)\right\rangle\right]
$$

$$
-\left[\varepsilon+\varepsilon_1 a\hbar^2\right]\left[\varepsilon+\varepsilon_1 b\hbar^2\right]\left\langle f_1\left(\hat{k}\right) f_2\left(\hat{k}\right)\right\rangle=0. \tag{3.34}
$$

Let $m_1=m_2=m$ and substitute l_s in eq.(3.34), we get

$$
\hbar^2 e^2(1+\chi)^2\left(\frac{1}{m^2}-4\varepsilon_1^2\Omega\right)\mu_0^2 H_{c2}^2+\hbar e(1+\chi)\left(\frac{(\alpha_1(T)+\alpha_2(T))}{m}+4\varepsilon\varepsilon_1\Omega\right)\mu_0 H_{c2}
$$

$$
+\left(\alpha_1(T)\alpha_2(T)-\Omega\varepsilon^2\right)=0. \tag{3.35}
$$

From quadratic equation $Ax^2+Bx+C=0$, $x=\dfrac{-B\pm\sqrt{B^2-4AC}}{2A}$ where A,B and C are constant, we can rewrite in form

$$
x=\frac{-B\pm B\left(1-\dfrac{4AC}{B^2}\right)^{\frac{1}{2}}}{2A} \tag{3.36}
$$

Let $\dfrac{4ac}{b^2}<<1$ and from binomial series as

$$
(a+x)^n=a^n+na^{n-1}x+\frac{n(n-1)}{2!}a^{n-2}x^2+\frac{n(n-1)(n-2)}{3!}a^{n-3}x^3+\ldots,
$$

we get

$$\left(1-\frac{4AC}{B^2}\right)^{\frac{1}{2}} \cong 1-\frac{1}{2}\left(\frac{4AC}{B^2}\right). \tag{3.37}$$

Substitute eq.(3.37) in eq.(3.36), one obtains

$$x^{(+)} \cong -\frac{C}{B} \tag{3.38}$$

and

$$x^{(-)} \cong -\frac{B}{A}+\frac{C}{B}. \tag{3.39}$$

We choose solution as eq.(3.39), then the upper critical magnetic field of two-band anisotropic magnetic superconductors is

$$\mu_0 H_{c2} \cong -\frac{\left(\frac{(\alpha_1(T)+\alpha_2(T))}{m}+4\varepsilon\varepsilon_1\Omega\right)}{\hbar e(1+\chi)\left(\frac{1}{m^2}-4\varepsilon_1^2\Omega\right)}+\frac{(\alpha_1(T)\alpha_2(T)-\Omega\varepsilon^2)}{\hbar e(1+\chi)\left(\frac{(\alpha_1(T)+\alpha_2(T))}{m}+4\varepsilon\varepsilon_1\Omega\right)}. \tag{3.40}$$

Eq.(3.40) can be reduced to one-band case if the parameter of the second band and coupling term are set to be zero.

2.2 Lower critical field

Consider the uniform applied magnetic field just above lower critical field, there are the fluxes penetration the superconductors. For the lower critical field's calculation that keeping as much symmetry as possible, we choose a flux line in cylindrical coordinate $\vec{B}=B(r)\hat{z}$ where $B(r)$ has its maximum value at the core and tends to zero at large radial. A vector potential in cylindrical coordinate is chosen in the same manner of upper critical field's calculation as

$$\vec{A}=[\mu_0(\chi-\chi')H_{c2}r+\mu_0(1+\chi')(H+M_{sc})r]\hat{\theta}=A(r)\hat{\theta}. \tag{3.41}$$

The solution of the order parameters in this form $\psi_1(T)$ and $\psi_2(T)$ are provided by

$$\psi_1(T)=|\psi_1(T)|e^{i\phi_1} \text{ and } \psi_2(T)=|\psi_2(T)|e^{i\phi_2},$$

where ϕ_1 and ϕ_2 are the phases of the order parameters. By Maxwell's equation, $\vec{j}=\frac{1}{\mu_0}\vec{\nabla}\times\vec{B}$ for the magnetostatic case; $\frac{\partial\vec{D}}{\partial t}=0$ and 2nd Ginzburg-Landau equation in uniform field, the vector potential is

$$\vec{A}=-\frac{w_1\vec{\nabla}\phi_1+w_2\vec{\nabla}\phi_2+w_3\left(\vec{\nabla}\phi_1+\vec{\nabla}\phi_2\right)}{w_4}. \tag{3.42}$$

Here, $\Phi=\frac{2\pi}{w_4}$ is the magnetic flux quantum with $w_4=-\frac{2e}{\hbar}(w_1+w_2+2w_3)$ and

$$w_1 = -\frac{\left\langle f_1^2(\hat{k})\right\rangle}{2m_1} 4e\hbar \left(-\frac{\left(\alpha_1 \left\langle f_1^2(\hat{k})\right\rangle + \frac{\varepsilon\left\langle f_1(\hat{k}) f_2(\hat{k})\right\rangle}{\theta}\right)}{\beta_1 \left\langle f_1^4(\hat{k})\right\rangle} \right),$$

$$w_2 = -\frac{\left\langle f_2^2(\hat{k})\right\rangle}{2m_2} 4e\hbar \left(-\frac{\left(\alpha_2 \left\langle f_2^2(\hat{k})\right\rangle + \varepsilon\left\langle f_1(\hat{k}) f_2(\hat{k})\right\rangle \theta\right)}{\beta_2 \left\langle f_2^4(\hat{k})\right\rangle} \right),$$

$$w_3 = -\varepsilon_1 \left\langle f_1(\hat{k}) f_2(\hat{k})\right\rangle 4e\hbar \left(\frac{\alpha_1 \left\langle f_1^2(\hat{k})\right\rangle + \frac{\varepsilon\left\langle f_1(\hat{k}) f_2(\hat{k})\right\rangle}{\theta}}{\beta_1 \left\langle f_1^4(\hat{k})\right\rangle} \right)^{\frac{1}{2}} \left(\frac{\alpha_2 \left\langle f_2^2(\hat{k})\right\rangle + \varepsilon\left\langle f_1(\hat{k}) f_2(\hat{k})\right\rangle \theta}{\beta_2 \left\langle f_2^4(\hat{k})\right\rangle} \right)^{\frac{1}{2}},$$

where $\theta = \frac{\psi_1}{\psi_2}$. The equation for the vector potential takes the form $\vec{j} = \vec{\nabla} \times \vec{b}$, where $\vec{b}$ is the internal field. So we can get

$$\vec{\nabla} \times \vec{\nabla} \times \vec{b} = \frac{1}{-\gamma_2(1+\chi')} \left[\frac{\left\langle f_1^2(\hat{k})\right\rangle}{2m_1} 8e^2 |\psi_1|^2 + \frac{\left\langle f_2^2(\hat{k})\right\rangle}{2m_2} 8e^2 |\psi_2|^2 + \varepsilon_1 \left\langle f_1(\hat{k}) f_2(\hat{k})\right\rangle 8e^2 \left(\psi_1^* \psi_2 + \psi_2^* \psi_1\right) \right] \quad (3.43)$$

Let our sample has a surface perpendicular to the x-axis and the external magnetic field be $\vec{B} = B_0 \hat{z}$, and the internal magnetic field be of the form $\vec{b} = b(x)\hat{z}$. Then, the London equation is of the form $\frac{d^2 b(x)}{dx^2} - \frac{1}{\lambda^2} b(x) = 0$. Here, λ is the anisotropic magnetic London penetration depth of the following form

$$\lambda^{-2} = \frac{1}{\gamma_2(1+\chi')} \left[\frac{2e}{\hbar} w_1 + \frac{2e}{\hbar} w_2 + \frac{4e}{\hbar} w_3 \right] \quad (3.44)$$

The lower critical field can be obtained as

$$B_{c1} = \frac{\gamma_2(1+\chi')\hbar}{8e\mu_0 \lambda^2} \ln\kappa \quad (3.45)$$

Here, κ is the Ginzburg-Landau parameter; $\kappa = \frac{\lambda}{\xi}$, ξ is coherence length where

$$\xi^2 = \frac{\left\langle f_1^2(\hat{k})\right\rangle \hbar^2}{2m_1|\alpha_1|} - E\frac{\left\langle f_2^2(\hat{k})\right\rangle \hbar^2}{2m_2|\alpha_2|} \quad \text{with} \quad E = -\frac{|\alpha_1|}{|\alpha_2|}.$$

We find that our lower critical field, eq.(3.45) has the similar form to Abrikosov but there is the difference in London penetration depth and coherence length. Moreover our results show the anisotropy and magnetic parameters dependence.

