

**THE DEVELOPMENT OF A PROBLEM-SOLVING INSTRUCTIONAL PROGRAM
TO DEVELOP PRESERVICE TEACHERS' COMPETENCE
IN SOLVING MATHEMATICAL PROBLEMS
AND THEIR BELIEFS RELATED TO PROBLEM SOLVING**

**A DISSERTATION
BY
RUNGFA JANJARUPORN**

**Presented in Partial Fulfillment of the Requirements for the
Doctor of Education Degree in Mathematics Education
at Srinakharinwirot University**

May 2005

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The purpose of the study was to develop a problem-solving instructional program for mathematics preservice teachers and study its effects on competence in solving mathematical problems, beliefs related to problem solving, and sociomathematical norms. This program provided the preservice teachers to learn the process of problem solving, and various problem-solving strategies. The program based on the belief that preservice teachers themselves should solve lots of problems and solve problems with more than one strategy. Posttest-only control group design was used for this study. Twenty-six second-year students in the mathematics teacher education program of Srinakharinwirot University were subjects for this study, thirteen students in each group, control group and experimental group. Three out of 13 students of the experimental group were purposefully chosen as target students for case study.

Results of the study were as follows :

1. The median score on problem-solving competence of the experimental group was significantly higher than that of the control group at the 0.05 level.
2. The median score on appropriate beliefs related to problem solving of the experimental group was not significantly higher than that of the control group at the 0.05 level.
3. Sociomathematical norms emerged in this problem-solving classroom were normative understanding of what counts as a reasonable solution process, a sophisticated representation, and an acceptable mathematical explanation and justification. The experimental preservice teachers agreed on that a mathematical solution process was called '*a reasonable solution process*' if that solution process used correct mathematical principles, facts, properties or theories to support each procedure, and the procedure was logical reasoning with connection between each other. The representation was called '*a sophisticated representation*' if that mathematical representation used appropriate mathematical symbols or signs along with clear and systematic representation. The

explanation was called '*an acceptable mathematical explanation and justification*' if the language of mathematics was precise, and correct.

4. While this program was conducted, there were evidences that when the experimental preservice teachers worked on more problems, they spent more time on trying to understand the problem, analyzing the problems, and group discussion before attempting to solve it. On the aspect of strategies in solving problems, the preservice teachers made clearer drawings or diagrams, and had more systematic on accounting for all possibilities. They also defined variables, set equation more appropriately. They gave more logical reasoning to convince for their arguments. On the aspect of solution process, the preservice teachers gave more reasons to support their written explanation of solution process and working systematic on solution process. These performances supported the finding that the preservice teachers who had experience on the problem-solving instructional program could develop their competence in solving mathematical problems.

การพัฒนาโปรแกรมการเรียนการสอนการแก้ปัญหา
เพื่อพัฒนาศักยภาพในการแก้ปัญหาทางคณิตศาสตร์ และความเชื่อที่เกี่ยวข้อง
กับการแก้ปัญหาของนักศึกษาครูคณิตศาสตร์

บทคัดย่อ
ของ
รุ่งฟ้า จันทจุฑาภรณ์

เสนอต่อบัณฑิตวิทยาลัย มหาวิทยาลัยศรีนครินทรวิโรฒ เพื่อเป็นส่วนหนึ่ง
ของการศึกษาตามหลักสูตรปริญญาการศึกษาดุขบัณฑิต
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ดร.ปรีชา เนาว์เย็นผล

การวิจัยนี้มีวัตถุประสงค์เพื่อพัฒนาและศึกษาผลของโปรแกรมการเรียนการสอนการแก้
ปัญหาสำหรับนักศึกษาครุคณิตศาสตร์ที่มีต่อศักยภาพในการแก้ปัญหาทางคณิตศาสตร์ ความเชื่อที่
เกี่ยวข้องกับการแก้ปัญหา และบรรทัดฐานทางสังคมที่เกี่ยวกับการทำกิจกรรมทางคณิตศาสตร์ ใน
โปรแกรมนี้ นักศึกษาครูได้เรียนรู้กระบวนการแก้ปัญหาและยุทธวิธีการแก้ปัญหาที่หลากหลาย
โปรแกรมนี้อยู่บนความเชื่อที่ว่า นักศึกษาครูควรมีโอกาสลงมือแก้ปัญหาเป็นจำนวนมากและได้ลง
มือแก้ปัญหามากกว่าหนึ่งยุทธวิธี โดยมีรูปแบบการวิจัยแบบมีกลุ่มควบคุมและการทดสอบหลังเรียน
เท่านั้น กลุ่มตัวอย่างที่ใช้ในการวิจัยนี้ได้แก่นักศึกษาชั้นปีที่ 2 ในหลักสูตรการศึกษาบัณฑิต (กศ.บ.)
สาขาวิชาคณิตศาสตร์ ของมหาวิทยาลัยศรีนครินทรวิโรฒ จำนวน 26 คน โดยแบ่งเป็นกลุ่มทดลอง
และกลุ่มควบคุมที่มีความสามารถใกล้เคียงกัน กลุ่มละ 13 คน และมีนักศึกษา 3 คนในกลุ่มทดลอง
เป็นนักศึกษาเป้าหมายเพื่อใช้เป็นกรณีศึกษา

ผลการวิจัยพบว่า

1. คะแนนมาตรฐานในด้านศักยภาพในการแก้ปัญหาทางคณิตศาสตร์ของกลุ่มทดลองสูง
กว่าคะแนนมาตรฐานของกลุ่มควบคุมอย่างมีนัยสำคัญทางสถิติ
2. คะแนนมาตรฐานในด้านความเชื่อที่เหมาะสมเกี่ยวข้องกับการแก้ปัญหาของกลุ่ม
ทดลองไม่สูงกว่าคะแนนมาตรฐานของกลุ่มควบคุมอย่างมีนัยสำคัญทางสถิติ
3. บรรทัดฐานทางสังคมที่เกี่ยวกับการทำกิจกรรมทางคณิตศาสตร์ที่เกิดขึ้นในชั้นเรียน
ได้แก่ บรรทัดฐานที่เกี่ยวกับกระบวนการหาผลเฉลยที่สมเหตุสมผล การนำเสนอที่สง่างาม การ
อธิบายและการให้เหตุผลที่ยอมรับได้ ซึ่งนักศึกษาครูในกลุ่มทดลองได้ตกลงกันว่า กระบวนการหา
ผลเฉลยเรียกว่าเป็น “กระบวนการหาผลเฉลยที่สมเหตุสมผล” ถ้ามีการใช้หลักการทางคณิตศาสตร์ที่
ถูกต้อง มีข้อเท็จจริงสมบัติหรือทฤษฎีบทสนับสนุนทุกขั้นตอน และในระหว่างขั้นตอนมีเหตุผล
เชิงตรรกะเชื่อมโยงด้วย สำหรับการนำเสนอผลเฉลยนั้นเรียกว่าเป็น “การนำเสนอที่สง่างาม” ถ้ามีการ
ใช้สัญลักษณ์ทางคณิตศาสตร์ที่เหมาะสม ชัดเจนและมีระบบ และการอธิบายผลเฉลยเรียกว่าเป็น
“การอธิบายและให้เหตุผลที่ยอมรับได้” ถ้าภาษาทางคณิตศาสตร์ที่ใช้นั้นเที่ยงตรงและถูกต้อง
4. ระหว่างที่โปรแกรมนี้ดำเนินการอยู่ เมื่อนักศึกษาครูในกลุ่มทดลองมีประสบการณ์แก้
ปัญหามากขึ้น นักศึกษาครูได้ใช้เวลามากขึ้นในการทำความเข้าใจปัญหา วิเคราะห์ปัญหา และ
อภิปรายกลุ่มก่อนที่จะลงมือแก้ปัญหา ในการปรับและเลือกใช้ยุทธวิธีแก้ปัญหานั้น นักศึกษาครู

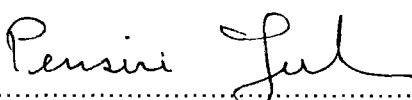
สามารถสร้างแผนภาพประกอบหรือแผนผังได้ชัดเจนขึ้น มีระบบในการแจกแจงกรณีที่เป็นไปได้ทั้งหมดและมีการนิยามตัวแปรและสมการที่เหมาะสมขึ้น อีกทั้งยังมีการใช้เหตุผลเชิงตรรกมาสนับสนุนมากขึ้นด้วย สำหรับกระบวนการหาผลเจลยนั้นนักศึกษาครูมีการนำเหตุผลมาสนับสนุนกระบวนการหาผลเจลยมากขึ้น อีกทั้งมีระบบในกระบวนการหาผลเจลยมากขึ้น ซึ่งพฤติกรรมเหล่านี้สนับสนุนผลการวิจัยที่ว่านักศึกษาครูซึ่งมีประสบการณ์ในโปรแกรมการเรียนการสอนการแก้ปัญหาสามารถพัฒนาศักยภาพในการแก้ปัญหาทางคณิตศาสตร์

**The Institute for the Promotion of Teaching Science and Technology
and Srinakharinwirot University Awarded the Scholarship
for this Dissertation**

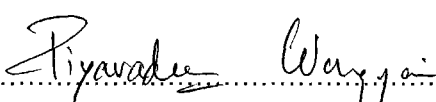
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and Their Beliefs Related to Problem Solving”

by
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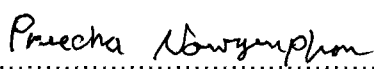
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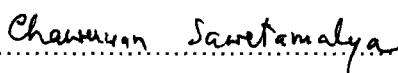

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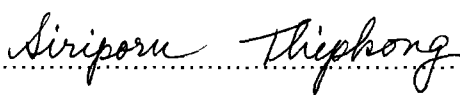
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TABLE OF CONTENTS

Chapter	Page
1 INTRODUCTION	1
Background	1
Research Questions	3
Purposes of the Study	3
Significance of the Study	4
Scope of the Study	4
Definitions	5
Research Hypotheses	7
2 REVIEW OF THE LITERATURE	8
Part 1: Literature on Mathematical Problem Solving	9
Meaning of a Mathematical Problem and Problem Solving	9
Types of Mathematical Problems	10
Process of Problem-Solving	12
Problem-Solving Strategies	15
Beliefs Related to Problem Solving	19
Factors Influencing Success Problem Solving	22
Part 2: Literature on Problem -Solving Instruction	24
Goals for Teaching Problem Solving	24
Tasks and Materials for Problem Solving	26
Teaching Approaches for Problem Solving	28
Teacher Roles in Problem-Solving Instruction	32
Organizing and Implementing Problem-Solving Instruction	33
Sociomathematical Norms	34
Evaluating Progress in Problem Solving	35
Research Related to Problem-Solving Instruction	43

TABLE OF CONTENTS (continued)

Chapter	Page
3 METHODOLOGY	46
Selecting Sample and Research Design	46
Developing the Framework of the Instructional Program	47
Developing Instruments	54
Experimenting and Collecting Data	56
Analyzing the Data	59
4 FINDINGS	60
Part 1: Descriptive Statistics for the Studied Variables	60
Part 2: Effects on Competence in Solving Mathematical Problem	62
Comparison of Preservice Teachers' Competence in Solving Mathematical Problems	62
Analysis of Experimental Students' Problem-Solving Competence	63
Part 3: Effects on Beliefs Related to Problem Solving	88
Comparison of Preservice Teachers' Beliefs Related to Problem Solving	88
Comparison between Pre- and Post- Assessment of the Experimental group	89
Part 4: Effects on Sociomathematical Norms	90
5 CONCLUSION AND DISUSSION.....	94
Summary of the Study	95
Conclusion and Discussion	96
Suggestions	100
BIBLIOGRAPHY.....	102

TABLE OF CONTENTS (continued)

Chapter	Page
APPENDIX	107
APPENDIX A : Name of the Experts in this Study	108
APPENDIX B : An Example of Lesson Plan	110
APPENDIX C : Examples of Problem	116
APPENDIX D : Posttest on Problem Solving	121
APPENDIX E : Questionnaire on Beliefs Related to Problem Solving	126
APPENDIX F : Observation Forms	131
APPENDIX G : Data Analysis	134
CURRICULUM VITAE	141

LIST OF TABLES

Table	Page
1 Schedule of lesson plans with major instructional activities	52
2 Schedule for problem-solving instruction	58
3 Descriptive statistics for scores on problem-solving competence of the experimental group and the control group	60
4 Descriptive statistics for scores on appropriate beliefs related to problem solving of the experimental group and the control group	61
5 Results of comparing the median scores on problem-solving competence of the experimental group and that of the control group	62
6 Strategies students used	73
7 Results of comparing the median scores on appropriate beliefs related to problem solving of the experimental group and that of the control group	88
8 Summary of sociomathematical norms emerged in the problem-solving classroom	93

LIST OF FIGURES

Figure		Page
1	Linear models of problem-solving process found in textbooks	14
2	The dynamic problem-solving process	15
3	An example of a problem-solving observation comment card	36
4	An example of a problem-solving observation checklist	36
5	An example of a problem-solving observation rating scale	37
6	An example of the question guide for a student report	38
7	An example of an attitude inventory	39
8	An example of a problem-solving strategy inventory	40
9	An example of an analytic scoring scale	41
10	An example of a holistic scoring scale	42
11	The framework of the problem-solving instructional program	47
12	The dynamic problem-solving process promoted in problem-solving activity.	48
13	The cycle of developing lesson plans	51
14	The lesson in each group problem solving	53
15	Target students' answer on the digit 8 and + problem	64
16	Chaiwat's written work on the mystery number problem	64
17	Chaiwat's underlined part on the digit 8 and + problem	65
18	Kanda's underlined part on the digit 8 and + problem	65
19	Target students' underlined parts on the mystery number problem	66
20	Nunya's underlined parts on the apple puzzle problem	67
21	Chaiwat's underlined parts on the apple puzzle problem	67
22	Chaiwat's restate the problem on the apple puzzle problem	68
23	Kanda's diagram on the magic traffic light problem	68
24	Nunya's used formula on the magic traffic light problem	68
25	Chaiwat's diagram on the locker puzzle problem	69
26	Chaiwat group's diagram on the green tea puzzle problem	75
27	Kanda group's drawing on the perfect square problem	75

LIST OF FIGURES (continued)

Figure		Page
28	Nunya group's drawing on the magic traffic light problem	76
29	Kanda's answers on the digit 8 and + problem	77
30	Chaiwat's diagram on the perfect square problem	78
31	Chaiwat group's solution on the apple puzzle problem	79
32	Nunya group's writing equations on the green tea puzzle problem	80
33	Nunya group's writing equations on the apple puzzle problem	80
34	Kanda group's reasoning on the mystery number problem	81
35	Kanda group's reasoning on the segment in a circle problem	82
36	Chaiwat group's explanation on the green tea puzzle problem	83
37	Kanda group's explanation on the perfect square problem	84
38	Chaiwat group's explanation on the apple puzzle problem	85
39	Kanda's written work on the digit 8 and + problem	86
40	Nunya group's solution process on the apple puzzle problem	87

CHAPTER 1

INTRODUCTION

Background

The National Education Act of A.D.1999 (B.E.2542), an Education Reform Act for Further Development of the Thai People, has provided national education guidelines on organizing the learning process. These guidelines have important implications for educational institutions and agencies. Two of the six items in these guidelines are as follows : (a) provide training for all students in thinking process, management, how to face various situations and the application of knowledge for obviating and solving problems; (b) organize activities for students to draw from authentic experience, drill in practical work for completing mastery, enable students to think critically and acquire the reading habit and continuous thirst for knowledge (Office of the National Education Commission [ONEC]. 1999 : 11). Indeed, the National Education Act of A.D.1999 (B.E.2542) has provided the national education guidelines to focus on thinking, problem-solving process, and learning from authentic experience.

Mathematics is an essential subject promoting students to do reasoning, critical thinking, and systematic thinking, especially problem-solving skill / process. As a result of the National Education Act and the Basic Education Reforms of A.D.2001 (B.E.2544), the current reform movements in Thai school mathematics have focused not only on the mathematics content, but also on mathematical process. The mathematics content has occupied a central place in the Thai school mathematics curriculum since antiquity, but mathematical process has not been emphasized. Problem solving is one mathematical process which educators have advocated for special attention.

Not only in Thailand have there been reform movements in education, there is also worldwide educational reform in school mathematics (Australian Education Council [AEC]. 1991 ; 1994 ; National Council of Teachers of Mathematics [NCTM]. 1989 ; 2000). These educational reforms have recommended the study of problem solving at all levels of the mathematics curriculum. The reform documents indicate that problem solving should be the central focus of these mathematics curricula. It should not be an isolated part of mathematics instruction but it should be an integrated part of mathematics learning.

Moreover, there is an expectation in these reform documents that even young students will improve their mathematical knowledge and procedures with understanding of problem solving. As a result of these reforms, problem solving has occupied a major focus in worldwide school mathematics curricula.

Problem solving is both a major goal of instruction and a principal activity in mathematics teaching and learning. By learning problem solving in mathematics, students acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom (NCTM. 2000 : 52). In addition, problem solving is one of the basic skills that students must take along with them throughout their lives and use long after they have left school. Thus, for success in learning problem solving, teachers should know how to improve their students' problem-solving competence and enhance their mathematical thinking.