2.3 Critical magnetic field ratio

Substitute eq.(3.44) and eq.(3.53) in eq.(3.24), we get the critical magnetic field ratio of two-band anisotropic magnetic superconductors as

$$\eta = \left[\frac{\mu_0}{(1+\chi)} \cdot \frac{\ln \kappa_0}{\ln \Theta \kappa_0} \right] \eta_0 , \tag{3.46}$$

where η_0 is the anisotropic non-magnetic superconductors that $\gamma_2 = \mu_0 = 1$, $\chi = \chi' = 0$ and

$$\eta_0 = \frac{8\lambda_0^2 m}{\hbar^2 \ln \kappa_0} \left[\frac{\alpha_1 + \alpha_2 + 4m\varepsilon\varepsilon_1\Omega}{1 - 4m^2\varepsilon_1^2\Omega} + \frac{\alpha_1\alpha_2 - \Omega\varepsilon^2}{\alpha_1 + \alpha_2 + 4m\varepsilon\varepsilon_1\Omega} \right]. \tag{3.47}$$

Here, $\Theta^2 = \gamma_2(1+\chi')$ is the magnetic parameter, κ_0 is the anisotropic non-magnetic Ginzburg-Landau parameter; $\kappa_0 = \frac{\lambda_0}{\xi_0}$ where λ_0 and ξ_0 is the penetration depth and coherence length of an anisotropic non-magnetic superconductors, respectively.

As a result, the upper critical field equation can be simplified as

$$B_{c2} = \left[\frac{\mu_0}{(1+\chi)} \cdot \frac{\ln \kappa_0}{\ln \Theta \kappa_0} \right] \eta_0 \cdot B_{c1} \tag{3.48}$$

3 Temperature-dependence upper critical magnetic field

To investigate the temperature dependence on the upper critical magnetic field of two-band superconductor, especially in Fe-based superconductors, we consider the temperature dependent on eq.(3.46). In GL-theory, the temperature-dependence parameter occur in α and ε that are in ξ and λ. In case 1, we modify the temperature -dependence parameter on α and ε by adding the constant, which in our assump -tion, there are 2 kinds in this case, i.e. linear and non-linear GL theory. In case 2, we use the Zhu et al. assumption (Zhu et al.2008:105001) of the temperature- dependence on α, and in case 3, we use the Shanenko et al. assumption (Shanenko et al. 2011:047005). So there are 4 cases of the temperature dependence on α and ε, and on upper critical magnetic field.

Case1. Modified GL theory: there are 2 cases of consideration.

Case 1.1) Linear GL theory

In GL theory, α is proportional to $1-t$ or $\alpha \equiv 1-t$ where $t = \frac{T}{T_c}$, the coherence length ξ is found to be $\xi^2 \equiv \frac{1}{\alpha}$ or $\xi^2 \equiv \frac{1}{1-t}$ (Fetter & Walecka, 1995:433). For one-band calculation, $B_{c2} \equiv \alpha$, so we find that $B_{c2}(t) = B_{c2}(0)(1-t)$, where $B_{c2}(0)$ and $B(t)$ are the zero temperature and temperature-dependent upper critical magnetic field, respectively. However, the equation $\alpha \equiv 1-t$ is the approximation equation in the vicinity of critical temperature. To extend the validity of this equation, we make the assumption that $\alpha \equiv 1-at$, then we get

$$B_{c2}(t) = B_{c2}(0)(1-at), \tag{3.49}$$

where a is an arbitrary constant that can be fit out from the experimental data.

Case 1.2) Non-linear GL theory.

Because the temperature-dependent of upper critical field in experimental result are in the form of t^2. Then, we make the assumption that $\xi^2 \equiv \dfrac{1}{\alpha}$ and $\xi^2 \equiv \dfrac{1}{1+at^2}$. The second order terms are found in BCS's upper critical magnetic field. We then get

$$B_{c2}(t) = B_{c2}(0)(1+at^2). \tag{3.50}$$

Case 2.The Zhu et al. assumption.

Zhu et al. (Zhu et al.2008:105001) make the assumption that $\alpha \equiv \dfrac{1-t^2}{1+t^2}$ or $\xi^2 \equiv \dfrac{1+t^2}{1-t^2}$. They claimed that this relation can be used for all of the temperature range. For one-band calculation, Zhu et al. (Zhu et al.2008:105001) found that $B_{c2}(t) = B_{c2}(0)\dfrac{1-t^2}{1+t^2}$. In two-band model (eq.(3.40)) and Udomsamuthirun et.al (Udomsamuthirun et.al 2006:62), we have $\xi^2 \equiv \dfrac{1}{\alpha}$ and $\xi^2 \equiv \dfrac{1}{\varepsilon}$, then $\alpha \equiv \dfrac{1-t^2}{1+t^2}$ and $\varepsilon \equiv \dfrac{1-t^2}{1+t^2}$. Our two-band upper critical magnetic field is

$$B_{c2}(t) = B_{c2}(0)\frac{1-t^2}{1+t^2}. \tag{3.51}$$

This is the same form of the one-band model.

Case 3.The Shanenko et al. assumption.

Shanenko et al. (Shanenko et al. 2011:047005) derived the extended GL-formalism for two-band superconductor that improved the validity of GL theory below T_c. They showed $\alpha \equiv (1-t) + \dfrac{1}{2}(1-t)^2$. To find $B_{c2}(T)$ that valid close to $T=0$, we make the assumption that $\alpha \equiv p(1-t)+\frac{q}{2}(1-t)^2$ and $\varepsilon \equiv p(1-t)+\frac{q}{2}(1-t)^2$, where p,q are the constant. The critical magnetic field ratio is the temperature-independent parameter and we can find the temperature-dependent upper critical magnetic field as

$$B_{c2}(t)=B_{c2}(0)\{p(1-t)+\frac{q}{2}(1-t)^2\}. \qquad (3.52)$$

In the next chapter, we will compare our numerical calculation with experimental data of Fe-based superconductors.

CHAPTER 4
RESEARCH RESULTS

The research carried out in chapter 3 is to calculate the critical magnetic field of one-band and two-band anisotropic magnetic superconductors analytically. In this chapter, we would like to obtain the numerical result from them and then compare to the experimental data.

1 Critical magnetic field ratio results

1.1 One-band superconductors

In one-band calculation, we find the upper critical magnetic field of one-band anisotropic magnetic superconductors as $\mu_0 H_{c2} = \frac{\phi_0}{2\pi\xi^2(1+\chi)} = \frac{\mu_0 H_{c2}(\chi = 0, T)}{(1+\chi)}$. The lower critical magnetic field of one-band anisotropic magnetic superconductors is $B_{c_1} = \frac{\hbar}{4e\lambda^2}\ln\kappa$. According to these critical magnetic fields, we have the critical magnetic field ratio of one-band anisotropic magnetic superconductors as

$$\eta = \frac{4\theta\kappa_0^2}{(1+\chi)\ln\theta\kappa_0^2}. \tag{3.20}$$

All parameters are the same as in chapter 3.

The effect of κ_0, χ and θ on the critical magnetic field ratio (η) for anisotro--pic one-band superconductors are shown in figure 18. Here, we use the κ_0 = 57, and κ_0 = 118 that are Ginzburg–Landau parameter of Y123 (Hao et al.1991: 2884) and Hg1223 (Mun-Seog Kim et al. 1995:691) superconductors, respectively. There are three cases in figure 18: the non-magnetic superconductors case $(\chi = 0)$, the diamag--netic case $(\chi < 0)$ and ferromagnetic case $(\chi > 0)$. The η must be positive so that the value of γ_2 is between 0 and 1. The θ represents the ratio of magnetic parameter to anisotropic parameter; θ = 1, θ > 1, θ < 1 are the isotropic non-magnetic case, the highly anisotropic case and highly magnetic case, respectively. Figure 18 shows that the value of critical field ratio does increase with increasing in κ and θ. The value of critical field ratio does decrease with increasing χ. The highest value of critical field ratio is found in the diamagnetic superconductors (χ < 0) with highly anisotropy and the lowest value is found in the ferromagnetic superconductor (χ > 0) with highly magnetism.

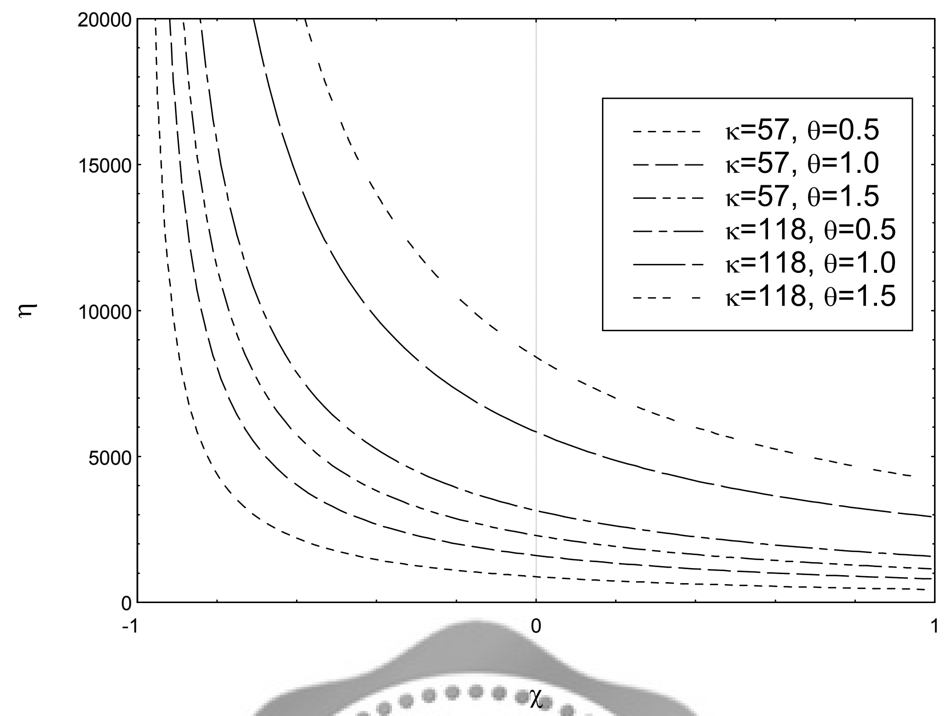


Figure 18 The critical magnetic field ratio versus the magnetic susceptibility of the One-band anisotropic magnetic superconductors.

1.2 Two-band superconductor

The upper critical magnetic field of two-band anisotropic magnetic superconductors are

$$\mu_0 H_{c2} \cong -\frac{\left(\frac{(\alpha_1(T)+\alpha_2(T))}{m}+4\varepsilon\varepsilon_1\Omega\right)}{\hbar e(1+\chi)\left(\frac{1}{m^2}-4\varepsilon_1^2\Omega\right)}+\frac{(\alpha_1(T)\alpha_2(T)-\Omega\varepsilon^2)}{\hbar e(1+\chi)\left(\frac{(\alpha_1(T)+\alpha_2(T))}{m}+4\varepsilon\varepsilon_1\Omega\right)}.$$

The lower critical magnetic field of two-band anisotropic magnetic superconductors is $B_{c1}=\frac{\gamma_2(1+\chi')\hbar}{8e\mu_0\lambda^2}\ln\kappa$. We obtain the critical magnetic field ratio of two-band anisotropic magnetic superconductors as $\eta=\left[\frac{\mu_0}{(1+\chi)}\cdot\frac{\ln\kappa_0}{\ln\Theta\kappa_0}\right]\eta_0$. We use above equation to investigate the critical magnetic field of Fe-base superconductors (two-band magnetic superconductor). After taking the numerical calculation on above equation, we show these results in figure 19-20. All parameters used are shown in the inset. Figure 19 shows that the higher value of critical magnetic field ratio may be found in superconductor with diamagnetism ($\chi<0$). However, the Fe-base super-conductors show the antiferromagntism and paramagnetism that $\chi\approx 0$, we think that the differential susceptibility (χ') may affect to this property. So the critical magnetic field ratio versus the differential susceptibility (χ') is investigated and shown in figure 20. The higher value of critical magnetic field ratio can be found in the negative differential susceptibility region $(\chi' < 0)$. Then, we can conclude that the very high upper critical magnetic field of Fe-base superconductor ($\chi \approx 0$) can be found in the negative differential susceptibility region.

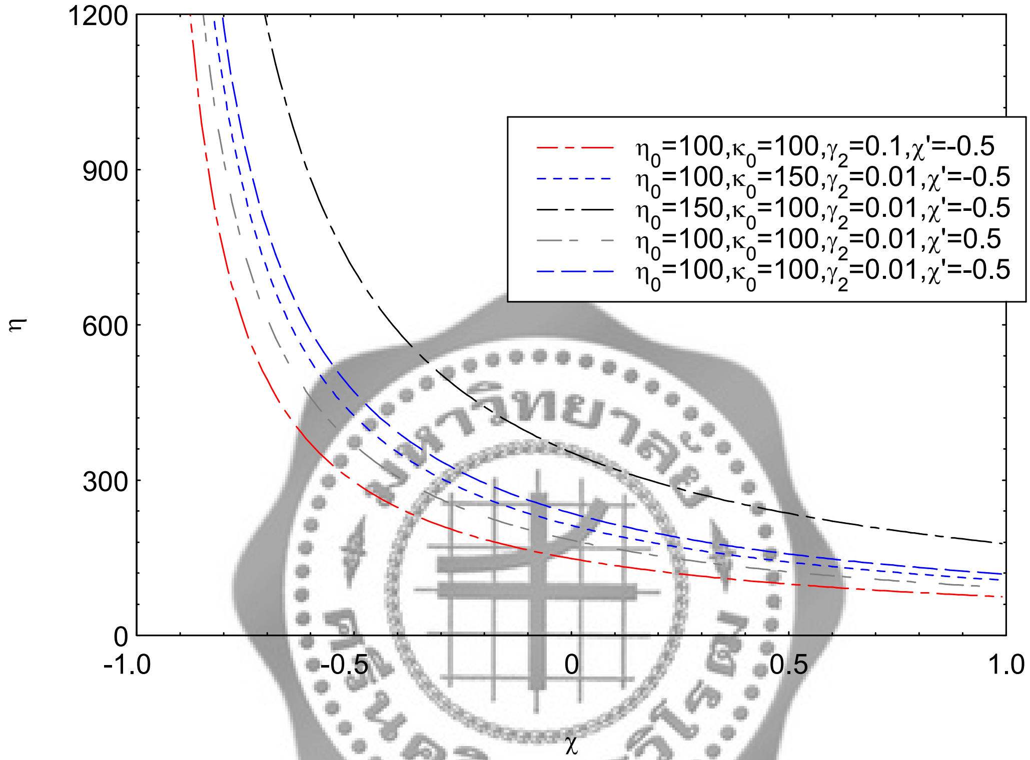


Figure 19 The critical magnetic field ratio (η) versus susceptibility (χ).