However, problem solving is an unfamiliar idea for most Thai teachers. The question, "How should we go about helping teachers increase their knowledge and confidence to teach problem solving?" has often been asked and it is timely to ask whether mathematics education programs in Thailand prepare teachers, especially preservice teachers, to teach problem solving. In seeking answers to these questions, educators are looking for the best way to prepare preservice teachers to teach problem solving. Charles (1988), Thompson (1985), and Taplin and Chan (2001) suggested the answer to such questions lies in finding a teacher education course that focuses on mathematical problem solving for preservice teachers such that they become good problem solvers and better teachers of problem solving.

If teacher education programs are to do more in preparing preservice teachers to be problem-solving practitioners, it is important to establish just what these preservice teachers should learn about the problem-solving process and strategies for problem solving. It is also important to determine what they should believe about mathematics, about problem solving and about teaching and learning mathematics. There are several ways of changing the thinking of these preservice teachers to be problem-solving practitioners. One of the best ways is to develop preservice teachers' problem-solving skill and process, and enhance their strategies and beliefs about problem solving (Schoenfeld. 1985). Although numerous studies have examined the problem-solving development of students (Cooney. 1985; Thompson. 1985; Schroeder; & Lester. 1989; Lester; et al. 1994; Lubinski; & Mariani. 1999; Taplin; & Chan. 2001), only a few studies have investigated beliefs about mathematics, beliefs about problem solving, beliefs about teaching and learning

mathematics, and even the program for developing competence in problem solving for the preservice teachers. This is especially the case in Thailand where there is little research on these issues. Therefore, the researcher as an instructor in a mathematics teacher education program sought to help preservice teachers to become good problem solvers by developing an effective problem-solving instructional program. In such an effective instructional program, preservice teachers had experienced in problem solving and were able to develop their problem-solving competence along with appropriate beliefs related to problem solving. The researcher developed a *problem-solving instructional program* for preservice teachers, implemented the *problem-solving* experimental instructional program and studied the effects of such a program.

Research Questions

Because the researcher sought to develop an instructional program that provided experience for preservice teachers in problem solving, promoted competence in solving mathematical problems, and produced appropriate beliefs related to problem solving, the researcher needed to examine the effectiveness of such a program. The specific research questions of this study were:

1. What were the effects of the problem-solving instructional program on preservice teachers' competence in solving mathematical problems?
2. What were the effects of the problem-solving instructional program on preservice teachers' beliefs related to problem solving?
3. What kinds of sociomathematical norms and classroom mathematical practices emerged in the experimental problem-solving class?

Purposes of the Study

1. To study the effects of the problem-solving instructional program on preservice teachers' competence in solving mathematical problems.
2. To study the effects of the problem-solving instructional program on preservice teachers' beliefs related to problem solving.
3. To study preservice teachers' sociomathematical norms in the experimental problem-solving class.

Significance of the Study

1. From the study, we had a problem-solving instructional program that developed preservice teachers' competence in solving mathematical problems, produced appropriate beliefs related to problem solving, and were available to other teacher education institutions.

2. The study provided research for teacher education programs to equip their preservice teachers to build competence in solving mathematical problems.

3. The study provided research for teacher education programs to equip their preservice teachers with appropriate beliefs related to problem solving.

Scope of the Study

Sample

The sample for the study was 26 *second-year* preservice teachers in the mathematics teacher education program of Srinakharinwirot University. These students were separated into two balanced groups : the experimental group and the control group. Partition of these students based on their score in the course MA 241, Principles of Mathematics. Random sampling were used for assigning the experimental group and the control group (see detail on page 46). Three students of the experimental group were purposefully chosen as *target students* for in-depth studying on problem-solving development and beliefs related to problem solving. Selection of these target students also based on students' scores in the course MA 241 : one from each category low, medium and high ability.

Mathematical content areas

Mathematical contents used in this study based on secondary school mathematics.

Variables

Independent variable. The independent variable of the study was *the problem-solving instructional program* developed by the researcher. The independent variable was divided into two categories : used and non-used.

Dependent variables. Dependent variables of the study were preservice teachers' competence in solving mathematical problems and beliefs related to problem solving.

Definitions

Mathematical problem

A mathematical problem is a mathematical situation in which an individual or group is called upon to perform a task for which there is no readily accessible algorithm that completely defines the method of solution.

Mathematical problem solving

Mathematical problem solving means that an individual or group is called upon to perform a task for which there is no readily accessible algorithm that completely defines the method of solution. The individual or group may adapt and apply previous knowledge and experience in problem solving and use a variety of appropriate strategies to solve the problem.

Problem-solving strategies

Problem-solving strategies refer to heuristics or strategies for effective problem solving (Schoenfeld, 1985). Strategies focused in this study are (1) looking for a pattern, (2) making a table, (3) making a drawing or diagram, (4) accounting for all possibilities, (5) guessing and checking, (6) working backward, (7) writing an equation, (8) adapting a different point of view, (9) examining a simpler case or solving a simpler analogous problem, (10) using logical reasoning, and (11) using indirect reasoning (The order of these does not reflect their relative importance).

Competence in solving mathematical problems

Competence in solving mathematical problems means preservice teachers' abilities to solve mathematical problems. In this study, indicators of competence are:

- Ability to *understand* the problem
- Ability to *adapt and apply* a variety of appropriate strategies
- Ability to *find the correct answer* to the problem with a clear explanation of the solution process

Each of their problem-solving outcomes is judged individually by the researcher and observers as to the degree to which the preservice teacher has mastered it.

Beliefs related to problem solving

Beliefs related to problem solving are an individual's perspectives on three aspects: mathematics, problem solving, and teaching and learning mathematics. The examples of appropriate beliefs related to problem solving enhanced in this program are:

- *Beliefs about mathematics*, such as some individuals believe that mathematics is a creative and useful discipline in which they learn to think.
- *Beliefs about problem solving*, such as some individuals believe that many mathematics problems can be solved in more than one way.
- *Beliefs about teaching and learning mathematics*, such as some individuals believe that the principal activity of mathematics is solving problems.

Sociomathematical norms

Sociomathematical norms mean normative aspects of mathematical discussions that are specific to preservice teachers' classroom activity. Sociomathematical norms which are focused in this study are normative understanding of what counts as (1) a reasonable solution process, (2) a sophisticated representation, and (3) an acceptable mathematical explanation and justification. These norms are often negotiated in small-groups and whole-class activities where preservice teachers justify their own solutions, present their solutions, ask questions, discuss their own ideas, listen to others ideas, and give constructive criticism.

The problem-solving instructional program

The problem-solving instructional program is the program for mathematics preservice teachers developed by the researcher. This program is designed to provide mathematical problem-solving experiences for preservice teachers, to develop their competence in solving mathematical problems, and to enhance appropriate beliefs related to problem solving and sociomathematical norms. Such a program equips preservice teachers with knowledge on problem solving, the problem-solving process and problem-solving strategies. This included practice problem solving and encouragement to use more than one strategy.

Research Hypotheses

1. The median score on problem-solving competence of the experimental group was significantly higher than that of the control group.
2. Following instruction, the median score on appropriate beliefs related to problem solving of the experimental group was significantly higher than that of the control group.

CHAPTER 2

REVIEW OF THE LITERATURE

The purpose of this study was to develop a problem-solving instructional program such that preservice teachers had experience in problem solving, and were able to develop their problem-solving competence and beliefs. The researcher studied and reviewed the relevant documents and research in the following topics:

Part 1: Literature on Mathematical Problem Solving

- 1.1 Meaning of a mathematical problem and problem solving
- 1.2 Types of mathematical problems
- 1.3 Process of problem solving
- 1.4 Problem-solving strategies
- 1.5 Beliefs related to problem solving
- 1.6 Factors influencing success in problem solving

Part 2: Literature on Problem-Solving Instruction

- 2.1 Goals for teaching problem solving
- 2.2 Tasks and materials for problem solving
- 2.3 Teaching approach for problem solving
- 2.4 Teacher roles in problem-solving instruction
- 2.5 Organizing and implementing problem-solving instruction
- 2.6 Sociomathematical norms
- 2.7 Evaluating progress in problem solving
- 2.8 Research related to problem-solving instruction

Part 1 : Literature on Mathematical Problem Solving

1.1 Meaning of a Mathematical Problem and Problem Solving

When the National Council of Teachers of Mathematics (NCTM) stated in the 1980 Yearbook that "problem solving is a critical focus of mathematics instruction" (Krulik. 1980), worldwide mathematics educators recommended the study of problem solving at all levels of the mathematics curriculum. Among these mathematics educators, Lester defined the meaning of problem as "a situation in which an individual or group is called upon to perform a task for which they determine completely the method of solution" (Lester. 1978 : 54). This is supported by the definition of Reys, Suydam, and Lindquist: "a situation in which a person wants something and does not know immediately what to do to get it" (Reys; Suydam; & Lindquist. 1989 : 27). Krulik and Rudnick also defined a problem as "a situation, quantitative or otherwise, that confronts an individual or group of individuals, that requires resolution, and for which no path to the answer is known" (Krulik; & Rudnick. 1993 : 6). The latter definition is related to the definition of Posamentier and Krulik as "a situation that confronts a person that requires resolution, and for which the path to the solution is not immediately known" (Posamentier; & Krulik. 1998 : 1).

These different mathematics educators agreed that "a problem involves a situation in which an individual or group want resolution but does not know immediately what to do to get it." If a problem is so easy that the person knows how to obtain the solution or know the solution immediately, there is really no problem at all. A problem for one student may not be a problem for another student. Resulting from these meanings, the term *problem* was defined for this study as "a situation in which an individual or group is called upon to perform a task for which there is no readily accessible algorithm that completely defines the method of solution." It followed that a *mathematical problem* may be defined as "a mathematical situation in which an individual or group is called upon to perform a task for which there is no readily accessible algorithm that completely defines the method of solution."

Besides the meaning of problem, mathematics educators also defined "problem solving" in many ways. Lester defined *problem solving* as "the set of actions taken to perform the task of solving the problem" (Lester. 1978 : 54). Polya defined it as "finding a way out of a difficulty, a way around on obstacle, attaining an aim which was not

immediately attainable” (Polya. 1981 : ix). Whereas the National Council of Supervisors of Mathematics defined problem solving as “the process of applying previously acquired knowledge to new and unfamiliar situations” (National Council of Supervisors of Mathematics. 1988), the NCTM defined a different aspect of problem solving: “engaging in a task for which the solution method is not known in advance” (NCTM. 2000 : 52).

For this study, the term *mathematical problem solving* was defined as “an individual or group is called upon to perform a task for which there is no readily accessible algorithm that completely defines the method of solution.” The individual or group may adapt and apply previous knowledge and experience in problem solving and a variety of appropriate strategies to attack the problem.

1.2 Types of Mathematical Problems

In order to have competence in problem solving, a person must have experiences in solving various types of problems. Mathematics educators categorized types of mathematical problems in several ways, some categorized according to the solver, some categorized according to the aim of problems, and some categorized according to the characteristics of problems. Those types of mathematical problems are,

1.2.1 According to the solver, mathematical problems are categorized into two types, routine problems and non-routine problems (Lester. 2001 : 570; Mayer; & Hegarty. 1996 : 32; NCTM. 1989 : 76).

1.2.1.1 Routine problems. These problems include any of the variety of mathematical tasks that students perform in school. Students are familiar with their structure and the kind of problem-solving strategies needed to solve them. These routine problems exist when students suddenly know how to carry out the correct solution procedure and recognize that the solution procedure is appropriate for the problem (Mayer; & Hegarty. 1996 : 32).

1.2.1.2 Non-routine problems. These problems present unfamiliar problem situations that students have not previously analyzed, or have not met regularly in the curriculum. Such problems demand some flexibility of thinking, and adaptation or extension of previous knowledge, principles and procedures. Because what to do is not obvious, the non-routine problems have often been used to evaluate the problem-solving process of students (NCTM. 1989 : 76; Mayer; & Hegarty. 1996 : 33).

1.2.2 According to the aim of problems, mathematical problems are categorized into two types, problems to find an answer and problems to prove (Polya. 1957 : 154).

1.2.2.1 *Problems to find an answer.* These problems require students to find a certain object, the unknown of the problem. The problems to find may be theoretical or practical, abstract or concrete, serious problems or only a puzzle. For example, "find the solution of the equation $x^2 - 1 = 0$."

1.2.2.2 *Problems to prove.* These problems require students to show conclusively that a certain clearly stated assertion is true, or else to show that it is false. For example, "prove the theorem of Pythagoras."

1.2.3 According to the characteristics of problems, mathematical problems are categorized into six types.

1.2.3.1 *One-step problems* (or simple translation problems). These problems require students to transform the words of a story problem into some kind of mathematical sentence or operation. These problems require just a one-step operation. The primary strategy used to solve these problems is choosing the operation (Charles; Lester; & O'Daffer. 1987 : 11; Hyde; & Hyde. 1991 : 17).

1.2.3.2 *Multiple-step problems* (or complex translation problems). These problems require students to transform the words of a story problem into some kinds of mathematical sentence or operation. But these problems require multiple-step operations. The primary strategy used to solve these problems is choosing the operations (Charles; Lester; & O'Daffer. 1987 : 12; Hyde; & Hyde. 1991 : 17).

1.2.3.3 *Open-ended problems.* These problems require students to pose some questions. Open-ended problems allow for various acceptable answers and for multiple approaches to an effective solution. The open-ended problems engage students in interesting situations and allow them to make their own assumptions, develop creative responses, and effectively communicate their solutions (NCTM. 1995 : 89). In responding to such problems, students not only show their work, but also are asked to explain how they got their answer or why they chose the method they did. Additionally, these problems provide the students with a wide range of possibilities for choosing and making decisions, and may allow for a variety of problem-solving strategies.

1.2.3.4 *Process Problems.* These problems describe mathematical situations or actions that cannot be readily transformed into a mathematical sentence. Students must

create a working process to find a solution that use strategies such as guessing and checking, looking for a pattern, working backward, using logical reasoning, and solving a simpler problem (Charles; Lester; & O'Daffer. 1987 : 12; Hyde; & Hyde. 1991 : 17).

1.2.3.5 *Applied Problems* (or situational problems). These problems require students to collect data outside the problem statement and then make a decision. Many applied problems also require students to formulate a clear statement of the problem and sub-problem, and identify assumptions and the needed data for finding a solution. The solution may require the use of one or more operations or strategies. These problems reflect realistic situations and they often have more than one correct answer (Charles; Lester; & O'Daffer. 1987 : 13).

1.2.3.6 *Puzzle problems*. These problems are based on a hidden assumption or trick. They allow students an opportunity to perform critical thinking, not specific to the mathematical content. Solutions to puzzle problems often require students to look at the problem from a different perspective (Billstein; Libeskind; & Lott. 1997 : 36).

For this study, a number of *non-routine problems and open-ended problems* were presented to preservice teachers to solve in the experimental classroom. Such problems were chosen because they (1) provided unfamiliar structure and problem-solving strategies, (2) provided a wide range of possibilities for choosing and making decisions, (3) allowed a variety of problem-solving strategies, and (4) might have multiple solutions. In order to improve problem-solving competence, the preservice teachers must solve these non-routine problems and open-ended problems on a regular basis and over a prolonged period of time.

1.3 Process of Problem Solving

Problem solving is a mathematical process; it is necessary that the teacher lets students view problem solving as a process in which they organize their actions in a logical and sequential way. Although some students may intuitively follow a process, most students need to be taught how to proceed through steps to reach a solution (O'Connell. 2000 : 2).

The best-known problem-solving process is that of George Polya, called *Polya's four-stage problem-solving process*. This problem-solving process consists of four important stages as follows: (Polya. 1957 : 5-19)

Stage 1 : Understanding the problem.

This stage requires students to “think about the problem” and decide what they are being asked to solve. In this stage, students not only have to “understand the problem,” but also have to “point out the principal parts of the problem,” that is, the unknown, the data, and the conditions. To understand the problem, students may consider the principal parts of the problem attentively, repeatedly, and from various sides. If there is a figure connected with the problem he or she should “draw a figure” and show on it the unknown and the data (Polya. 1957 : 6-7).

Stage 2 : Devising a plan

This stage requires students to “find the connection” between the data and the unknown; then “plan” how to attack the problem or “find” a strategy to solve the problem. Students may be obliged to consider auxiliary problems if an immediate connection is not found. At the end, the students must “identify” what problem-solving strategies might be used to solve the problem (Polya. 1957 : xvi).

Stage 3 : Carrying out the plan

This stage requires students to “implement the strategy from Stage 2,” and perform any necessary actions or computations. In this stage, the students carry out their plan of the solution, and check each step of the plan. This may be intuitive checking or a formal proof of each step (Polya. 1957 : 12-13). If the chosen strategy is not applicable, the students may try to devise a new strategy. In case when the chosen strategy is unsuccessful, planning and finding another strategy to solve the problem, is an important skill in developing an effective problem solver.

Stage 4 : Looking back

This stage requires students to “look back” or “check” the solution that has been obtained (Polya. 1957 : xvii). In this step, if students are able to predict an answer before trying their plan, they should compare their predicted and actual answers and see if both are reasonable.

According to Polya's four-stage problem-solving process, Wilson and his colleagues mentioned that numerous textbooks misinterpreted Polya's process as a linear process and as a series of steps (Wilson; et al. 1993 : 60). Moreover, these textbooks implied that solving mathematical problems is a procedure to be memorized, practiced, and habituated. This led to an emphasis on answer getting rather than using an algorithm of

resolution. The following figure illustrates the linear models of problem solving found in these textbooks.