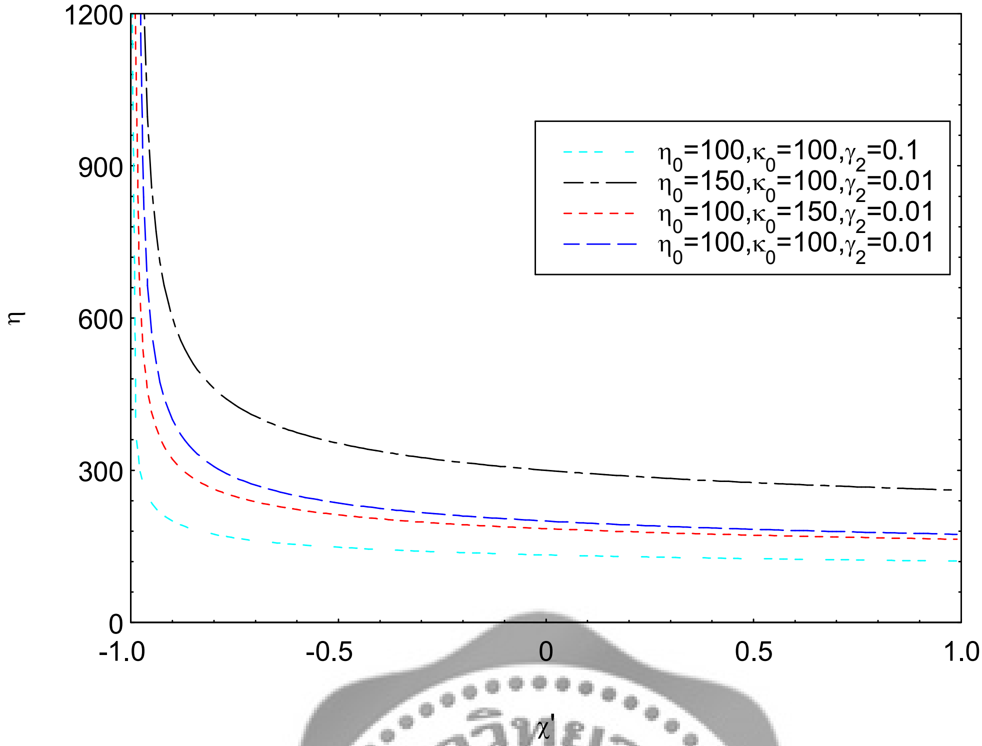


Figure 20. The critical magnetic field ratio(η) versus differential susceptibility (χ') with $\chi = 0$.

2 Temperature-dependence upper critical magnetic field

To investigate the temperature-dependent upper critical magnetic field of two band superconductor, especially in Fe-based superconductors, we modify eq.(3.46) to be the temperature-dependent equation in 4 cases.

Case1. Modify GL theory

Case 1.1) Linear GL theory

$$B_{c2}(t) = B_{c2}(0)(1 - at), \tag{3.49}$$

where a is an arbitrary constant that can be fit out from the experimental data.

Case 1.2) Non-linear GL theory

$$B_{c2}(t) = B_{c2}(0)(1 + at^2) \tag{3.50}$$

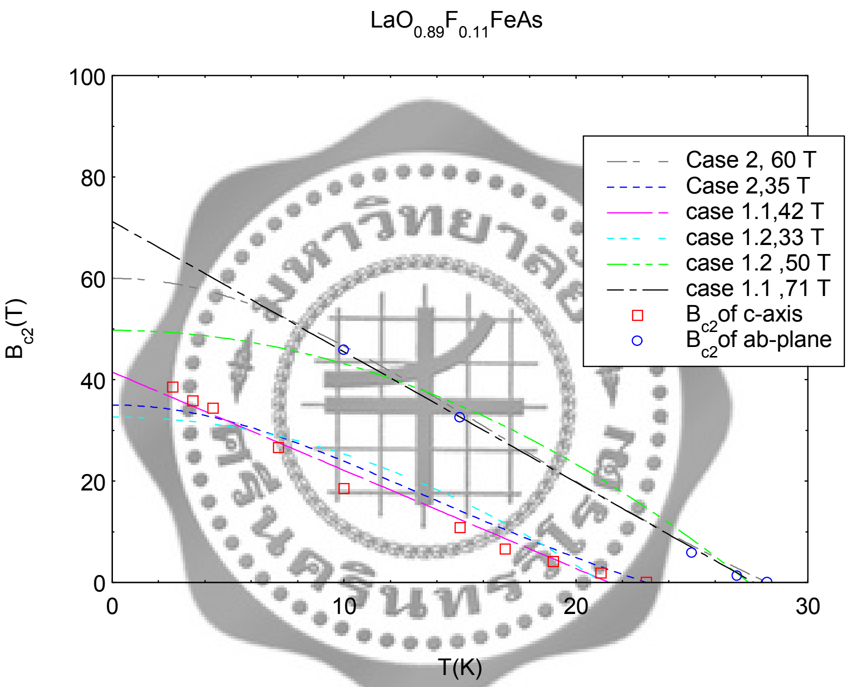
Case 2.The Zhu et al. assumption

$$B_{c2}(t) = B_{c2}(0)\frac{1 - t^2}{1 + t^2} \tag{3.51}$$

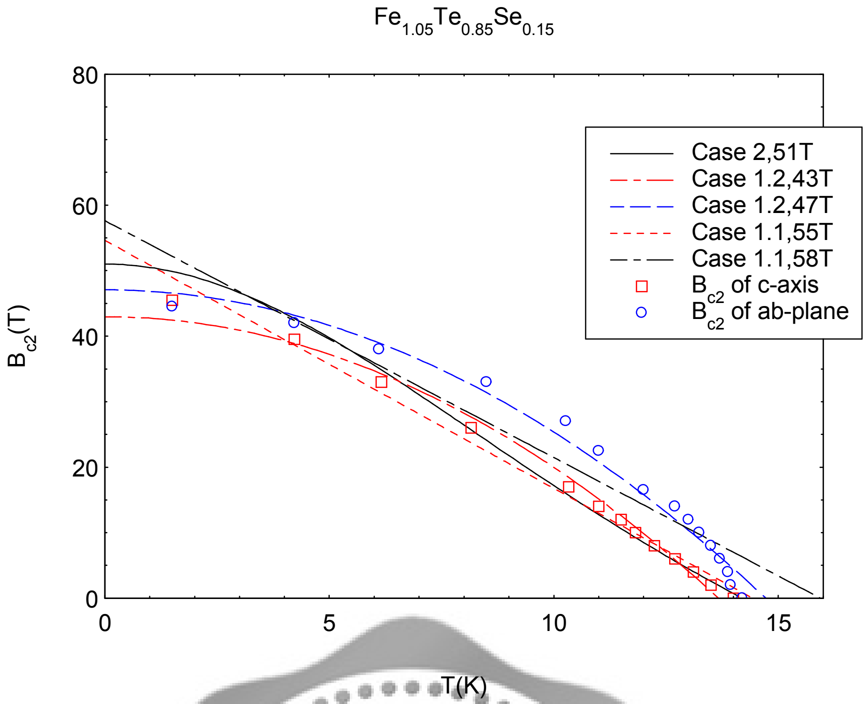
Case 3.The Shanenko et al. assumption

$$B_{c2}(t) = B_{c2}(0)\{p(1-t) + \frac{q}{2}(1-t)^2\} \tag{3.52}$$

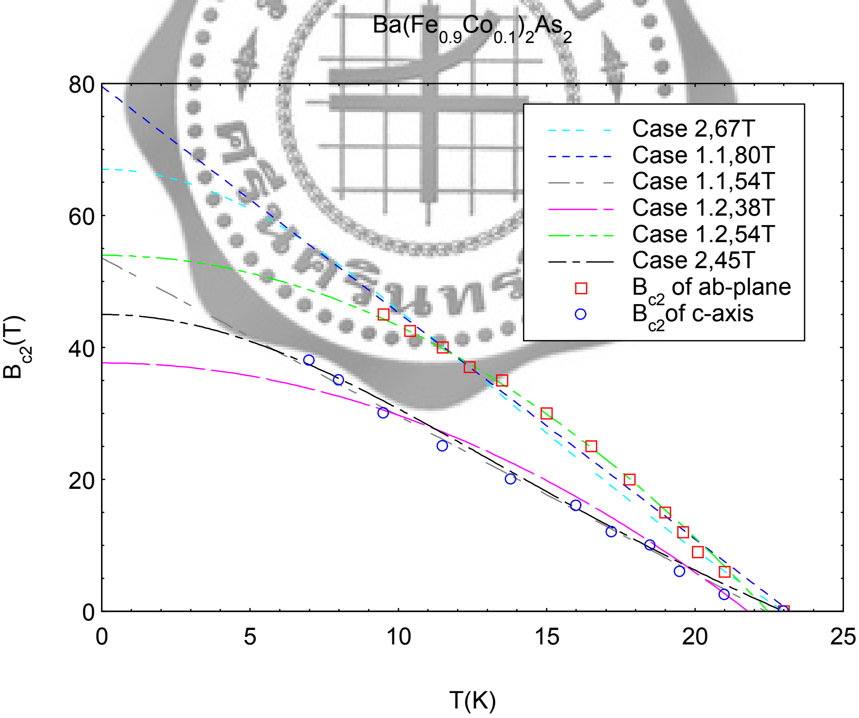
We compare our numerical calculations to the experimental data of $LaO_{0.89}F_{0.11}FeAs$ (Yamamoto et.al. 2008:252501), $Fe_{1.05}Te_{0.85}Se_{0.15}$ (Kida et.al. 2010: 074706), and $Ba(Fe_{0.9}Co_{0.1})As_2$ (Yamamoto et.al. 2009: 062511) as shown in Figure 21. The B_{c2} of c-axis and ab-plane is fitted separately with case 1.1, 1.2 and case 2. The best fit values of zero temperature upper critical magnetic fields are summarized in Table 1.



(a)



(b)



(c)

Figure 21. Comparing of the theoretical curves from Case 1.1, 1.2 and 2 to the experimental data of B_{c2} in c-axis and in ab-plane:(a) $LaO_{0.89}F_{0.11}FeAs$, (b) $Fe_{1.05}Te_{0.85}Se_{0.15}$ and (c) $Ba(Fe_{0.9}Co_{0.1})As_2$. The inset display the case of calculation and the fit of zero-temperature upper critical magnetic field $B_{c2}(0)$.

Table 1 The best fit values of zero temperature upper critical magnetic fields $B_{c2}(0)$ of samples of interested.

Samples	Direction	$B_{c2}(0)$ Tesla	Best fit with case
$LaO_{0.89}F_{0.11}FeAs$	ab-plane	71	1.1
	c-axis	42	1.1
$Fe_{1.05}Te_{0.85}Se_{0.15}$	ab-plane	47	1.2
	c-axis	43	1.2
$Ba(Fe_{0.9}Co_{0.1})As_2$	ab-plane	54	1.2
	c-axis	45	2

From eq.(3.51) (case 2) and eq.(3.52) (case 3), we calculate the temperature-dependence upper critical field showing in figure 22. These curves are constraint with the experimental data that $\frac{B_{c2}(T=0)}{B_{c2}(0)}=1$ and $\frac{B_{c2}(T=T_c)}{B_{c2}(0)}=0$. We consider in the region that $0<T<T_c$ and the case 2 can be included in case 3 depending on the parameters p and q. The experimenter can use eq.(3.51) and eq.(3.52) to predict the zero-temperature upper critical field of the Fe-based superconductors and the other two-band superconductors.

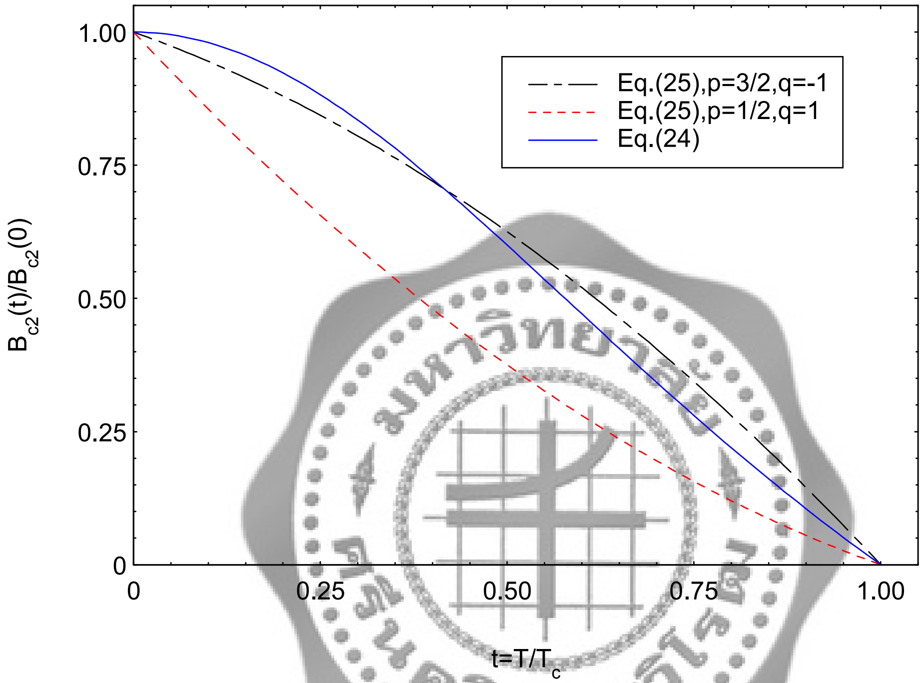


Figure 22. The upper critical magnetic field versus temperature.

CHAPTER 5
CONCLUSIONS DISCUSSION AND SUGGESTION

In this research, we study the critical magnetic field of magnetic superconductors, by using the two-band anisotropic Ginzburg-Landau theory. Research's results are summarized as follows.

1 Conclusions

In one-band calculation, the upper critical field, the lower critical field and the critical magnetic field ratio of anisotropic magnetic superconductors are calculated by Ginzburg–Landau theory analytically. The effects of the Ginzburg–Landau parameter, magnetic susceptibility and magnetic-to-anisotropic parameter ratio on the critical field ratio are considered. We find that the value of critical field ratio increases with increasing Ginzburg–Landau parameter and magnetic-to-anisotropic parameter ratio decreases with increasing magnetic susceptibility. The highest and the lowest value of critical field ratio are found in the diamagnetic superconductors and the ferromagnetic superconductors, respectively.

Based on two-band anisotropic Ginzburg-Landau theory, we calculate analytically the upper and lower critical field and the critical field ratio. The effect of the Ginzburg–Landau parameter, magnetic susceptibility and magnetic to anisotropic parameter ratio on the critical field ratio is considered. Although there are many parameters in two-band model, our results look simple. The arbitrary constants are grouped; temperature-dependent and non temperature-dependent. We make the assumption that they can separated into temperature-dependent and non-dependent parts.