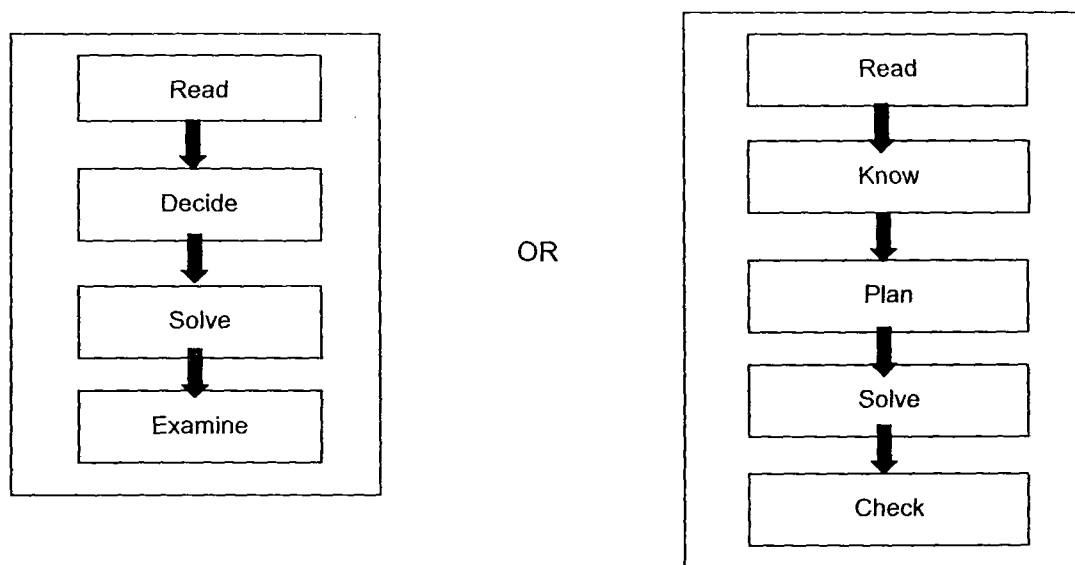


FIGURE 1 LINEAR MODELS OF PROBLEM-SOLVING PROCESS FOUND IN TEXTBOOKS

From : Wilson; et al. (1993). *Mathematical Problem Solving*. p. 60.

These linear formulations in Figure 1 do not promote the spirit of Polya's stages and his goal of teaching students to think. They are not consistent with genuine problem-solving activity process. Wilson and his colleagues modified the framework, called *the dynamic problem-solving process*, to emphasize the dynamic and cyclic nature of genuine problem solving (Wilson; et al. 1993 : 61). Figure 2 illustrates the dynamic problem-solving process.

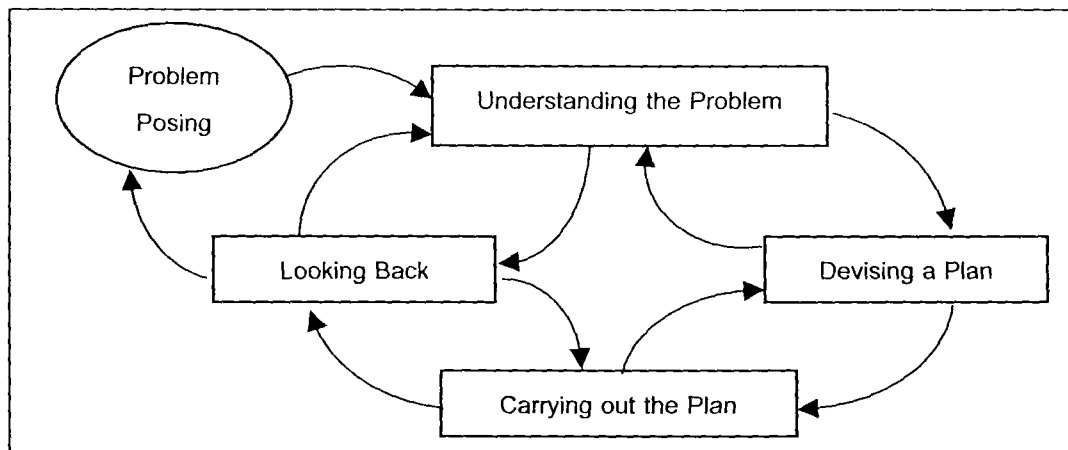


FIGURE 2 THE DYNAMIC PROBLEM-SOLVING PROCESS

From : Wilson; et al. (1993). *Mathematical Problem Solving*. p. 61.

The framework in Figure 2 is useful for illustrating the dynamic, cyclic interpretation of Polya's stages. In the dynamic model, a student may start a problem and engage in thought and activity to understand it, then attempt to make a plan, and in the process may discover a better route to solve the problem. Or, when a plan has been formed, the student may attempt to carry it out and be unable to do so. The next action may be attempting to make a new plan, or going back to develop a new understanding of the problem, or posing a new, possibly related, problem to work on. Any of the arrows could describe student activities (thought) in the process of solving mathematical problems (Wilson; et al. 1993 : 61).

For this study, Polya's four-stage problem-solving process was introduced to students at the beginning of the class, along with the dynamic problem-solving process.

1.4 Problem-Solving Strategies

In solving mathematical problems, students should have not only resources or knowledge of mathematics and understanding of the problem-solving process, but also a variety of appropriate strategies for solving problems effectively. If the students are familiar with possible strategies, they can select and draw on them to solve the problems. Schoenfeld mentions that "strategies (or heuristics) are rules of thumb for successful problem solving, general suggestions that help an individual to understand a problem or to make progress toward its solution" (Schoenfeld. 1985 : 23). They are fairly broad strategies for making progress on unfamiliar or difficult problems. In Polya's brief reference,

a heuristic is “serving to discover” (Polya. 1957 : 113). The above two educators used the term *heuristic* and *strategies* with the same meaning as were used in this study. Many descriptions of problem-solving strategies exist, some of the best known can be found in the works of Billstein; Libeskind; & Lott (1997); Polya (1957); O’Connell (2000); Posamentier; & Krulik (1998); Reys and Suydam; & Lindquist (1989). These problem-solving strategies consist of acting it out, using objects, looking for a pattern, making a table, making a drawing or diagram, accounting for all possibilities, guessing and checking, working backward, writing an equation, adapting a different point of view, examining a simpler case or solving a simpler analogous problem, using logical reasoning, and using indirect reasoning and conjecture.

For this study, *problem-solving strategies* were defined as *heuristics or strategies for effective problem solving*. Problem-solving strategies which were considered in this problem-solving instructional program were (1) looking for a pattern, (2) making a table, (3) making a drawing or diagram, (4) accounting for all possibilities, (5) guessing and checking, (6) working backward, (7) writing an equation, (8) adapting a different point of view, (9) examining a simpler case or solving a simpler analogous problem, (10) using logical reasoning, and (11) using indirect reasoning. These strategies are explained briefly here.

Strategy 1 : Looking for a pattern

This strategy requires students to analyze the problem, and then identify (passively observe) a pattern in the problem. Once the pattern is discovered, the students should describe it and try to show how it is applicable. Looking for a pattern is not only a tool in solving the problem, but also is able to help students to develop number sense and communication skills (Billstein; Libeskind; & Lott. 1997 : 4; Posamentier; & Krulik. 1998 : 41; Reys; Suydam; & Lindquist. 1989 : 34).

Strategy 2 : Making a table

This strategy requires students to construct a table, and then organize the data into the table. A table is used to summarize data and help students discover a pattern and identify information that is missing. It also can help students see all possible cases in the given problem (Billstein; Libeskind; & Lott. 1997 : 23; Reys; Suydam; & Lindquist. 1989 : 34).

Strategy 3 : Making a drawing or diagram

This strategy requires students to depict the information in the given problem to make the relationships apparent. Drawing a picture makes the description clearer and easier to follow. There is no need to draw detailed pictures, draw only what is essential to tell about the problem (Posamentier; & Krulik. 1998 : 139; Reys; Suydam; & Lindquist. 1989 : 34).

Strategy 4 : Accounting for all possibilities

This strategy requires students to account for all possibilities in some systematic way. The students may be able to organize the possibilities into categories, and then dismiss some classes of possibilities before beginning a systematic search of the remaining ones. If students do not have an organized procedure to account for all the possibilities, the strategy does not work and some possibilities may be omitted. This strategy is sometimes used with looking for a pattern and making a table (Posamentier; & Krulik. 1998 : 187; Reys; Suydam; & Lindquist. 1989 : 35).

Strategy 5 : Guessing and checking

This strategy requires students to make a guess, and then test it against the conditions of the problem. Each succeeding guess is based upon information the students obtain from checking the previous guess. An educated guess is based on careful attention to pertinent aspects of the problem, plus knowledge from previously related experiences. This is often applied when a student does not know how to solve the problem more efficiently or if the student does not yet have the tools to solve the problem in a faster way (Billstein; Libeskind; & Lott. 1997 : 33; Posamentier; & Krulik. 1998 : 165; Reys; Suydam; & Lindquist. 1989 : 36).

Strategy 6 : Working backward

This strategy requires students to begin with what they know and reverse their actions to find out how the situation of the problem should begin. Most other strategies involve working through the information forwards, but in using this strategy students have to change their focus and consider the whole problem in reverse. Working backward is used when we know how a situation ends, but we do not know how it started. This is a very useful strategy in certain situations. One of the most difficult parts of working backwards is keeping track of the information and organizing it in a meaningful way (Billstein; Libeskind;

& Lott. 1997 : 33; Herr; & Johnson. 1994 : 294; Posamentier; & Krulik. 1998 : 17; Reys; Suydam; & Lindquist. 1989 : 36).

Strategy 7 : Writing an equation

This strategy requires students to write an equation related to the information of the given problem, in some cases writing an inequality is needed. In solving equations, the students may use the properties of equality. It is also important after solving an equation that the students should substitute the answer into the original context rather than substitute into the equation, because there may be a mistake in setting up the equation. A problem-solving strategy commonly used in algebra consists of writing an equation (Billstein; Libeskind; & Lott. 1997 : 30).

Strategy 8 : Adapting a different point of view

This strategy requires students to adapt the problem in a completely different way. This strategy is often used after several other strategies have been tried without success. To address the complexity of a problem, not only requires an understanding of the problem, but also a consideration of different points of view and perspectives of the problem (Posamentier; & Krulik. 1998 : 70; Reys; Suydam; & Lindquist. 1989 : 38).

Strategy 9 : Examining a simpler case (or solving a simpler analogous problem)

This strategy requires students to break the given problem down into manageable parts, or change the given problem into a simpler analogous problem that may be easier to solve. Some problems are made difficult by large numbers or complicated patterns, so that the way to solve them is unclear. For such a problem, breaking the problem down, or making an analogous but simpler one may aid in ascertaining how to solve it (Billstein; Libeskind; & Lott. 1997 : 24; Posamentier; & Krulik. 1998 : 97; Reys; Suydam; & Lindquist. 1989 : 37).

Strategy 10 : Using logical reasoning

This strategy requires students to use logical reasoning to convince people that their arguments are true. In this strategy, the students must analyze clues or bits of information presented in the problem, and then use that information to help solve the problem. Students sometimes need to use logical reasoning when adjusting their guesses in guess and check, and analyzing their pictures or diagrams, so that it is hard to separate

logical reasoning from other strategies (O'Connell. 2000 : 49-50; Posamentier; & Krulik. 1998 : 97).

Strategy 11 : Using indirect reasoning

This strategy requires students to show that it is impossible for the statement to be false. In assuming that the statement is false, there exists something that contradicts or is impossible. This approach is useful when the problem is difficult to start a direct argument and, when negating the given statement, gives us something tangible with which to work (Billstein; Libeskind; & Lott. 1997 : 35).

Problem-solving strategies are tools that help students success in solving mathematical problems, and there are various appropriate strategies for making progress in effective problem solving. To help students learn and use these problem-solving strategies, teachers must have a collection of problems that draw on these strategies (Posamentier; & Krulik. 1998). In addition to changing students' faulty belief that "there is just one right way to solve any mathematical problem," teachers should have problems that can be solved in more than one way and provide students with a wide range of possibilities for choosing and making decisions.

1.5 Beliefs Related to Problem Solving

In the last few decades, worldwide mathematics educators have increased their attention to teachers' and students' beliefs, given their impact on the learning and teaching of mathematics (Ernest. 1989; Lubinski; & Mariani. 1999; Schoenfeld. 1985 ; 1992; Thompson. 1992). These research results indicated that "an individual's beliefs could determine the problem-solving performance and strategy, which applies to each problem." In addition, the research results indicated that "an important difference between a successful and an unsuccessful problem solver lies in their beliefs about problem solving, about themselves as problem solvers, and about ways to approach solving problems" (Kroll; & Miller. 1993 : 64). For teachers, to develop students' competence in solving problems, it is important to develop teacher's beliefs regarding problem solving as well.

Diaz-Obande and his colleagues mentioned a *belief* as "one of the components of the implicit knowledge of an individual," and particularly *mathematics beliefs* as "the knowledge that students or teachers have about mathematics and process of learning and teaching" (Diaz-Obande; et al. 2003 : 162). Such knowledge is based on experiences that an individual has had. On the other hand, Schoenfeld referred to belief systems as "the set

of understandings about mathematics that establish the psychological context within individuals who do mathematics.” He defined the terms as follows:

“**Belief systems**” are one’s mathematical worldview, the perspective with which one approaches mathematics and mathematical tasks. One’s beliefs about mathematics can determine how the individual chooses to approach a problem, which techniques will be used or avoided, how long and how hard one will work on it, and so on. These beliefs establish the context in which resources, strategies, and control operates (Schoenfeld. 1985 : 45).

For this study, the researcher used some of aspects of beliefs related to problem solving. The term *belief system* was broken down as *beliefs related to problem solving*, which meant “*an individual’s perspectives on three aspects: (1) mathematics, (2) problem solving, and (3) teaching and learning mathematics.*”

Beliefs about mathematics

Mathematics educators attempted to investigate teachers’ and students’ beliefs about mathematics (Ernest. 1989; Frank. 1988; Lubinski; & Mariani. 1999; Schoenfeld. 1985; Thompson. 1992). In these attempts, these educators focused on the question “what is mathematics?” Examples of inappropriate beliefs about mathematics are listed below :

- Mathematics is computation.
- Mathematics is using numbers, equations and shapes to solve problems (Frank. 1988 : 33; Lubinski; & Mariani. 1999 : 156).

Mathematics, for these students is what they called “the four basics : addition, subtraction, multiplication, and division. These basics involve the memorization of arithmetic facts and algorithms. If these students confront multiple or non-routine problems, they will be reluctant to pursue the problems” (Frank. 1988 : 33). Thus to make students be more competent in problem solving, teachers should promote students’ appropriate beliefs about mathematics. Examples of appropriate beliefs about mathematics include the following :

- Mathematics is a discipline characterized by accurate results and infallible procedures, whose basic elements are arithmetic operations, algebraic procedures, and geometric terms and theorems.
- Mathematics is a creative and useful discipline in which students learn to think.

- Mathematics is continually expanding and is created by individuals as they add new knowledge to their existing knowledge (Ernest. 1989 : 13-33; Steele; & Laberge. 2002 : 37; Thompson. 1992 : 127).

Beliefs about problem solving

Several mathematics educators had been conducting research about teachers' and students' beliefs about problem solving (Ernest. 1989; Frank. 1988; Kroll; & Miller. 1993; Lubinski; & Mariani. 1999; Schoenfeld. 1985; Thompson. 1992). Such research results indicated that many students develop inappropriate beliefs :

- All mathematical problems could be solved quickly and directly.

If these students do not immediately know how to solve a problem, they often develop images of themselves as incompetent problem solvers (NCTM. 2000 : 259).

- There is only one right way to solve any mathematical problem.
- An algebraic approach is the only procedure that will work.

Such students not only fail to appreciate ways to solve problems, but also become dependent on a teacher's answer for verification of their success (Kroll; & Miller. 1993 : 64-65).

To make students more competent in problem solving, teachers should promote students' appropriate beliefs about problem solving. Examples of appropriate beliefs about problem solving include the two identified below :

- Many mathematics problems can be solved in more than one way.
- Many mathematics problems have more than one solution.

Beliefs about teaching and learning mathematics

Beliefs about teaching and learning mathematics in general, and problem solving in particular, are other aspects that concern mathematics educators. What teachers believe about teaching and learning mathematics influences how they make instructional decisions, what they do in their classrooms, and what and how their students learn (Lubinski; & Mariani. 1999 : 149). For example, if teachers believe that "mathematics is continually expanding and is created by individuals as they add new knowledge to their existing knowledge" (Ernest. 1989 : 13-33; Steele; & Laberge. 2002 : 37; Thompson. 1992), they allow students to construct their own knowledge of mathematics. Furthermore, if teachers believe that "the principal activity of mathematics is solving problems, a central point of mathematics is problem solving, and a central point of teaching problem solving is teaching

heuristics.” Such teachers provide opportunities for their students to solve a variety of problems (Cooney. 1985 : 328).

Additionally, many students believe about teaching and learning mathematics that:

- The mathematics learned in school has little or nothing to do with the real world.
- The goal of doing mathematics is to receive mathematical knowledge and to verify that students have received this knowledge.
- Mathematics textbook exercises can be solved only by the methods presented in the book.
- Only the mathematics to be tested is important and worth knowing.