We apply our calculation to Fe-based superconductors that are two-band superconductors. We find that the very high value of critical magnetic field ratio of Fe-base superconductors can be found in the negative differential susceptibility region. The temperature-dependent upper critical magnetic field of our two-band model looks the same as of the one band GL model. The temperature behaviors of the upper critical magnetic field are also studied. The very high value zero- temperature upper critical magnetic fields can be estimated from the experimental data and our temperature-dependent upper critical field equation.

2 Discussion

We calculate one-band and two-band GL theory and apply to magnetic superconductors. In conventional GL theory, they propose the model for one-band s-wave non- magnetic superconductors. Hampshire (Hampshire. 1998: 1-11) extend GL model to describe the one-band magnetic superconductors. However, there are many two-band superconductors found today; MgB_2 and Fe-based superconductors. Especially, Fe-based superconductor is the two-band magnetic superconductors. In this research, we extend the GL theory to describe the two-band magnetic superconductors. We also add the anisotropic parameter in our model. Our model can describe anisotropic two-band magnetic superconductors.

We show the effect of κ_0, χ and θ on the critical magnetic field ratio (η) for anisotropic one-band magnetic superconductors. We use κ_0 = 57, and κ_0 = 118 that are Ginzburg–Landau parameter of Y123 (Z. Hao et al.1991: 2884) and Hg1223 (Mun-Seog Kim et al. 1995:691) superconductors, respectively. Figure 18 shows that the value of critical field ratio does increase with increasing in κ and θ. The value of critical field ratio does decrease with increasing in χ. The highest value of critical field ratio is found in the diamagnetic superconductors (χ < 0) with highly anisotropy and the lowest value is found in the ferromagnetic superconductor (χ > 0) with highly magnetism.

The higher value of critical magnetic field ratio of anisotropic two-band magnetic superconductors may be found in superconductor with diamagnetism ($\chi < 0$) as shown in figure 19. The critical magnetic field ratio versus the differential susceptibility (χ') is investigated and shown in figure 20. The higher value of critical magnetic ratio can be found in the negative differential susceptibility region $(\chi' < 0)$. Then, we can conclude that the very high upper critical magnetic field of Fe-base superconductor ($\chi \approx 0$) can be found in the negative differential susceptibility region.

We compared our numerical calculations to the experimental data of $LaO_{0.89}F_{0.11}FeAs$ (Yamamoto et.al. 2008:252501), $Fe_{1.05}Te_{0.85}Se_{0.15}$ (Kida et.al. 2010: 074706), and $Ba(Fe_{0.9}Co_{0.1})As_2$ (Yamamoto et.al. 2009:252501). The upper critical field of c-axis and ab-plane is fitted separately with cases 1.1, 1.2 and case 2. The best fit value of zero temperature upper critical magnetic field are summarized in Table 1 that the upper critical field of c-axis are 42-45 Tesla and the upper critical field of ab-plane are 47-71 Tesla. The best fit model of all materials is the case 1.2: non-linear GL theory. The Case 1.2 can fit well on $Fe_{1.05}Te_{0.85}Se_{0.15}$ and $Ba(Fe_{0.9}Co_{0.1})As_2$ as show in figure 21(b) and 21(c). In $LaO_{0.89}F_{0.11}FeAs$ material (figure 21(a)), there are less experimental data than the others, and then the best fit curve is the linear model. If we ignore the curve fit of $LaO_{0.89}F_{0.11}FeAs$ material, the upper critical field of c-axis are 43-45 Tesla and the upper critical field of ab-plane are 47-54 Tesla that are in the range of Zhu et al. report (Zhu et al.2008:105001).

3 Suggestion

1. In our calculation, we consider Fe-based superconductors that are the two-band superconductors. There are many experiments claim that there are more than two-band in these materials but the dominate properties is two-band superconductors. However, the more than two-band model must be considered to improve the result of calculation.

2. The temperature-dependent parameter "α" is the assumption in our model. To complete the calculation, the exact calculation of "α" should be done.

3. More experimental data is needed in order to make a decision on the validity of our temperature-dependent model.

BIBLIOGRAPHY

BIBLIOGRAPHY

Askerzade, Ì. N.;Gencer A.; & Güçlü N. (2002).“On the Ginzbueg – Landau analysis of the upper critical field H_{C2} in MgB_2”, *Superconductor,Science&Technology*. **15** : L13 – L16.

Askerzade, Ì.; N.;Gencer A.; Güçlü N.; & Kiliç A. (2002). “Two - band Ginzburg – Landau theory for the lower critical field H_{C1} in MgB_2”, *Superconductor, Science&Technology*. **15** : L17 – L20.

Bardeen, J.; Cooper, N.; & Schrieffer, J.R. (1959). “Theory of Superconductivity”, *Physical Review*. **108**(5) :175 – 1204.

Bednorz,J.G.; & Muller,K.A. (1986). “Possible high T_c superconductivity in the Ba−La−Cu−O system”. *Z. Physik, B* **64** (1): 189–193.

Bouquet, F; et al.(2002). “Specific Heat of Single Crystal MgB_2: A Two-Band Superconductor with Two Different Anisotropies”. *Phys. Rev. Lett*. **89**: 257001

Buckel, W. (1991). “*Superconductivity*”. New York ;VHC.

Changjan, A.; & Udomsamuthirun, P. (2010). “Critical magnetic field ratio of anisotropic magnetic superconductors”, *Physica C: Superconductivity* **471**: 23-25.

Changjan, A.; & Udomsamuthirun, P. (2011). “The critical magnetic field of anisotropic two- band magnetic superconductors”, *Solid State Communications* **151**: 988-992

Chen, X. K.; et al.(2001). “Evidence for Two Superconducting Gaps in MgB_2”. *Phys. Rev. Lett.* **87**:157002.

Doh, H.; et al. (1999). “Phenomenological Theory of Superconductivity and Magnetism in $Ho_{1-x}Dy_xNi_2B_2C$”, *Physical Review Letter* **83** (25) : 5350 – 5353.

Fetter, A.L.; & Walecka.J.D. (1995). “*Quantum theory of Many – Particle system*”. Taiwan ; MaGraw – Hill.

Fischer-Cripps,A.C.(2008). “*The Materials Physics Companion*”. New York ; Taylor & Francis.

Haas, S.; & Maki, K. (2001). “Anisotropic s – wave superconductivity in MgB_2”, *Phys. Rev. B* **65**, 020502(R).

Hampshire, D.P. (1998). “Ferromagnetic and antiferromagnetic superconductivity”, *Physica C: Superconductivity* **304** :1-11.

Hao, Z.;et al.(1991). “Model for the reversible magnetization of high-κ type-II superconductors: Application to high-T_c superconductors”, *Phys. Rev. B* **43**:2844.

Kamihara, Y.; et al.(2008). “Iron-Based Layered Superconductor $La[O_{1-x}F_x]FeAs$ (x = 0.05−0.12) with T_c = 26 K”, *J. Am. Chem. Soc.***130**(11): 3296–3297.

Ketterson, J.B.; & Song, S.N. (1998) “*Superconductivity*” : Cambridge.

Kida, T. ; et.al. (2010) “Weak superconducting fluctuations and small anisotropy of the upper critical fields in an $Fe_{1.05}Te_{0.85}Se_{0.15}$ single crystal”, *Phys. Soc. Jpn.*, **79** :074706

Kitel, C. (1997) “*Introduction to solid state Physics*”. 7th edition; John Wiley & Sons.

Mun-Seog Kim.;et al.(1995). “Study of the Thermodynamic Parameters of $HgBa_2Ca_2Cu_3O_{8-x}$ by Using Hao and Clem's Model”, *Chinese J. Phys.* **33**:691

Nakamura, H; et al.(2008). “First-Principle Electronic Structure Calculations for Iron-Based Superconductors: An LSDA+U Study”, *Journal of the Physical Society of Japan.* **77** : 153-154.

Omar, M.A. (1975). "*Elementary Solid State Physics* :*Principles and Applications*". Singpore: Addison-Wesley.

Paul, C.C.; & George, W.C. (2003). "Magnesium Diboride: Better Late than Never" , *Physics Today* : **37**

Poulter, J. (1991). "*Lectures on Ginzburg – Landau (GL) Theory in The Theory of Superconductivity* " ; Faculty of Science Chulalongkorn University.

Ren,Z.A.; et al.(2008). "Superconductivity at 55K in Iron-Based F-Doped Layered Quaternary Compound $Sm[O_{1-x}F_x]FeAs$", *Chin.Phys.Lett.* **25**(6): 2215-2216

Shanenko, A.A.; et al.(2011). "Extended Ginzburg-Landau Formalism for Two-Band Superconductors", *Phys. Rev. Lett.* **106**(4): 047005

Singh, J. (1996). "*Quantum Mechanics : Fundamentals & applications to Technology*". USA; Wiley Interscience.

Tsuda, S.; et al.(2001). "Evidence for a Multiple Superconducting Gap in MgB_2 from High-Resolution Photoemission Spectroscopy", *Phys. Rev. Lett*. **87**: 177006.

Udomsamuthirun, P.; et al.(2006). "H_{c2} of anisotropy two-band superconductors by Ginzburg–Landau approach ", *Physica C: Superconductivity* **434** (1): 62-66.

Ummarino G.A.(2010). "The Phenomenology of Iron Pnictides Superconductors Explained in the Framework of s-Wave Three-Band Eliashberg Theory" *Advances in Condensed Matter Physics*. **2010**:167985

Yamamoto, A.; et.al.(2008) "Evidence for electromagnetic granularity in the polycrystalline iron-based superconductor $LaO_{0.89}F_{0.11}FeAs$", *Applied Physics Letters* **92**, 252501.

Yamamoto, A.; et.al.(2009) "Small anisotropy, weak thermal fluctuations, and high field superconductivity in Co-doped iron pnictide $Ba(Fe_{1-x}Co_x)_2As_2$", *Applied Physics Letters* **94**, 062511.

Zhu, X.; et al.(2008). "Upper critical field, Hall effect and magnetoresistance in the iron-based layered superconductor $LaFeAsO_{0.9}F_{0.1-\delta}$", *Supercond. Sci. Technol.* **21**:105001.

Appendix

Appendix A

Appendix A

The 1st Ginzburg-Landau equation

From eq.(2.7) and eq.(2.8), we get the free energy density

$$g_s = f_n + a|\psi|^2 + \frac{1}{2}b|\psi|^4 + \psi^* \frac{1}{2m^*}\left(-i\hbar\vec{\nabla} + \frac{e^*}{c}\vec{A}\right)^2 \psi + \frac{h^2}{8\pi} - \frac{\vec{h}\cdot\vec{H}}{4\pi}. \qquad (2.9)$$

The free energy is

$$G_s = \int d\vec{r}\, g_s . \qquad (2.10)$$

The variation of G_s with respect to ψ^* is

$$\delta G_s = \int d\vec{r}\, \frac{\partial g_s}{\partial \psi^*}\delta\psi^* .$$

From $\frac{\partial g_s}{\partial \psi^*} = 0$, let consider

$$\frac{1}{2m^*}\int\left|\left(-i\hbar\vec{\nabla} + \frac{e^*}{c}\vec{A}\right)\psi\right|^2 d\vec{r}$$

$$= \frac{1}{2m^*}\int d\vec{r}\left[\hbar^2\vec{\nabla}\psi^*\cdot\vec{\nabla}\psi + i\hbar\frac{e^*}{c}\vec{A}\psi\cdot\vec{\nabla}\psi^* - i\hbar\frac{e^*}{c}\vec{A}\psi^*\cdot\vec{\nabla}\psi + \frac{e^{*2}}{c^2}A^2|\psi|^2\right]. \qquad (A.1)$$

From the divergence theorem is

$$\int_v d\vec{r}\,\vec{\nabla}\cdot\vec{P} = \oint_s \vec{P}\cdot d\vec{s} ,$$

where s is the closed surface enclosing the volume v. Let $\vec{P} = f\vec{Q}$, where f is scalar quantity and $\vec{P},\vec{Q}$ are vector quantity. We get

$$\vec{\nabla}\cdot\vec{P} = \vec{\nabla}\cdot f\vec{Q} = f\vec{\nabla}\cdot\vec{Q} + \vec{Q}\cdot\vec{\nabla}f$$

then

$$\int_v d\vec{r} f\vec{\nabla}\cdot\vec{Q} + \int_v d\vec{r}\,\vec{Q}\cdot\vec{\nabla}f = \oint_s f\vec{Q}\cdot d\vec{s} .$$

Let $f = \psi^*$ and $\vec{Q} = \vec{\nabla}\psi$, we get

$$\int_v d\vec{r}\,\psi^*\nabla^2\psi + \int_v d\vec{r}\,\vec{\nabla}\psi\cdot\vec{\nabla}\psi^* = \oint_s \psi^*\vec{\nabla}\psi\cdot d\vec{s} \qquad (A.2)$$

Let $f = \psi^*$ and $\vec{Q} = \vec{A}\psi$, we get

$$\int_v d\vec{r}\,\psi^*\vec{\nabla}\cdot\vec{A}\psi + \int_v d\vec{r}\,\vec{A}\psi\cdot\vec{\nabla}\psi^* = \oint_s \psi^*\vec{A}\psi\cdot d\vec{s} \qquad (A.3)$$

Substitution eq.(A.2) and eq.(A.3) in (A.1), we have

$$\frac{1}{2m^*}\int\left|\left(-i\hbar\vec{\nabla} + \frac{e^*}{c}\vec{A}\right)\psi\right|^2 d\vec{r}$$

$$= \frac{1}{2m^*}\int d\vec{r}\left[-\hbar^2\psi^*\nabla^2\psi - i\hbar\frac{e^*}{c}\psi^*\vec{\nabla}\cdot\vec{A}\psi - i\hbar\frac{e^*}{c}\vec{A}\psi^*\cdot\vec{\nabla}\psi + \frac{e^{*2}}{c^2}A^2|\psi|^2\right]$$

$$+ \frac{1}{2m^*}\oint_s\left[\hbar^2\psi^*\vec{\nabla}\psi + \frac{i\hbar e^*}{c}\psi^*\vec{A}\psi\right]\cdot d\vec{s} . \qquad (2.11)$$

From

$$\left(-i\hbar\vec{\nabla}+\frac{e^*}{c}\vec{A}\right)^2=-\hbar^2\nabla^2-\frac{i\hbar e^*}{c}\vec{\nabla}\cdot\vec{A}-\frac{ie^*\hbar}{c}\vec{A}\cdot\vec{\nabla}+\frac{e^*}{c^2}A^2,$$

we can be rewritten as

$$\frac{1}{2m^*}\int\left|\left(-i\hbar\vec{\nabla}+\frac{e^*}{c}\vec{A}\right)\psi\right|^2 d\vec{r}$$

$$=\frac{1}{2m^*}\int_v d\vec{r}\left[\psi^*\left(-i\hbar\vec{\nabla}+\frac{e^*}{c}\vec{A}\right)^2\psi\right]+\frac{i\hbar}{2m^*}\oint_s\left[\psi^*(-i\hbar\vec{\nabla}+\frac{e^*}{c}\vec{A})\psi\right]\cdot d\vec{s}$$