For this study, the following examples of appropriate beliefs related to problem solving were promoted with preservice teachers :

- Mathematics is a creative and useful discipline in which they learn to think.
- Mathematics is continually expanding and is created by individuals as they add new knowledge to their existing knowledge.
- Many mathematics problems can be solved in more than one way.
- Many mathematics problems have more than one solution.
- A central point of mathematics is problem solving and a central point of teaching problem solving is teaching heuristics.

1.6 Factors Influencing Success in Problem Solving

One of students' difficulties in learning mathematics is doing problem solving. Mathematics educators have identified factors influencing students' success in problem solving as below :

1.6.1 According to the nature of problem solving, Lester has identified four types of factors associated with problem solving as follows (Lester. 1980 : 288-289) :

1.6.1.1 Task variables. These factors deal with the problem itself, or the nature of the problem. Such factors include the problem's related mathematical content, method of presentation, and structure.

1.6.1.2 Subject variables. These factors deal with the various characteristics of the individual or what the individual brings to the problem. Such characteristics include the individual's conceptual style, previous mathematical background, reaction under stress or pressure, and field independence.

1.6.1.3 *Process variables*. These factors deal with the individual's behavior during problem solving. These behaviors include the manner of organizing and processing information, the types of cognitive strategies used to plan and carry out the solving, and the methods used to evaluate what was done.

1.6.1.4 *Instruction variables*. These factors deal with the features of the problem-solving environment external to the problem and the problem solver. Particularly, these factors are a feature of the problem-solving instruction that helps students to become better problem solvers.

1.6.2 According to the necessary knowledge and behavior for problem solving, Schoenfeld explained that much of the complexity of learning and teaching problem solving is due to the fundamental interconnections that an individual must make in relation to several cognitive process (Schoenfeld. 1985 : 44-45 ; 1992 : 348):

1.6.2.1 His/her mathematical *resources*, the body of knowledge that an individual is capable of bringing to bear in a particular mathematical situation. They are the factual, procedural and knowledge possessed by the individual;

1.6.2.2 *Heuristics*, rules of thumb for effective problem solving. They are fairly broad strategies for making progress on non-routine mathematical problems;

1.6.2.3 *Control*, global decisions regarding the selection and implementation of resources and strategies. That is, decisions about when and what resources and strategies to use. They include planning, monitoring and assessment, decision-making, and conscious metacognitive acts;

1.6.2.4 *Beliefs*, one's mathematical world views, the perspectives with which one approaches mathematics and mathematical problems. Beliefs establish the context within which resources, strategies, and control operates;

1.6.2.5 A variety of *contextual and affective factors* that bear on problem solving performance.

Similarly, Kroll and Miller explained that students' difficulties with problem solving relate to one or more of the following four major factors (Kroll; & Miller. 1993 : 61-65);

Knowledge factors. These factors deal with algorithmic knowledge (e.g., computational skills), linguistic knowledge (e.g., reading skills), conceptual knowledge, schema and strategic knowledge.

Beliefs and affective factors. These factors deal with emotions, preferences, attitudes, and beliefs.

Control factors. These factors deal with decisions about planning, evaluating, monitoring, and regulating thinking.

Sociocultural factors. These factors deal with out-of-class experiences, and the social context of the classroom.

The above statements show that there are numerous factors influencing students' success in problem solving. To solve problems, students have to engage in a variety of cognitive actions, and each of them requires some knowledge and skill. As a result, problem solving is an extremely complex activity for most students to learn and for teachers to develop their students' problem-solving competence.

To support his idea about factors influencing success in solving problems, Schoenfeld constructed a problem-solving course focusing on resources, heuristics, control, and belief systems (Schoenfeld. 1985). He found that this course had significant impact on the way undergraduate students solved mathematical problems. Consequently, current researchers tend to focus their attention on questions in the above five broad interdependent factors : resources, heuristics, control, beliefs, and contextual and affective factors (Lester. 2001 : 571).

Part 2 : Literature on Problem Solving Instruction

Since problem solving had been identified as both a major goal of instruction, and a principal activity in mathematics teaching and learning, worldwide mathematics educators attempted to invent ways of developing effective instructional programs for problem solving. These educators have recommended several essential components of teaching problem solving, which will be discussed in the following paragraphs.

2.1 Goals for Teaching Problem Solving

In teaching problem solving, Charles, Lester, and O'Daffer recommended that the teacher should identify important goals in order to develop students' problem-solving competence and assess their progress in problem solving (Charles; Lester; & O'Daffer. 1987 : 7-11). They have stated seven goals for teaching problem solving (their order does not reflect their relative importance).

Goal 1 : To develop students' problem-solving *thinking skills*. This goal comprises seven important thinking skills :

- Understand / formulate the question in the problem
- Understand the conditions and variables in the problem
- Select or find the data needed to solve the problem
- Formulate sub-problems and select appropriate solution strategies to pursue
- Correctly implement the solution strategy or strategies and solve subproblems
- Give an answer in terms of the data in the problem
- Evaluate the reasonableness of the answer

Goal 2 : To develop students' abilities to *select and use problem-solving strategies*. Students' confidence and abilities are greatly enhanced when they have mastered a repertoire of strategies for solving problems.

Goal 3 : To develop *appropriate attitudes and beliefs about problem solving*. Students' attitudes and beliefs about problem solving and about themselves can influence their performance greatly. Examples of appropriate attitudes and beliefs are presented below :

- Many problems can be solved in more than one way.
- Many problems have more than one answer.
- If the first strategy I try doesn't help, I'll try to find one that might help.

Attitudes and beliefs that should be discouraged are as follows:

- All problems can be solved in only one way.
- If I can't get the answer right away, I'll never get it.

Goal 4 : To develop students' abilities to *use related knowledge*. A student's ability to accurately recall and apply specific knowledge often influences his/her successful problem-solving performance. This knowledge might be mathematical or relevant to the context of the problem. Students need to be taught *how and when* to use specific mathematical knowledge. Furthermore, they need to practice applying newly learned mathematical skills in problem-solving situations.

Goal 5 : To develop students' abilities to *monitor and evaluate their thinking and progress while solving problems*. Many students should learn that it is valuable to slow down periodically and reflect on what they are trying to do, what they have done, and

what they still need to do. An instructional program for problem solving should be developed in terms of monitoring and evaluating the students' thinking and progress while they are solving problems.

Goal 6 : To develop students' abilities to solve problems in cooperative learning situations. At one level, this instructional goal is concerned with the development of helpful social skills involved in cooperative learning situations. Additionally, it is also concerned with many kinds of intellectual skills that are needed for successful problem solving in groups. Cooperative learning could develop in the individual problem-solving skills and social skills.

Goal 7 : To develop students' abilities to find correct solutions to a variety of problems. The student who masters goals 1 through 6 but not goal 7 would certainly be deficient in problem solving. After all, the reason we try to solve a problem is to get the correct answer. An important goal for problem-solving instruction is for students to gain the ability to find correct answers to problems. However, in the classroom, teaching and evaluation should focus not only on their ability to get correct answers but also on the abilities reflected in all the six goals.

For this study, the major goal for teaching problem solving is to develop preservice teacher's competence and their beliefs. The above seven goals were also incorporated in the instructional program used to teach problem solving in this study.

2.2 Tasks and Materials for Problem Solving

Tasks and materials are considered as one essential component of a problem-solving instructional program. There is general agreement that "the task confronting a student has a significant influence on their problem-solving performance" (Kroll; & Miller. 1993 : 66; Lester. 1980 : 291). The necessary features of these tasks and these materials for this problem-solving instructional program are as follows:

2.2.1 Problems that are motivating. Research results indicate that "real-world problems," problems using names of students in class or famous people, problems using a realistic situation or popular context, or problems using real mathematics content, often seem more meaningful and interesting to students (Kroll; & Miller. 1993 : 66; Lester. 1980 : 291). Furthermore, to increase more interest, teachers should give students regular opportunities to compose (and share) their own problems.

2.2.2 Problems with missing, extraneous, or contradictory information.

Research results indicate that students should be confronted with problems that offer missing, extraneous, or contradictory information, because these are typical of real-world problems. To help students learn to identify missing, extraneous, or contradictory information, teachers should integrate instruction on reading comprehension with mathematics lessons on problem solving (Kroll; & Miller. 1993 : 66).

2.2.3 Using calculators, computers, and other technology. Using a calculator helps students improve overall problem-solving achievement because the students are able to shorten their time to calculate answers, and have more time to spend on the “thinking” part of the problem-solving process (Kroll; & Miller. 1993 : 66; O’Connell. 2000 : 6). Furthermore, students who use calculators have a more appropriate attitude. The computer is another potential tool, not only as a problem-solving tool but also as teaching aids in developing problem solving abilities.

2.2.4 Activities focusing on phases of the problem-solving process. Problem solving is a mathematical process, requiring students to follow a series of steps to find a solution (O’Connell. 2000 : 2). Highly recognized famous problem-solving process include Polya’s four-stage problem-solving process and Wilson’s dynamic model for the problem-solving process. To help students see this progression, activities focusing on phases of the problem-solving process should be included in the problem-solving instruction.

2.2.5 Activities utilizing a variety of problem-solving strategies. Experience with using different strategies gives students a better understanding of the problem-solving process and of possible strategy choices (Kroll; & Miller. 1993 : 68).

2.2.6 Activities enhancing appropriate problem-solving beliefs. Appropriate attitudes and beliefs can influence students’ problem-solving performance greatly (Ernest. 1989; Lubinski; & Mariani. 1999; Schoenfeld. 1985; Thompson. 1992). Thus teachers should organize and implement problem-solving instruction having appropriate beliefs concerning problem solving. Those beliefs are, “Many mathematics problems can be solved in more than one way”; “ Many mathematics problems have more than one solution”; and “Mathematics is a creative and useful discipline in which they learn to think.”

2.2.7 Activities promoting metacognition. Metacognition is the word which refers to what one knows or believes about oneself as a learner and how one controls and adjusts one’s behaviors (Reys; Suydam; & Lindquist. 1989 : 39). Competent problem solvers are efficient at keeping track of what they know, and of how well or poorly their

attempt to solve a problem is proceeding. In order to develop metacognitive actions, students are required to observe what they do, and to reflect on what they observe. One idea for developing metacognition is to ask students to make up questions focusing on both “what” and “why” (Kroll; & Miller. 1993 : 68; Reys; Suydam; & Lindquist. 1989 : 39).

2.3 Teaching Approaches for Problem Solving

In teaching problem solving, it is essential that the teacher should know how to teach problem solving and how to approach problem-solving instruction. Schroeder and Lester (1980) recommended three important teaching approaches to problem-solving instruction :

2.3.1 Teaching about problem solving. This approach highlights the “problem-solving process,” and “problem-solving strategies.” At its best, teaching about problem solving also includes experience with actually solving problems, but it always involves a great deal of explicit discussion of, and teaching about, “how problems are solved.”

2.3.2 Teaching for problem solving. This approach highlights ways in which the mathematics being taught can be applied in the solution of both routine and non-routine problems. A strong advocate for this approach might argue that the sole reason for learning mathematics is to give students an opportunity to “apply” the concepts and skills recently learned to the solution of “real world problems.”

2.3.3 Teaching via problem solving. This approach highlights “using real-world problems to learn mathematics and do mathematics.”

For this study, the above teaching approaches for problem solving were conducted in the problem-solving classroom. Teaching focused on problem-solving process and strategies helps students to be able to solve systematically. Seeing that mathematics can apply to real world problems helps these students recognize the power and usefulness of mathematics in the world around them, and then have appropriate attitudes and beliefs in problem solving.

Teaching in the experimental classroom had three aspects: (1) teaching problem-solving process, (2) teaching problem-solving strategies, and (3) teaching real world problems.

Teaching problem-solving process

This approach highlights *the problem-solving process*. According to Polya's four-stage problem-solving process, and Wilson's dynamic problem-solving process, the problem-solving process is introduced at the beginning of the mathematical problem-solving class. Students are encouraged to become aware of their own progression through these stages while they themselves are solving mathematical problems. In this way, not only experience with actually solving problems, but also a great deal of explicit discussion of "*how problems are solved*" is found in this approach. The following will focus on teaching each of the four stages of the problem-solving process.

According to the understanding of the problem stage, the teacher should ask a question or lead a discussion to help students understand what the problem is asking, what constraints are given, and what assumptions they may need to consider (Kroll; & Miller. 1993 : 69). Making the problem interesting by making it concrete, asking students to restate the problem in their own words, and explaining what it is asking them to do will force them to understand the problem (Polya. 1957 : 7). Mathematics educators have offered questions and activities that may be useful in this preparatory stage (Billstein; Libeskind; & Lott. 1997 : 19; Polya. 1957 : xvi) :

- What is the unknown?
- What are the data?
- What is the condition?
- Is it possible to satisfy the condition?
- Is the condition sufficient to determine the unknown? Or is it insufficient?
Or redundant? Or contradictory?
- Can you state the problem in your own words?

Moreover, Polya suggested that ...

- If there is a figure connected with the problem the students should "draw a figure" and locate on it the unknown and the data.
- If it is necessary to give names to these objects the students should "introduce suitable notation;" devoting some attention to the appropriate choice of signs (Polya. 1957 : 7).

According to the planning stage, the teacher should engage students in an open-ended discussion of possible problem-solving strategies. There has been a recommendation that students may look for key concepts to help them decide on a strategy,

may relate the problem to other familiar problems in order to help them decide how to proceed, may look around for some other appropriate point of contact, or explore the various aspects of the given problem, and modify the problem (O'Connell. 2000 : 6; Polya. 1957 : 10). Polya offered questions that may be useful in this preparatory stage :

- Have you seen this before? Or have you seen the same problem in a slightly different form?
- Do you know a related problem?
- Do you know a theorem that could be useful?
- Here is a problem related to yours and solved before. Could you use it?
- Look at the unknown! Do you know a problem having the same or a similar unknown? (Polya. 1957 : 9-10)

According to the *carrying out stage* or as students are working in small groups, the teacher should circulate around the room and play the role of question asker, problem poser, and careful listener. The teacher should expect the students to be the problem solvers and the explainers. Polya offered questions that may be useful in this preparatory stage :

- Can you see clearly that the step is correct?
- Can you prove that it is correct? (Polya. 1957 : 12-13)

In this stage, using a calculator may help students reduce the amount of time needed to calculate answers, and give the students more time to spend on the "thinking" part of the process. Moreover, recognizing that a strategy was unsuccessful and deciding on another strategy are important skills in building effective problem solvers (O'Connell. 2000 : 6).

Finally, in the *looking back stage*, the teacher should lead students to discuss the problem under consideration, its solution strategies, the related problem, problem difficulties, and the entire problem-solving process. Polya offered questions that may be useful in this preparatory stage :

- Check the results in the original problem (In some cases, this will require a proof).
- Interpret the solution in terms of the original problem. Does your answer make sense? Is it reasonable? Does it answer the question that was asked? Determine whether there is another method of finding the solution.
- If possible, determine other related or more general problems for which the techniques will be applied (Polya. 1957 : 14-15).

In this step, if students are able to predict an answer before trying their plans, this is the time for them to compare their prediction and actual answers and see if they are compatible. In addition, asking students to write a summary sentence that relates their answer to the problem will force them to look at the problem and their answer together, and will often help them detect unreasonable answers (O'Connell. 2000 : 7).

Teaching problem-solving strategies

This approach highlights *a variety of problem-solving strategies*. Experience with using different strategies gives students a better understanding of the problem-solving process and of possible strategy choices (Kroll; & Miller. 1993 : 68). The teacher should build on the problem-solving techniques that many students use naturally and bring with them when they start school. As students move through the grades, their skill and understanding of strategies can be enhanced and deepened, and new, more sophisticated strategies can be introduced (Charles; Lester; & O'Daffer. 1987 : 9-10). In teaching problem-solving strategies, the teacher gives the students problems that provide them with a wide range of possibilities for choosing and making decisions and allow a variety of problem-solving strategies as well as having multiple solutions. After the problem has been solved, the teacher may lead students to present their solution and discuss their solution strategies to the class. A sociomathematical norm that often emerges in class is "what counts as a different solution." This generally occurs when a student has solved a problem in a different way or has different solution. In this case the student should explain the difference and justify the solution. In addition, the teacher should lead a discussion on an alternative solution, what counts as an efficient mathematical solution, and what counts as an acceptable mathematical explanation and justification.

Teaching real world problems

This approach highlights *applying the concepts and skills* recently learned in the solution of real world problems. In this kind of teaching, the teacher should concentrate on ways in which the mathematics being taught can be applied to real world problems. When these students recognize the power and usefulness of mathematics in the world around them, then they have helpful attitudes and beliefs in problem solving. In this teaching approach, after students have studied mathematical concepts and structures, teachers should give many opportunities to apply this mathematics in solving problems.