Using the surface condition as

$$\hat{n}\cdot\left(-i\hbar\vec{\nabla}+\frac{e^*}{c}\vec{A}\right)\psi=0, \tag{2.12}$$

for all points on the surface of the superconductor with normal vector $\hat{n}$.
Then, we get the 1st Ginzburg-Landau equation

$$\frac{1}{2m^*}\left(-i\hbar\vec{\nabla}+\frac{e^*}{c}\vec{A}\right)^2\psi+a\psi+b|\psi|^2\psi=0. \tag{2.13}$$

Appendix B

Appendix B

The 2nd Ginzburg-Landau equation

Consider the variation of eq.(2.9) with respect to $\vec{A}$. Since the free energy density is a function of $\vec{A}$ and its spatial derivatives $\left(\vec{h} = \vec{\nabla} \times \vec{A}\right)$, we need to consider

$$\int d\vec{r} \left[\frac{\partial g_s}{\partial A_i} \delta A_i + \sum_i \frac{\partial g_s}{\partial \left(\frac{\partial A_i}{\partial i} \right)} \frac{\partial (\delta A_i)}{\partial i} \right] = 0 \; . \tag{2.14}$$

We have three equations since $i = 1,2,3$ (or x,y,z). The second term, after integrated by parts, is

$$\left. \sum_i \frac{\partial g_s}{\partial \left(\frac{\partial A_i}{\partial i} \right)} \cdot \delta A_i \right|_s - \int d\vec{r} \sum_i \frac{\partial}{\partial i} \left[\frac{\partial g_s}{\partial \left(\frac{\partial A_i}{\partial i} \right)} \right] \delta A_i \; .$$

From surface condition the 1st term vanishes, then

$$\int d\vec{r} \left(\frac{\partial g_s}{\partial A_i} - \sum_i \frac{\partial}{\partial i} \left[\frac{\partial g_s}{\partial \left(\frac{\partial A_i}{\partial i} \right)} \right] \delta A_i \right) = 0 \tag{B.1}$$

and

$$\frac{\partial g_s}{\partial A_i} = \sum_i \frac{\partial}{\partial i} \left[\frac{\partial g_s}{\partial \left(\frac{\partial A_i}{\partial i} \right)} \right].$$

From

$$\frac{\partial g_s}{\partial A_i} = \frac{1}{2m^*} \left(i\hbar \frac{e^*}{c} \psi \frac{\partial \psi^*}{\partial i} - i\hbar \frac{e^*}{c} \psi^* \frac{\partial \psi}{\partial i} + 2 \frac{e^{*2}}{c^2} A |\psi|^2 \right) \tag{B.2}$$

and

$$\vec{h} = \vec{\nabla} \times \vec{A} = \hat{x} \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{y} \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{z} \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right), \tag{B.3}$$

we get

$$\frac{h^2}{8\pi} - \frac{\vec{h} \cdot \vec{H}}{4\pi} = \frac{1}{8\pi} \left[\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right)^2 + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right)^2 + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)^2 \right]$$

$$- \frac{1}{4\pi} \left[\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) H_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) H_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_X}{\partial Y} \right) H_Z \right].$$

Then

$$\frac{\partial g_s}{\partial\left(\frac{\partial A_x}{\partial x}\right)} = 0,$$

$$\frac{\partial g_s}{\partial\left(\frac{\partial A_x}{\partial y}\right)} = -\frac{1}{4\pi}\left(h_z - H_z\right),$$

$$\frac{\partial g_s}{\partial\left(\frac{\partial A_x}{\partial z}\right)} = -\frac{1}{4\pi}\left(h_y - H_y\right)$$

and

$$\frac{\partial}{\partial x}\left[\frac{\partial g_s}{\partial\left(\frac{\partial A_x}{\partial x}\right)}\right] + \frac{\partial}{\partial y}\left[\frac{\partial g_s}{\partial\left(\frac{\partial A_x}{\partial y}\right)}\right] + \frac{\partial}{\partial z}\left[\frac{\partial g_s}{\partial\left(\frac{\partial A_x}{\partial z}\right)}\right]$$

$$= -\frac{1}{4\pi}\frac{\partial}{\partial y}\left(h_z - H_z\right) + \frac{1}{4\pi}\frac{\partial}{\partial z}\left(h_y - H_y\right)$$

$$= -\frac{1}{4\pi}\left[\vec{\nabla}\times\left(\vec{h} - \vec{H}\right)\right]_x .$$

Then, the other component can write in form

$$\sum_i \frac{\partial}{\partial i}\left[\frac{\partial g_s}{\partial\left(\frac{\partial A_i}{\partial i}\right)}\right] = -\frac{1}{4\pi}\left[\vec{\nabla}\times\left(\vec{h} - \vec{H}\right)\right]_i . \tag{B.4}$$

Rewritten eq.(B.2) and eq.(B.4) in vector form as

$$\frac{1}{2m^*}\left(i\hbar\frac{e^*}{c}\psi\vec{\nabla}\psi^* - i\hbar\frac{e^*}{c}\psi^*\vec{\nabla}\psi + 2\frac{e^{*2}}{c^2}\vec{A}|\psi|^2\right) = -\frac{1}{4\pi}\vec{\nabla}\times\left(\vec{h} - \vec{H}\right)$$

$$-\frac{1}{4\pi}\vec{\nabla}\times\left(\vec{h} - \vec{H}\right) = \frac{e^*\hbar}{2m^* i}\left(\psi^*\nabla\psi - \psi\vec{\nabla}\psi^*\right) + \frac{e^{*2}}{m^* c}\vec{A}|\psi|^2 . \tag{2.15}$$

We will consider the external magnetic field to be uniform that we have $\vec{\nabla}\times\vec{H} = 0$.

From Maxwell's equation the current density is given by $\vec{j} = \frac{c}{4\pi}\vec{\nabla}\times\vec{h}$.

Now, we get

$$\vec{j} = -\frac{e^*\hbar}{2m^* i}\left(\psi^*\nabla\psi - \psi\vec{\nabla}\psi^*\right) - \frac{e^{*2}}{m^* c}\vec{A}|\psi|^2 . \tag{2.16}$$

This is the 2nd Ginzburg-Landau equation.

INDEX

INDEX

AUTHOR INDEX

AUTHOR INDEX

VITAE

VITAE

Name: Arpapong Changjan
Date of Birth: 26 April 1979
Place of Birth: Nonthaburi, Thailand
Address: 1792/70 Prachasongkroh Rd., Dindang, Bangkok, Thailand. 10400
Current Position: Instructor
Work Place: Phathumwan institute of technology, Bangkok, Thailand.

Educational Background:

2001 B.Sc. (Physics) from Silpakorn University
2005 M.Sc. (Physics) from Srinakharinwirot University
2011 Ph.D. (Physics) from Srinakharinwirot University

Conference Presentation:

2011 Oral presentation for "Investigate the upper critical magnetic field of Fe-based superconductors" at The 2nd International Conference on Applied Science (ICAS) & The 3 rd International Conference on Science and Technology for Sustainable Development of the Greater Mekong Sub-region (STGMS) Souphanouvong University, Luang Prabang, Lao PDR. 24-25 March, 2011.

Publications:

2006 P. Udomsamuthirun, A. Changjan , C. Kumvongsa , S. Yoksan, "H_{c2} of anisotropy two-band superconductors by Ginzburg–Landau approach", *Physica C* **434** (1): 62-66.

2010 A. Changjan. and P. Udomsamuthirun, "Critical magnetic field ratio of anisotropic magnetic superconductors", *Physica C:* **471**: 23-25.

2011 A. Changjan. and P. Udomsamuthirun, "The critical magnetic field of anisotropic two- band magnetic superconductors", *Solid State Communications* **151**: 988-992.