2.4 Teacher Roles in Problem-Solving Instruction

Knowing what teachers should do when teaching problem solving is one of the best ways of helping teachers increase their knowledge and confidence to teach problem solving. Mathematics educators have identified the teacher's necessary role in problem-solving instruction as follows (Artzt; & Newman. 1990 : 19; Kroll; & Miller. 1993 : 68-70):

2.4.1 Teacher as a guide. The role of the teacher in a problem-solving classroom as a guide is not easy. A teacher who gives detailed directions may be sending the message to students that they are to be dependent on the teacher for all knowledge. On the other hand, a teacher who gives almost no direction increases the possibility of student frustration rising to a debilitating level. Thus the teacher has to adopt a balance, and recognize when to intervene and redirect student exploration, and when to allow the student to stumble along. Furthermore, in choosing problems and activities, the teacher should choose which ones to use as a means for introducing material and guiding the discussion of these problems. However, the teacher should not give the solutions. This is somewhat contrary to what has been considered appropriate teacher behavior in the past.

2.4.2 Teacher as a leader of discussion. Before students start working on a problem, research results recommended that the teacher should lead a discussion to help students understand what the problem is, what constraints there are, and what assumptions they may need to consider (Kroll; & Miller. 1993 : 69). To provide some assistance in the planning stage, the teacher should engage students in an open-ended discussion of possible problem-solving strategies. Finally, after the problem has been solved, the teacher should lead students to discuss solution strategies, the problem-solving experience, problem difficulties, and the entire problem-solving process.

2.4.3 Teacher as a monitor. While students are working on the problem (either individually or in small groups), the teacher should circulate throughout the room, observe and question students to determine where they are in the problem-solving process, offer carefully chosen hints at key points and provide extension problems as appropriate.

2.4.4 Teacher as a facilitator. One responsibility of the teacher is "to present students with opportunities that will help them gain an in-depth understanding of the problem-solving process and strategies.

2.5 Organizing and Implementing Problem-Solving Instruction

The NCTM (1989 ; 2000) and Artzt and Newman (1990) recommended that cooperative learning is an important strategy for problem-solving instruction. Since cooperative learning is an instructional strategy that involves a small group of students, these students work together as a team to solve a problem, complete a task, or accomplish a common goal. In this way students can talk about the problem under consideration, discuss strategies and solutions, reflect on and think about the entire problem-solving process, and share their questions with the whole class and in small groups. Small groups provide a forum in which students ask questions, discuss ideas, make mistakes, learn to listen to others' ideas, offer constructive criticism and summarize their discoveries in writing. In addition, group assignments enable students to work together, helping one another integrate new knowledge with prior knowledge and discovering their own meanings as they explore, discuss, explain, relate and question new ideas that arise in the group (Artzt; & Newman. 1990 : 1-2; NCTM. 1989 : 75-76). With the above characteristics, the cooperative learning method has been credited with the promotion of critical thinking, higher-level thinking and an improved problem-solving ability of students (Artzt; & Newman. 1990 : 5).

The following procedures are suggested for implementing cooperative learning.

2.5.1 Forming cooperative groups

- Deciding on the size of the group, usually four persons
- Arranging the size of cooperative learning groups
- Assigning students to teams
- Preparing students for cooperative teamwork.
- Planning teaching materials
- Explanation of the team task for the day is vital for good learning (Foster; & Theesfeld. 1976 : 2-3).

2.5.2 Teacher responsibilities during cooperative team work

The role of the teacher in the problem-solving classroom is quite different from that in more traditional mathematics classes. The teacher is usually responsible for:

- Carefully explaining the task so that each team knows its responsibilities
- Monitoring student work and behavior.

- Answering question only when they are team questions, and providing assistance when necessary.
- Interrupting the group process to reinforce cooperation group skills or to provide direct instruction to all students.
- Providing closure for the lesson.
- Evaluating the group process by discussing the actions of team members at least twice each week.
- Helping students learn to be individually accountable for learning.
- Monitoring team progress and giving praise when appropriate (Foster; & Theesfeld. 1976 : 3).

For this study, *cooperative learning* was used to organize and implement problem-solving instruction.

2.6 Sociomathematical Norms

Sociomathematical norms differ from general social norms that constitute the classroom participation structure in that they concern the “*normative aspects of mathematical discussions that are specific to students’ mathematical activities*” (Yackel; & Cobb. 1996 : 458). These norms regulate classroom discourse and influence the learning opportunities that arise for both the students and the teacher (Cobb; & McClain. 2001 : 219). Examples of sociomathematical norms are what counts as a mathematically *different* solution, a mathematically *sophisticated* solution, a mathematically *efficient* solution, and a mathematically *elegant* solution in the classroom. In contrast, another sociomathematical norm, what counts as an *acceptable* mathematical explanation and justification, deals with the actual *process* of making a contribution. These norms all involve a taken-as-shared sense of when it is appropriate to contribute to a discussion. To further clarify the subtle distinction between social norms and sociomathematical norms, we offer the following examples. The understanding that students are expected to explain their solutions and their ways of thinking is a social norm, whereas the understanding of what counts as an acceptable mathematical explanation is a sociomathematical norm. Likewise, the understanding that, when discussing a problem, students should offer solutions different from those already contributed is a social norm, whereas the understanding of what constitutes a mathematical difference is a sociomathematical norm (Yackel; & Cobb. 1996 : 461).

For this study, **sociomathematical norms** mean *normative aspects of mathematical discussions that are specific to preservice teachers' mathematical activity*. The sociomathematical norms identified in this study are important because they will set forth a way of analyzing and concluding about mathematical aspects of teacher's and students' activities in the problem-solving classroom. Sociomathematical norms which be important for this study were a normative understanding of what counts as (1) a **reasonable solution process**, (2) a **sophisticated representation**, and (3) an **acceptable mathematical explanation and justification**. These norms are often negotiated in small-group working and in whole-class discussions when preservice teachers reason about their own solution, present their solution, ask questions, discuss their own ideas, listen to others' ideas, and give constructive criticism.

2.7 Evaluating Progress in Problem Solving

Because problem solving involves the recall of facts, the use of a variety of skills and procedures, the ability to evaluate one's own thinking and progress while solving mathematical problems, and many other capabilities, problem-solving assessment is a key task. The tasks used to assess problem solving are more complex. It is necessary that teachers know how to evaluate progress in problem solving. Students' progress in problem solving includes two major areas: (1) **performance** in using a variety of problem-solving skills and strategies, and (2) **beliefs** or attitudes related to problem solving. Charles, Lester, and O'Daffer (1987 : 15-61) recommended four evaluation techniques for problem solving : (1) observing and questioning students, (2) using assessment data from students, (3) using holistic scoring techniques, and (4) using multiple-choice and completion tests.

2.7.1 Observing and questioning

Observing and questioning is the valuable evaluation technique that proceeds while students are solving mathematical problems. The data obtained will inform students' performance, beliefs, and attitudes. Observations and questions can be handled informally as the teacher moves around the room while students are working, or formally through structured individual interviews. That is, there is both informal and formal observing and questioning. As the teacher observes and questions students in a problem-solving situation, the teacher should record the findings briefly and objectively. This record should be made as soon as possible after the observation, and can include both a description of

the situation and a teacher's interpretations of it. Methods for recording observations include using a comment card (Figure 3), a checklist (Figure 4), or a rating scale (Figure 5).

Problem-solving Observation Comment Card	
Student _____	Sue Trent _____ Date <u>10 / 5</u>
Comments: - Knows how and when to look for a pattern - Knows that a table will help her find a pattern - Keeps trying even when she has trouble finding a solution - Needs to be reminded to check her solutions.	

FIGURE 3 AN EXAMPLE OF A PROBLEM-SOLVING OBSERVATION COMMENT CARD

From : Charles; Lester; & O'Daffer. (1987). *How to Evaluate Progress in Problem Solving*.

p.18

Problem-solving Observation Checklist	
Student _____	Date _____
_____ 1. Likes to solve problems _____ 2. Works cooperatively with others in the group _____ 3. Contributes ideas to group problem solving _____ 4. Perseveres – sticks with a pattern _____ 5. Tries to understand what a problem is about _____ 6. Can deal with data in solving problems _____ 7. Thinks about which strategies might help _____ 8. Is flexible –tries different strategies if needed _____ 9. Checks solutions _____ 10. Can describe or analyze a solution	

FIGURE 4 AN EXAMPLE OF A PROBLEM-SOLVING OBSERVATION CHECKLIST

From : Charles; Lester; & O'Daffer. (1987). *How to Evaluate Progress in Problem Solving*.

p.18

Problem-solving Observation Rating Scale			
Student _____	Date _____		
	Frequently	Sometimes	Never
1. Selects appropriate solution strategies	_____	_____	_____
2. Accurately implements solution strategies	_____	_____	_____
3. Tries a different solution strategy when stuck (without help from the teacher)	_____	_____	_____
4. Approaches problems in a systematic manner (clarifies the question, identifies needed data, plans, solves, and checks)	_____	_____	_____
5. Shows a willingness to try problems	_____	_____	_____
6. Demonstrates self-confidence	_____	_____	_____
7. Perseveres in problem-solving attempts	_____	_____	_____

FIGURE 5 AN EXAMPLE OF A PROBLEM-SOLVING OBSERVATION RATING SCALE

From : Charles; Lester; & O'Daffer. (1987). *How to Evaluate Progress in Problem Solving*.
p.19

Prior to the lesson, it is important that the teacher should choose the aspects of performance and attitude that they wish to evaluate. Any checklists or rating scales should be prepared ahead of time. As the chosen students solve problems, observe them, listen as they talk to others, and pose questions according to what the teacher wants to evaluate. Then the teacher should record the observations on the spot if possible, rather than relying on memory. Limit the goals and try not to do too much. It is not intended that the teacher evaluates every student in every problem-solving experience. Rather, in a given problem-solving situation, the teacher may wish to focus on one to four students (Charles; Lester; & O'Daffer. 1987 : 20).

2.7.2 Using self-assessment data from students

Self-assessment is another technique to evaluate problem solving. The usefulness of such assessments depends on how candidly the students report their feelings, beliefs, intentions, thinking patterns, and so forth. Two useful techniques for collecting assessment data from students are the *student report* and *inventory*.

2.7.2.1 Student Reports. Students are asked to write or dictate on a tape recorder a retrospective report on a problem-solving experience that they have just completed. A general direction or question, such as “tell about your thinking as you describe how you solved the problem,” is useful for helping the student get started. To give more direction to the report, the student might be asked to respond to the focus questions in Figure 6.

Student Report : Focus Questions
Use the following questions to help you look back and describe your thinking as you worked toward a solution to the problem 1. What did you do when you first saw the problem? What were your thoughts? 2. Did you use any problem-solving strategies? Which ones? How did work out? How did you happen to find a solution? 3. Did you try an approach that didn't work and have to stop and try another approach? How did you feel about this? 4. Did you find a solution to the problem? How did you feel about this? 5. Did you check your answer in any way? Did you feel sure it was correct? 6. How did you feel, in general, about this problem-solving experience?

FIGURE 6 AN EXAMPLE OF THE QUESTION GUIDE FOR A STUDENT REPORT

From : Charles; Lester; & O'Daffer. (1987). *How to Evaluate Progress in Problem Solving*.
p.24

Before using this technique, try to develop a feeling among students that their honest and thorough responses will give you important information that will enable you to help them become better problem solvers. They should be asked to complete their reports immediately following a problem-solving experience. Some may find it natural and exciting to use a tape recorder, and others may be more effective preparing a written report.

2.7.2.2 Inventories. An *inventory* is a list of items a student checks selectively to give a systematic self-appraisal of performance or attitudes. An attitude inventory might be a simple yes-no (or true-false) checklist. Examples of inventories are Figure 7 and Figure 8:

Attitude Inventory Items

Pretend your class has been given some mathematical story problems to solve. Mark true or false depending on how the statement describes you. There are no right or wrong answers for this part.

- _____ 1. I will put down any answer just to finish a problem.
- _____ 2. It is no fun to try to solve problems.
- _____ 3. I will try almost any problem.
- _____ 4. When I do not get the right answer right away I give up.
- _____ 5. I like to try hard problems.
- _____ 6. My ideas about how to solve problems are not as good as other students' ideas.
- _____ 7. I can only do problems everyone else can do.
- _____ 8. I will not stop working on a problem until I get an answer.
- _____ 9. I am sure I can solve most problems.
- _____ 10. I will work a long time on a problem.
- _____ 11. I am better than many students as solving problems.
- _____ 12. I need someone to help me work on problems.
- _____ 13. I can solve most hard problems.
- _____ 14. There are some problems I will just not try.
- _____ 15. I do not like to try problems that are hard to understand.
- _____ 16. I will keep working on a problem until I get it right.
- _____ 17. I like to try solve problems.
- _____ 18. I give up on problems right away.
- _____ 19. Most problems are too hard for me to solve.
- _____ 20. I am a good problem solver.

FIGURE 7 AN EXAMPLE OF AN ATTITUDE INVENTORY

From : Charles; Lester; & O'Daffer. (1987). *How to Evaluate Progress in Problem Solving*.

Problem-solving Strategies Inventory			
Think about your use of strategies when solving the problem and check the following that apply.			
1. _____	I did not think about using strategies at all.		
2. _____	The idea of using strategies came to my mind, but I did not think about it much more.		
3. _____	I looked at a strategy list, but did not try a strategy.		
4. _____	I looked at a strategy list and picked a strategy, which I tried.		
5. _____	I did not look at a list, but just thought of a strategy to try.		
6. _____	I used at least one strategy and it helped me find a solution.		
7. _____	I tried the following strategies :		
_____	Guess and check	_____	Solve a simple problem
_____	Make a table	_____	Work backward
_____	Look for a pattern	_____	Draw a picture
_____	Make an organized list	_____	Write an equation
_____	Other _____		

FIGURE 8 AN EXAMPLE OF A PROBLEM-SOLVING STRATEGY INVENTORY

From : Charles; Lester; & O'Daffer. (1987). *How to Evaluate Progress in Problem Solving*.
p.28

Be sure to have a specific purpose for administering the inventory. Also, establish a climate in which the students feel that their honest responses are important and will enable the teacher to help them become better problem solvers (Charles; Lester; & O'Daffer. 1987 : 29).

2.7.3 Rubric scoring

Since students are often asked to show their solutions in writing, it is natural to consider how a student's written work on a problem can be used to help evaluate progress in problem solving. There are three methods for evaluating a student's written work on a problem:

2.7.3.1 Analytic scoring. This scoring involves the use of a scale to assign points to certain phases of the process. An example of an analytic scoring scale appears in Figure 9.

Analytic Scoring Scale		
Understanding the problem	0 :	Complete misunderstanding of the problem
	1 :	Part of the problem misunderstood or misinterpreted
	2 :	Complete understanding of the problem
Planing a Solution	0 :	No attempt, or totally inappropriate plan
	1 :	Partially correct plan based on part of the problem being interpreted correctly
	2 :	Plan could have led to a correct solution if implemented property
Getting an Answer	0 :	No answer, or wrong answer based on an inappropriate plan
	1 :	Copying error; computational error; partial answer for a problem with multiple answers
	2 :	Correct answer and correct label for the answer

FIGURE 9 AN EXAMPLE OF AN ANALYTIC SCORING SCALE

From : Charles; Lester; & O'Daffer. (1987). *How to Evaluate Progress in Problem Solving*.
p.30

2.7.3.2 Holistic scoring. This scoring enables the teacher to assign a numerical score to the total solution of a problem based on criteria related to a specific thinking process. An example of a holistic scoring scale appears in Figure 10.

2.7.3.3 Impression scoring. This scoring uses a general impression, based on the evaluator's implicit criteria, to rate a total solution numerically.

Finding ways to ensure that students record as much as possible of the results of their thinking enhances the success of each method. Since any analysis of a student's written work that does not include observing the student solve the problem requires some inference, all techniques for evaluating written work should be supplemented with observation and questioning or the collection of assessment data from students (Charles; Lester; & O'Daffer. 1987 : 30).

Focused Holistic Scoring Point Scale	
Point	These papers have one of following characteristics :
0	- They are blank. - The data in the problem may be simply recopied, but nothing is done with the data or there is work but on apparent understanding of the problem. - There is an incorrect answer and no other work is shown.
1	- There is a start toward finding the solution beyond just copying data that reflects some understanding, but the approach used would not have led to a correct solution. - An inappropriate strategy is started but not carried out, and there is no evidence that the student turned to another strategy. It appears that the student tried one approach that did not work and then gave up - The student tried to reach a subgoal but never did.
2	- The student used an inappropriate strategy and got an incorrect answer, but the work showed some understanding of the problem. - An appropriate strategy was used, but— a) it was not carried out far enough to reach a solution (e.g., there were only 2 entries in an organized list); b) it was implemented incorrectly and thus led to no answer or an incorrect answer. - The student successfully reached a subgoal, but went no further. - The correct answer is shown, but— a) the work is not understandable; b) no work is shown.
3	- The student has implemented a solution strategy that could have led to the correct solution, but he or she misunderstood part of the problem or ignored a condition in the problem. - Appropriate solution strategies were properly applied, but— a) the student answered the problem incorrectly for no apparent reason; b) the correct numerical part of the answer was given and the answer was not labeled or was labeled incorrectly; c) no answer is given. - The correct answer is given, and there is some evidence that appropriate solution strategies were selected. However, the implementation of the strategies is not completely clear.
4	- The student made an error in carrying out an appropriate solution strategy. However, this error does not reflect misunderstanding of either the problem or how to implement the strategy, but rather it seems to be a copying or computational error. - Appropriate strategies were selected and implemented. The correct answer was given in terms of the data in the problem.

FIGURE 10 AN EXAMPLE OF A HOLISTIC SCORING SCALE

From : Charles; Lester; & O'Daffer. (1987). *How to Evaluate Progress in Problem Solving*.

2.7.4 Multiple-choice and completion tests

Two types of written test items that are particularly useful in evaluating a student's thinking processes are multiple-choice and completion.

2.7.4.1 Multiple-choice tests. A multiple-choice test is made up of items that consist of a problem or question and a list of possible solutions or answers. The other possible answers, usually called distracters, often reflect common mistakes or misinterpretations, and are designed to entice students who are unsure about the correct response.

2.7.4.2 Completion tests. A completion test is made up of items that are answered by supplying requested information. The information may be simply a word, number, or phrase that will correctly complete a statement, or it may be a sentence or collection of symbols that satisfy the request.

With respect to the above four techniques, Charles, Lester, and O'Daffer (1987 : 15) claim that a teacher's choice of techniques might be based on such factors as:

1. The type of problem-solving outcome or skill being measured
2. The number of students being evaluated
3. The time available for evaluation
4. Teacher's experience in teaching and evaluating problem solving
5. How teacher intend to use the results of the evaluation
6. The availability of evaluation materials

2.8 Research Related to Problem-Solving Instruction

Because problem solving has been identified as both a major goal of instruction, and a principal activity in mathematics teaching and learning, there have been numerous attempts to invent ways of developing effective instructional programs in problem solving. Thompson developed a summer mathematics course focused on principles of heuristic teaching. This course was expected to enhance teachers' confidence and competence in solving problems, in the use of heuristics, and in the use of pedagogical techniques for enhancing students' problem-solving performance and mathematical thinking skills (Thompson. 1988 : 236). The research results of Thompson indicated that "one feature of the course that seemed essential in broadening the teachers' conceptions of the nature of problem solving was their active involvement in solving a wide variety of problems and reflecting on their attempts to solve them. The modeling of the teaching techniques was

followed by a discussion of the rationale for the teachers' role in teaching problem solving" (Thompson. 1988 : 242).

Lester and his colleagues developed a mathematics course for elementary school preservice teachers (PSTs) in order to assist them to become more mature mathematically by teaching via problem solving (Lester; et al. 1994 : 154). This course emphasized four practices; (1) helping PSTs become reflective about their own mathematical behavior, (2) assisting PSTs in building mathematical connections, (3) engaging PSTs in cooperative learning, and (4) fostering an alternative view of assessment. They suggested that "a mathematics course for prospective teachers should help them develop more mathematical power (ability to do mathematics and to have insight into the learning of it) but at the same time build a bridge to the development of teaching power (the ability to reflect on the complexities of learning and of designing and assessing instruction to facilitate it)" (Lester; et al. 1994 : 164).

Taplin and Chan developed a research project on the development of preservice mathematics teachers' skills and understanding of themselves as pedagogical problem solvers (Taplin; & Chan. 2001 : 285). This study described change in the students' attitudes towards problem-based learning (PBL) and examined the critical incidents that were catalysts for these changes to occur. The research results indicated that "by adopting a problem-solving approach to teaching, teachers would be better able to view themselves as competent problem solvers who are able to develop various strategies to deal with change" (Taplin; & Chan. 2001 : 285).

Cooney examined a mathematics teacher's beliefs about problem solving while the teacher was finishing his preservice teacher training and during his first three months of teaching (Cooney. 1985 : 324). The research results revealed conflicts between his idealism and the reality of classroom practice, as his students were not always receptive to his problem-solving teaching strategies (Cooney. 1985 : 324).

Lubinski and Mariani explored preservice teachers' beliefs and discussed how changes in beliefs can be influenced through a mathematics method course (Lubinski; & Mariani. 1999 : 150). In order to determine what impact the course had on the preservice teachers' beliefs, they used several written surveys, such as the Mathematics Belief Survey. The research results indicated that preservice teachers' beliefs can be influenced by reading and discussing articles on how children learn (Lubinski; & Mariani. 1999 : 159).

Somdech Boonprajak studied the development of mathematical power of Mathayonsuksa I students through cooperative learning. The mathematical power consisted of problem solving, reasoning and communication. The research results indicated that the experimental students had successfully attained higher level of mathematical power. The experimental students could approach a problem with better and systematic skills and reasons. They were able to explain the problem with clarity and conciseness and arrived at the conclusions with ease. As a whole, all three areas of mathematical power were reached as a result of the cooperative learning scheme (Boonprajak. 1997 : 339-340).

Preecha Nowyenphon developed mathematics teaching and learning activities through open-ended problem solving for Mathayomsuksa I students and studied their implementation effects on ability to solve problems, behaviors reflecting problem-solving thoughts, attitudes towards mathematics, and mathematical learning achievement in the course MATH 101 : Mathematics I. The research results indicated that the experimental students could improve their ability to solve problems, problem-solving behaviors, and positive attitudes towards mathematics. The experimental group students' mathematical learning achievement in the course MATH 101 was significantly higher than the achievement norm for the school (Nowyenphon. 2001 : 328-329).

In this study, the ideas of Polya's four-stage problem-solving process along with the dynamic problem-solving process, problem-solving strategies, beliefs related to problem solving, teaching approach for problem solving, organizing and implementing problem-solving instruction, and evaluating progress in problem solving were used in designing and implementing the instructional program for preservice teachers. Observation reference were also used to trace preservice teachers' competence and beliefs about problem solving.

Although numerous studies have developed problem-solving instructional programs and examined the problem-solving development of students and teachers, few studies in Thailand use the ideas in this review to analyze what courses have done on these issues for preservice teaching on problem solving.

CHAPTER 3

METHODOLOGY

The purpose of the study was to develop a problem-solving instructional program such that preservice teachers gained experience in problem solving and were able to develop their problem-solving competence. The effect on the preservice teachers' beliefs related to problem solving and sociomathematical norms were also studied. The data were analyzed both qualitatively and quantitatively. The *quantitative analysis* was used in assessing the preservice teachers' competence in solving mathematical problems and their beliefs related to problem solving after the experiment. A posttest on problem solving and a questionnaire on beliefs related to problem solving provided the quantitative data. The *qualitative analysis* was used to understand more deeply the experimental preservice teachers' development in problem solving. Additionally, sociomathematical norms of the experimental class were observed and identified. Observation forms and mathematical journals provided the qualitative data. The process used to accomplish the study is presented in a subsequent section of this chapter.

1. Selecting Sample and Research Design

Sample of the study

Twenty-six second-year students in the mathematics teacher education program of Srinakharinwirot University were subjects for this study. These students were partitioned into two balanced groups: the experimental group and the control group. Partition of the students was based on their scores in the course MA 241: Principles of Mathematics, which was the foundation course on basic mathematical knowledge and process. The researcher arranged such scores from the highest score to the lowest. Then students were paired according to their score (i.e., first and second rank, third and forth rank, and so on) and one from each pair was randomly sampled to the experimental group, and one to the control group. There were 13 students in the experimental group, and 13 students in the control group. Three students in the experimental group were purposefully chosen as target students for the case study on problem-solving competency and beliefs related to problem solving. The target students were selected one from each category: low, medium, and high ability on basic mathematical knowledge and process. Students whose scores from MA

241 were close to the 1st quartile, 2nd quartile and 3rd quartile were assigned to the low, medium and high ability categories, respectively.

Research Design

The study was an *experimental research, posttest-only control group design*.

Experimental group R X O₁

Control group R O₂

Where R referred to *random* assignment to separate groups

X referred to *treatment*, the problem-solving instructional program

O₁ referred to outcomes of the experimental group (*scores of problem-solving competence and belief related to problem solving*)

O₂ referred to outcomes of the control group (*scores of problem-solving competence and belief related to problem solving*)

(Campbell; & Stanley. 1963 : 13)

2. Developing the Framework of the Instructional Program

As a result of studying and analyzing the documents and related research in Chapter 2, the researcher developed a framework for the problem-solving instructional program. Figure 11 illustrates such framework.

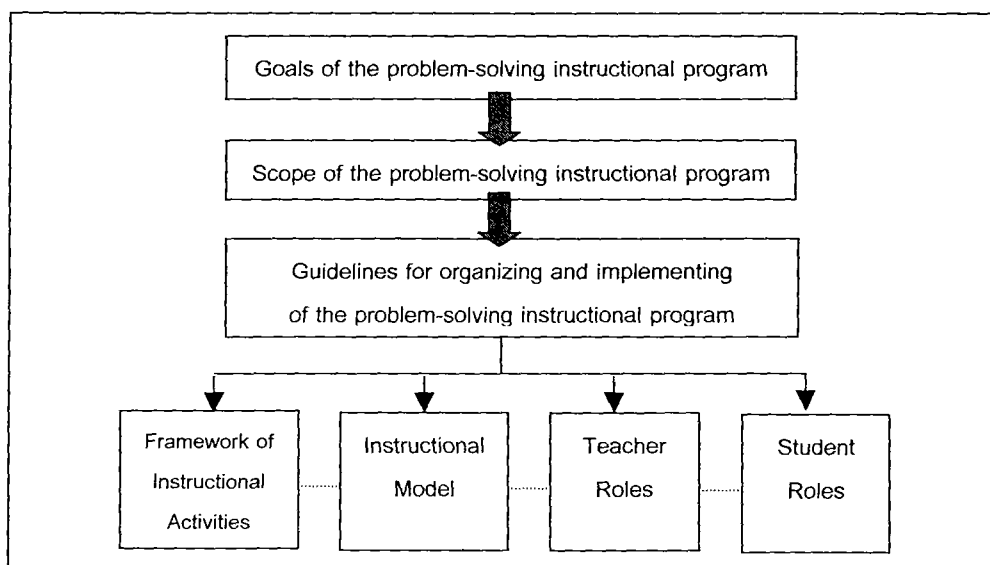


FIGURE 11 THE FRAMEWORK OF THE PROBLEM-SOLVING INSTRUCTIONAL PROGRAM

Goals of the problem-solving instructional program

1. To develop students' competence in solving mathematical problems
2. To enhance appropriate beliefs related to problem solving
3. To identify emerging sociomathematical norms.

Scopes of the problem-solving instructional program.

The problem-solving instructional program was designed for students in a mathematics teacher education program. To learn and experience solving problems, and using a variety of problem-solving strategies, this program emphasized problem solving by having the students *do extensive problem solving with using more than one method or strategy*. Mathematical contents used in the study were secondary school mathematics. Developing appropriate beliefs was also considered along with creating sociomathematical norms.

In the instructional program, it was expected that students would *use Wilson's dynamic problem-solving process*. This process incorporates the dynamic, cyclic interpretation of Polya's four-stage problem-solving process. Figure 12 illustrates the dynamic problem-solving process promoted in problem-solving activities.

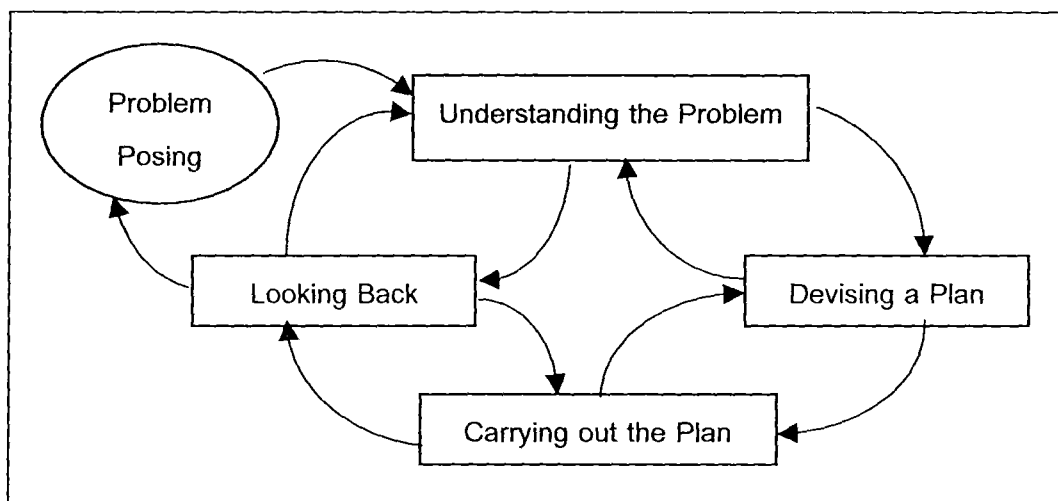


FIGURE 12 THE DYNAMIC PROBLEM-SOLVING PROCESS PROMOTED IN PROBLEM-SOLVING ACTIVITIES

Strategies to be covered in the program were (1) looking for a pattern, (2) making a table, (3) making a drawing or diagram, (4) accounting for all possibilities, (5) guessing and checking, (6) working backward, (7) writing an equation, (8) adapting a different point of view, (9) examining a simpler case or solving a simpler analogous problem, (10) using logical reasoning and (11) using indirect reasoning.

In order to have experience on problem solving, students were expected to solve various types of problems in their own way on a regular basis and over a prolonged period of time. *Non-routine problems and open-ended problems* were used in the instructional program. These non-routine problems and open-ended problems were motivating (interesting, fascinating) real world problems with missing, or extraneous information that provided students with a wide range of possibilities for choosing and making decisions and allowed a variety of problem-solving strategies and multiple solutions. During problem solving, students were *encouraged to monitor and evaluate their own and group members' thinking by using metacognition, and also to reflect on the process of problem solving*. Furthermore, students were *motivated to have appropriate beliefs related to problem solving*. Such beliefs consisted of aspects of beliefs about mathematics, beliefs about problem solving, and beliefs about teaching and learning mathematics. Those appropriate beliefs were:

(1) Mathematics is a creative and useful discipline in which they learn to think.

(2) Mathematics is continually expanding and is created by individuals as they add new knowledge to their existing knowledge.

(3) A mathematical problem is a situation that challenges a person to find a solution to solve, and for which the path to the solution is not immediately known.

(4) Many mathematics problems can be solved in more than one way.

(5) Many mathematics problems may have more than one solution.

(6) Finding the correct answer is not the most important purpose of solving mathematical problems.

(7) Solving a variety of types of problems is better than solving only one type of problems all the time.

(8) Problem solving should be a major goal of instruction and a principal activity in mathematics teaching and learning.

(9) A central point of teaching problem solving is teaching learners discover and learn problem-solving strategies.

(10) A central point in learning the problem-solving process and problem-solving strategies is learning by doing an extensive number of problems by themselves.

(11) In teaching problem solving, the teacher should use a variety of problem types and difficulty levels, using both a short and a long time to solve as well as using more than one strategy.

(12) A central point of promoting the learning of problem solving is providing opportunities for learners to present their own ideas, listen to others' ideas and discuss those ideas.

Guidelines for organizing and implementing the problem-solving instructional program

Framework of instructional activities and instructional model. The problem-solving instructional program consisted of twenty 2-hour-class periods. *Lesson plans* were developed, and instructional activities were designed with appropriate beliefs related to problem solving. The results from the pretest on fundamental background in mathematics, the questionnaire on beliefs related to problem solving, and the review of the literature were used in formulating conjectured learning process. The conjectured learning process was the basis for developing lesson plans. Cooperative learning was used in the instructional program. After teaching each lesson, the researcher used the results of previous lesson plans to modify the conjectured learning process for the next lesson plan. *Each lesson plan was revised as needed according to the results of the previous instructional period.* Figure 13 illustrates the cycle of developing lesson plans in this program.

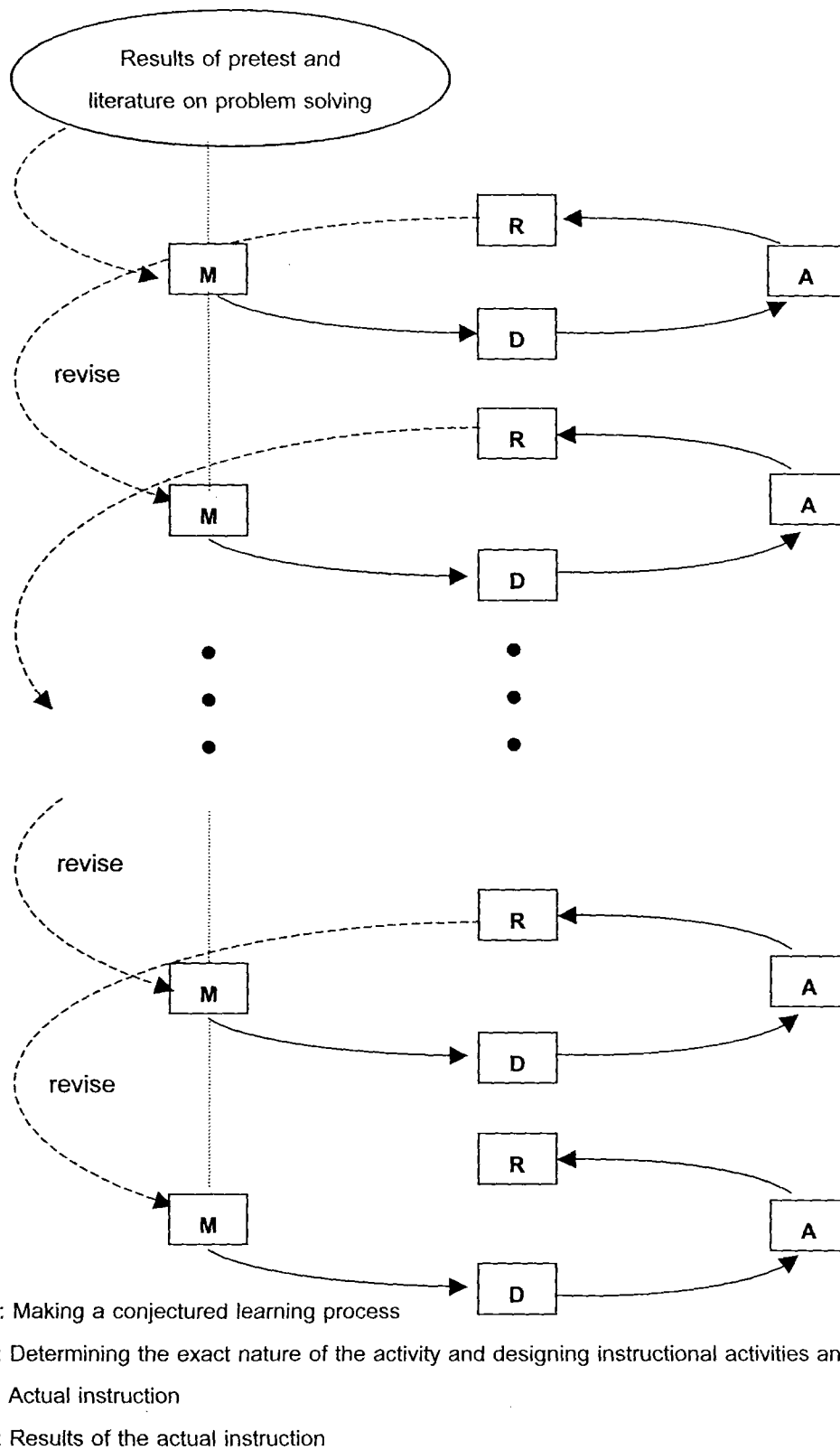


FIGURE 13 THE CYCLE OF DEVELOPING LESSON PLANS

Table 1 illustrates schedule of lesson plans and their major instructional activities.

TABLE 1 SCHEDULE OF LESSON PLANS WITH MAJOR INSTRUCTIONAL ACTIVITIES

Lesson plan No.	Instructional Activity
1	- Introduction to the problem-solving process
2 – 5	- Group problem solving focusing on the first 4-strategies: looking for a pattern, making a table, making a drawing or diagram, and accounting for all possibilities
6	- Individual problem solving focusing the first 4-strategies - Making an individual reflection journal
7 – 10	- Group problem solving focusing on the second 4-strategies: working backward, guessing and checking, using logical reasoning, and using indirect reasoning
11	- Individual problem solving focusing on the second 4-strategies - Making an individual reflection journal
12 – 15	- Group problem solving with problems focusing on the rest of the strategies: writing an equation, adapting a different point of view, and examining a simpler case or solving a simpler analogous problem
16	- Individual problem solving focusing on the rest of the strategies - Making an individual reflection journal
17 – 19	- Group problem solving focusing on the previous learning strategies
20	- Concluding the previous learning problem-solving process and strategies - Making an individual reflection journal

The first lesson plan began with an introduction to the problem-solving process. Four class periods involved group problem solving focusing on 4-strategies. This was followed by one class period of individual problem solving focusing on learning one of the previous 4-strategies and students were asked to write their reflection journal about problem solving. Then the cycle was repeated until all strategies had been considered. Figure 14 illustrates a lesson in each group problem solving.

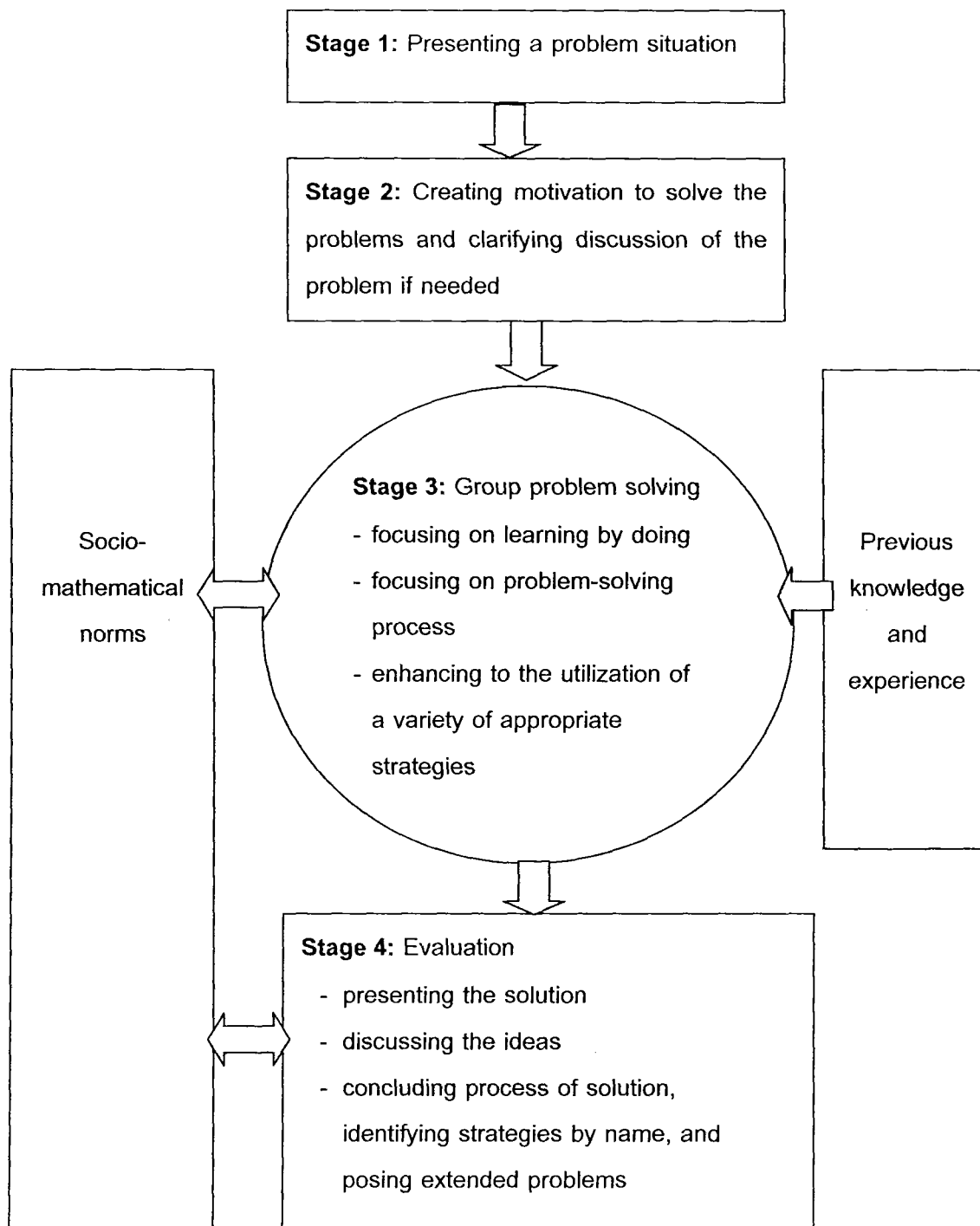


FIGURE 14 THE LESSON IN EACH GROUP PROBLEM SOLVING

Teacher roles. In this instructional program, the teacher-researcher played the roles of a *guide, external monitor, facilitator, and leader of discussion*. This included,

- providing opportunities for students to solve problems in their own ways
- listening to students' explanation of their thinking
- providing a hint by asking questions if necessary
- observing and questioning to examine their problem-solving thinking
- encouraging students to select and use a variety of appropriate problem-solving strategies
- encouraging students to monitor and evaluate their own thinking
- providing a forum to present their work, ask questions, discuss their ideas, and render constructive criticism of their solution
- using students' mathematical thinking in making instructional decisions

Student roles. In this instructional program, each student played the roles of a *cooperative learner, internal monitor, reflective thinker, and problem solver*. That incorporated,

- participating in cooperative learning
- listening to, responding to and questioning one another
- participating in group discussion and whole class discussion
- monitoring themselves or using metacognition
- reflecting to become aware of how they themselves learn to solve problems
- solving mathematical problems in their own ways

3. Developing Instruments

There were two types of instruments used in the study: instruments for problem-solving instruction and instruments for evaluation.

Instrument for Problem-Solving Instruction.

Lesson plans. The lesson plan was the instrument for this problem-solving instruction used in the study. There were *twenty 2-hour lesson plans* for the problem-solving instructional program. Each lesson plan included *learning objectives, instructional activities, materials, and evaluation*. After the researcher constructed the first draft of 20 lesson plans that related to the goals of this instructional program, three educational experts reviewed and commented on the draft of such lesson plans with respect to their content validity, and appropriateness. These educational experts commented on sequence and

difficulty of the problems in the lesson plans. Then, the draft of the lesson plans was revised according to the experts' comments. The revised draft of the lesson plans was tried out with twelve preservice teachers at Suan Sununta Rajabhat University.

Instruments for Evaluation

Pretest on fundamental background. The pretest on fundamental background was used with the experimental group and control group in order to assess the secondary school mathematical knowledge and problem-solving ability the students brought to this experiment. This test was a one-hour test consisting of two parts: four questions on mathematical content and two non-routine problems. The test was tried out with four second-year students in a mathematics teacher education program at Suan Sununta Rajabhat University in order to ascertain whether the test questions were understandable, and to determine if they were too easy or too difficult. Then problems and questions were chosen and adjusted. Three educational experts evaluated the test for the content validity, and commented on the appropriateness and difficulty of the test questions. These educational experts also commented on clarity and conceptual understanding of the test questions. Then, the test was revised and adjusted for clarity and wording according to those experts' comments.

Posttest on problem solving. The posttest on problem solving was administered to assess the experimental and the control groups' competence in solving mathematical problems. The 2-hour test consisted of *four non-routine* problems. A *rubric scoring scale* from 0 to 4 was used. In developing the posttest, six non-routine problems were tried out with the same four second-year students in a mathematics teacher education program at Suan Sununta Rajabhat University as in the process of developing the pretest. In order to find if the test questions were understandable, and to determine if they were too easy or too difficult. Four of them were chosen as the test problems. Three educational experts evaluated the test for content validity, and commented on the appropriateness and difficulty of the test questions. These educational experts commented on clarity and conceptual understanding of the test questions. After the test was revised according to the comments of the experts, the test was tried out with other twelve second-year students in a mathematics teacher education program at Suan Sununta Rajabhat University in order to examine the possibility and reliability of the test.

Questionnaire on beliefs related to problem solving. The questionnaire on beliefs related to problem solving was adapted from the *Mathematics Belief Scale* of Fennema, Carpenter, and Peterson (1987). This questionnaire consisted of 30 items using *Likert scales*. The Likert scaling consisted of four score levels; 1:strongly disagree, 2:disagree, 3:agree, and 4:strongly agree. If any item reflected a negative belief, its score was reversed; that is, 1:strongly agree, 2:agree, 3:disagree, and 4:strongly disagree. In developing this questionnaire, the same four second-year students in a mathematics teacher education program at Suan Sununta Rajabhat University as in the process of developing the pretest were used to check whether the questions were understandable. Often the test was translated into Thai. Then, the questionnaire was revised and adjusted. Three educational experts evaluated these questionnaires on the content validity, and commented on the appropriateness and understanding. The questionnaire was revised and adjusted according to these experts' comments. Then the questionnaire was tried out with the same twelve second-year students in mathematics teacher education program at Suan Sununta Rajabhat University as in the process of developing the posttest in order to examine the possibility and the reliability.

Observation forms. The observation forms were constructed to assess target students' competence in mathematical problem solving and the overview of the whole class's competence in mathematical problem solving as well as the whole class' s sociomathematical norms. Two observation forms were developed by the researcher, an observation checklist and an observation field note. *The observation checklist* was used with target students and problem-solving groups. Every target student and problem-solving group were observed on their ability to *understand the problem, to adapt and apply appropriate strategies and to find the correct answer*. These abilities were recorded on the checklist forms. *The observation field note* was used with individual target students, group problem solving, and whole class discussion when a remarkable situation occurred or a sociomathematical norm emerged.

4. Experimenting and Collecting Data

One week before the teaching experiment started the mathematical knowledge and ability on problem solving of the experimental group and the control group were assessed. The questionnaire on beliefs related to problem solving was administered. The results of this investigation were used for conjecturing the learning process and formulating

the lesson plans. The teaching experiment was conducted in the first semester of the academic year 2004 at Srinakharinwirot University, during July 1 – August 25. The researcher played the roles of the teacher of the teaching experiment for *the problem-solving instructional program*. The experimental group participated as students of this instructional program. Whereas the control group did not participate in the experimental program, they took the same other regular courses as the experimental group. Twenty 2-hour class periods were conducted in this program, 3 times a week. The researcher grouped the 20 lessons into 4 units with lesson number 1-6, 7-11, 12-16, and 17-20 as the 1st, 2nd, 3rd, and 4th units respectively Table 2 illustrates schedule for problem-solving instruction.

After every group problem solving activity, the Observation Checklist (as noted earlier) was collected along with the students' *written works* and their *presentation*. *Sociomathematical norms* were observed during presentation, discussion, and recorded on the Observation Field Note. The researcher did not only observe target students' competence in solving mathematical problems, but also overviewed the group's competence in solving mathematical problems, and identified whole class's sociomathematical norms. Two graduate students also observed the process. Before the experiment these observers were trained in observation procedures and practiced this in the prior study at Suan Sununta Rajabhat University. After observation, the observers discussed the results and their understanding of the observations with each other and the researcher.

In the 6th, 11th, and 16th Session, students *individually solved* a non-routine problem, read an article related to mathematical problem solving, and then *reflected their thinking and problem solving* in their own reflection journal. This information provided insights for the researcher on how students thought and approached that problem. The gained data were used to revise conjectured learning process and planning activities. During each individual problem solving activity, *three target students* were observed with respect to their ability to understand the problem, to adapt and apply appropriate strategies, and to find the correct answer to the problem. The Observation Checklists were used for this purpose. After class, the target students were *interviewed* about the above progress. These interview questions were constructed from their responses in their individual written work. The target students' reflection of their thinking collected from individual reflection journals was used to support the analysis and conclusion about the students' problem-solving progress.

TABLE 2 SCHEDULE FOR PROBLEM-SOLVING INSTRUCTION

Session No.	Instructional Activity	Approximate used time per period (min.)
1	- Introduction to problem solving and the problem-solving process	10
	- Group problem solving with simple problems	60
	- Presenting and discussing the solution	30
	- Concluding process of problem solving and process of solution, identifying strategies, name, and posing extended problems	20
2 – 5	- Group problem solving with problems covering the first 4-strategies	60
	- Presenting and discussing the solution	40
	- Concluding process of solution, identifying strategies, name, and posing extended problems	20
6	- Individual problem solving with a problem covering the first 4-strategies	60
	- Reading an article related to solving problem or extending the problem	40
	- Make an individual reflection journal	20
7 – 10	- Group problem solving with problems covering the second 4-strategies	60
	- Presenting and discussing the solution	40
	- Concluding process of solution, identifying strategies, name, and posing extended problems	20
11	- Individual problem solving with a problem covering the second 4-strategies	60
	- Reading an article related to solving problem or extending the problem	40
	- Make an individual reflection journal	20
12 – 15	- Group problem solving with problems covering the rest of the strategies	60
	- Presenting and discussing the solution	40
	- Concluding process of solution, identifying strategies, name, and posing extended problems	20
16	- Individual problem solving with a problem covering the rest of the strategies	60
	- Reading an article related to solving problem or extending the problem	40
	- Make an individual reflection journal	20
17 – 19	- Group problem solving with problems covering the previous learning strategies or unseen strategies	60
	- Presenting and discussing the solution	40
	- Concluding process of solution, identifying strategies' name, and posing extended problems	20
20	- Concluding the previous learning problem-solving process and strategies	80
	- Make an individual reflection journal	20

After the teaching experiment, the posttest on problem solving and the questionnaire on beliefs related to problem solving were given to the experimental group and the control group on August 25, 2004. The results were used to compare the median scores on problem-solving competence and on beliefs related to problem solving for the experimental group and those of the control group.

5. Analyzing the Data

There were three categories of data analyzed in this study: problem-solving competence, beliefs related to problem solving, and sociomathematical norms. In analyzing problem-solving competence, the *median score on problem-solving competence* of the experimental group was compared with that of the control group. In order to find if *the median score on problem-solving competence of the experimental group was significantly higher than that of the control group*, the Wilcoxon Signed Ranks Test was used. Additionally, the group written works, presentations along with the information from the Observation Checklists, and students' reflection journals were used to qualitatively analyze the corpus of data on the groups' problem-solving competence. Target students' individual written work, explanations in problem solving from the Observation Checklists, the reflection journals about their thinking and problem solving, and interviews about the problem-solving progress were also qualitatively analyzed as a case study.

Following the teaching experiment, the median score on appropriate beliefs related to problem solving of the experimental group was compared with that of the control group. The Wilcoxon Signed Ranks Test was used to determine if *the median score on appropriate beliefs related to problem solving of the experimental group was significantly higher than that of the control group*. For sociomathematical norms, the researcher identified the sociomathematical norms that emerged in the experimental problem-solving class during the group *presentation* and whole class *discussion*. These observations together with data recorded in the Observation Field Notes were descriptively analyzed.

CHAPTER 4

FINDINGS

The purpose of the study was to develop a problem-solving instructional program for preservice teachers, and then to study its effects on the preservice teachers' problem-solving competence, their beliefs related to problem solving, and sociomathematical norms that emerged in the experimental problem-solving class. Findings of this study are reported in four parts in this chapter.

Part 1 : Descriptive Statistics for the Studied Variables

Two variables to be studied in this study were students' competence in solving mathematical problem and their beliefs related to problem solving. The research design of this study was posttest-only control group design. Posttest on problem-solving competence was conducted to evaluate competence in solving mathematical problems. This posttest consisted of four non-routine problems. Rubric scoring scale from 0 to 4 was used. Table 3 illustrates descriptive statistics for scores on problem-solving competence of the experimental group and the control group.

TABLE 3 DESCRIPTIVE STATISTICS FOR SCORES ON PROBLEM-SOLVING
COMPETENCE OF THE EXPERIMENTAL GROUP AND THE CONTROL GROUP

Group	Competence in Solving Mathematical Problems						
	N	Mean	Median	Range	SD	Skewness	Kurtosis
Experimental Group	13	2.44	2.25	2.25	0.639	0.906	0.397
Control Group	13	1.40	1.25	1.50	0.389	2.018	5.180

From Table 3, the mean and median of the experimental group are 2.44 and 2.25 whereas those of the control group are 1.40 and 1.25. It is noticeable that the mean and median of the experimental group are much higher than that of the control group. Additionally, the standard deviation and range of the experimental group were wider than that of the control group. The standard deviation and range were wider for the experimental group, this indicated that the scores were more widely spread about the mean

center. While the Skewness and Kurtosis for the control group indicated that score distribution deviated from normal distribution. In addition, as the result of the Kolmogorov-Smirnov Test and Shapiro-Wilk Test indicated that the competence scores of the control group significantly deviated from normal distribution (see Appendix G Table a). Since the normality assumption was not met, a non-parametric method was used for hypothesis testing in this study. The results were shown in Part 2.

In studying the effects on beliefs related to problem solving, the results of the questionnaire were studied. This questionnaire evaluated beliefs about mathematics, beliefs about problem solving, and beliefs about teaching and learning mathematics. Four scales (1 to 4) of Likert scaling were used. Table 4 illustrates descriptive statistics for scores on appropriate beliefs related to problem solving of the experimental group and the control group.

TABLE 4 DESCRIPTIVE STATISTICS FOR SCORES ON APPROPRIATE BELIEFS
RELATED TO PROBLEM SOLVING OF THE EXPERIMENTAL GROUP AND THE
CONTROL GROUP

Group	Beliefs Related to Problem Solving.						
	N	Mean	Median	Range	SD	Skewness	Kurtosis
Experimental Group	13	3.35	3.40	0.56	0.212	0.011	-1.428
Control Group	13	3.19	3.23	0.87	0.261	0.650	0.234

From Table 4, the mean and median of the experimental group are 3.35 and 3.40 whereas those of the control group are 3.19 and 3.23. It is noticeable that the mean and median of the experimental group were slightly higher than that of the control group, while the standard deviation and range of the experimental group were narrower than that of the control group. The Skewness and Kurtosis indicated that score distribution of the experimental group and the control group was approximately normal distribution. Additionally, the results of the Kolmogorov-Smirnov Test and Shapiro-Wilk Test also indicated that the belief scores were normal distribution (see Appendix G Table b).

Since this study, each group consisted of 13 students who were dependently paired and the normality assumption was not met for problem-solving competence scores. And even though for the belief variable, the evidences showed that the belief scores were

normality distributed, the researcher decided to use the Wilcoxon Signed Ranks Test for hypothesis testing for both competence and belief variables.

Part 2 : Effects on Competence in Solving Mathematical Problems

Comparison of Preservice Teachers' Competence in Solving Mathematical Problems

In studying the effects on competence in solving mathematical problems, the median scores on problem-solving competence of the experimental group and that of the control group were compared. The Wilcoxon Signed Ranks Test was used, as indicated in Part 1, to test the hypothesis on competence in solving mathematical problems. Table 5 illustrates results of comparing the median scores on problem-solving competence of the experimental group and that of the control group.

TABLE 5 RESULTS OF COMPARING THE MEDIAN SCORES ON PROBLEM-SOLVING COMPETENCE OF THE EXPERIMENTAL GROUP AND THAT OF THE CONTROL GROUP

		N	Mean Rank	Sum of Rank	z-score	p-value
Score of the experimental group – Score of the control group	Negative Ranks	1 ^a	1.50	1.50	3.087	p< 0.002
	Positive Ranks	12 ^b	7.46	89.50		
	Ties	0 ^c				
	Total	13				

a. Score of the experimental group < Score of the control group

b. Score of the experimental group > Score of the control group

c. Score of the experimental group = Score of the control group

It was found that the median scores on problem-solving competence of the experimental group was significantly higher than that of the control group at the 0.05 level ($p < 0.002$), the hypothesis on competence in solving mathematical problems was accepted in this study.

Analysis of Experimental Students' Problem-Solving Competence

In studying the development of problem-solving competence of the experimental preservice teachers, a qualitative analysis was also conducted. For every class period during the teaching experiment, the experimental students' competence in solving mathematical problem was observed by the researcher and two assistants. Competence in solving mathematical problems in this study was categorized as the ability to (a) *understand the problem*, (b) *adapt and apply appropriate strategies*, and (c) *find the correct answer to the problem with a clear explanation on the solution process*. The researcher looked at group's competence in solving mathematical problems, and three target students' were studied as case studies. The three target students were Kanda, Chaiwat, and Nunya. These names were pseudonym. Kanda was among the top students and usually played a leader role in a group discussion. Chaiwat was the average student but also seldom played a role as leader in a group discussion. While Nunya who had low ability on mathematics and usually listen to other students' idea before attempting to solve the problem.

As the researcher grouped the 20 lessons into 4 units with lesson number 1-6, 7-11, 12-16, and 17-20 as the 1st, 2nd, 3rd, and 4th units respectively. To illustrate students' development of problem-solving competence, students' performance in one or two sessions of each unit was chosen to describe their competence. In the 1st unit, session 1, 2 or 5 was chosen to describe what students exhibited in the beginning of the program. In the 2nd unit, session 8 or 9 was chosen to describe what performance students changed after having experience on problem solving. In the 3rd and 4th units, session 13, 16, or 17 was chosen to describe what performance students changed or be consistent. Such problems which students' work were analyzed are in Appendix C.

Ability to understand the problem

In this study, the researcher looked at ability to understand the story of the problem and the mathematical concepts in the problem, and to identify the important data, the conditions, the unknown, and the relationships among the data. Three performance patterns were discerned during classroom observation. These patterns based on students' written works, checklists and field notes from observation. These patterns were (a) students spent more time on trying to understand the problem; (b) students showed more evidence of analyzing the problem to help them understand the problem; and (c) students spent more time in a group discussion on the problem and its concepts before attempting to

solve it. In the following sections the researcher analyzed and elaborated on each of these patterns. This analysis focused on three target students in the experimental class.

Students spent more time on trying to understand the problem. For the digit 8 and + problem in the first session (see Appendix C), students spent a short time on reading the problem (less than 1 minute) and started working on the problem right away. All target students, Chaiwat, Kanda and Nunya spent 10-20 seconds reading the problem. Then, they worked out to finish the first given example of the solution (see Figure 15). After that, they stopped working on the solution and reread the problem for more details. Such behaviors suggested that the students did not pay much attention on understanding the problem in the first time.

$8 + 8 + 8 + \dots + 8 = 125 \text{ and}$ Chaiwat's answer	$\begin{array}{r} 125 \\ 8 \overline{) 1000} \\ \underline{8} \\ 20 \\ \underline{16} \\ 40 \end{array}$ Kanda's answer
$\frac{8 + 8 + \dots + 8}{125 \text{ and}} = 3000$ Nunya's answer	

FIGURE 15 TARGET STUDENTS' ANSWER ON THE DIGIT 8 AND + PROBLEM

In Session 8, students worked on *the mystery number* problem (see Appendix C). Students started spending more time on reading and understanding the problem (3-5 minutes). They read carefully and analyzed the data and the conditions of the problem before starting to work on the problem. Chaiwat, a target student, spent 3 minutes on reading and analyzing the problem. In analyzing the problem, he noted mathematical concepts that helped him to interpret conditions of the problem (see Figure 16).

FIGURE 16 CHAIWAT'S WRITTEN WORK ON THE MYSTERY NUMBER PROBLEM

As students progressed about half way through the program, in Session 13, they began to be consistent in spending more time on reading and carefully analyzing the problem (about 4-5 minutes). On *the apple puzzle* problem (see Appendix C), Chaiwat and Nunya, two target students, spent about 4 and 5 minutes on reading and analyzing the problem respectively. This pattern of spending more time on understanding the problem was consistent till the last unit of the program.

In summary, at the beginning of the program the experimental students spent a very short time reading the problem before starting to work on the problem, then they had to come back to read the problem again for more details. Later, students began to recognize the importance of understanding the problem. They spent more time on trying to understand the problem in more details around the middle of the program. There were evidences that students paid more attention on understanding and analyzing the problem.

Students showed more evidence of analyzing the problem to help them understand the problem. Students underlined different parts of the problem, or drew diagrams or representations to help them understand the problem. In Session 1 (*the digit 8 and +* problem, see Appendix C), students rarely underlined parts of the problem, or did not show evidence on analyzing the problem. Six students (46%) underlined 1-2 parts of the problem. Four of them recognized the importance of the questions in the problem and underlined it. Chaiwat, a target student, underlined only the two questions of the problem (see Figure 17). Two of those six students underlined parts of the problem that were not important. Kanda, a target student, underlined one of the given examples, which was not on important part of the problem (see Figure 18).

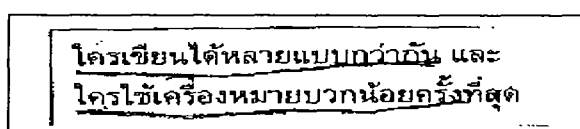


FIGURE 17 CHAIWAT'S UNDERLINED PART ON THE DIGIT 8 AND + PROBLEM

<u>$8 + 8 + 8 + \dots$</u>	=	1000
$8 + 88 + \dots$	=	1000
$8 + 888 + \dots$	=	1000
$8 + 88 + 888 + \dots$	=	1000

FIGURE 18 KANDA'S UNDERLINED PART ON THE DIGIT 8 AND + PROBLEM

In Session 8, when students worked on *the mystery number* problem (see Appendix C), they began to pay more attention to the principal ideas of the problem and underlined them. Seven students (54%) focused on the given conditions of the mystery number. But they underlined only some of the conditions. Chaiwat and Nunya, two target students, underlined 2 out of 4 conditions of the mystery number while Kanda underlined 3 out of 4 conditions (see Figure 19).

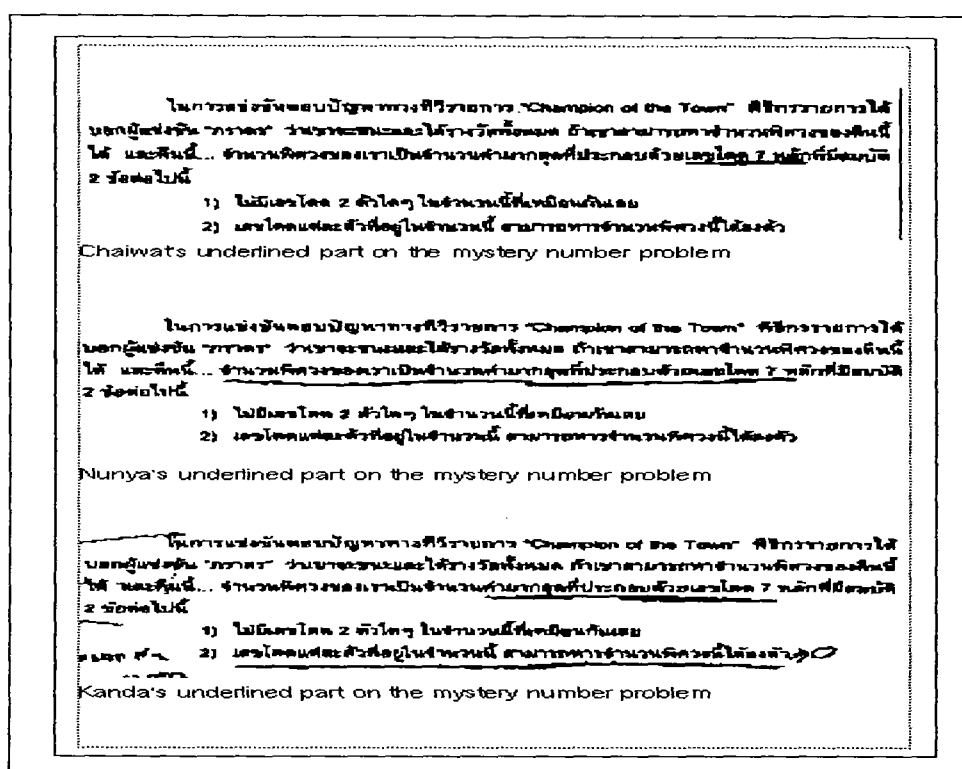


FIGURE 19 TARGET STUDENTS' UNDERLINED PARTS ON THE MYSTERY NUMBER PROBLEM

As students progressed about half way through the program, in Session 13 (*the apple puzzle* problem, see Appendix C), students underlined different parts of the problem and showed evidence of analyzing the problem. Eleven students (85%) underlined 3-10 important ideas of the problem. Among these students, eight of them (62%) underlined some data and conditions of the problem whereas three of them underlined increasingly of the given data, conditions, and questions. Again, from underlining the important parts of the problem, students also made some notes from the problem to understand the problem better as Chaiwat did. The examples of students' work are shown in Figure 20, 21 and 22.

แอปเปิลปริศนา

เจ้ามิดของทุกวัน ชาวสวนจะนำผลไม้มาขายให้แม่ค้าเจ้าประจำที่ตลาดไท ซึ่งเป็นศูนย์กลางค้าขายมีผลไม้แทบทุกชนิด นายอินทร์เป็นชาวสวนคนหนึ่งก็นำแอปเปิลมาขายที่ตลาดไททุกวันเช่นกัน ซึ่งก่อนที่จะมาขายนั้นนายอินทร์ได้ทำการคัดแยกแอปเปิลออกเป็น 3 ขนาดก่อนจะบรรจุใส่กล่องๆ ละ 1 ขนาด ซึ่งได้แก่

ขนาด A ซึ่งแอปเปิลแต่ละใบหนัก 60 กรัม 1 ใบ 60 กรัม
 ขนาด B ซึ่งแอปเปิลแต่ละใบหนัก 50 กรัม
 และขนาด C ซึ่งแอปเปิลแต่ละใบหนัก 45 กรัม

เจ้ามิดวันหนึ่ง นายอินทร์ได้ทำการคัดแยกแอปเปิลไว้ 3 ขนาดเช่นทุกวัน แล้วบรรจุใส่กล่อง 14 กล่อง ซึ่งมีเบอร์ 1 ถึง 14 ติดอยู่ กล่องละ 20 ใบ แต่เจ้ากรรม... นายอินทร์กลับลืมไปแล้วว่า... กล่องใดบรรจุแอปเปิลขนาดใดอยู่บ้าง จำได้เพียงว่า...

มีแอปเปิลขนาด A อยู่หนึ่งกล่อง และแอปเปิลขนาด C อยู่หนึ่งกล่อง ส่วนที่เหลือเป็นแอปเปิลขนาด B

นายอินทร์แก้ปัญหาโดยการเอาแอปเปิลจากกล่องเบอร์ 1 มา 1 ใบ กล่องเบอร์ 3 มา 3 ใบ... และกล่องเบอร์ 12 มา 12 ใบ ซึ่งก็คือ เอาแอปเปิลออกจากแต่ละกล่องเท่ากับจำนวนเบอร์ของกล่อง แล้วนำมาซึ่งรวมกัน ปากกฎว่าหนัก 5,300 กรัม ~~27.15 กิโลกรัม~~ 105 กรัม

ด้วยวิธีแก้ปัญหานี้ข้างต้น อยากทราบว่า...

- (1) -กล่องเบอร์ไหนบ้างที่เราสามารถบอกได้แน่ๆ ว่ามีแอปเปิลขนาด B อยู่
- (2) -กล่องเบอร์ไหนบ้างที่มีแอปเปิลขนาด A และขนาด C อยู่ เพราะเหตุใด

FIGURE 20 NUNYA'S UNDERLINED PARTS ON THE APPLE PUZZLE PROBLEM

แอปเปิลปริศนา

เจ้ามิดของทุกวัน ชาวสวนจะนำผลไม้มาขายให้แม่ค้าเจ้าประจำที่ตลาดไท ซึ่งเป็นศูนย์กลางค้าขายมีผลไม้แทบทุกชนิด นายอินทร์เป็นชาวสวนคนหนึ่งก็นำแอปเปิลมาขายที่ตลาดไททุกวันเช่นกัน ซึ่งก่อนที่จะมาขายนั้นนายอินทร์ได้ทำการคัดแยกแอปเปิลออกเป็น 3 ขนาดก่อนจะบรรจุใส่กล่องๆ ละ 1 ขนาด ซึ่งได้แก่

ขนาด A ซึ่งแอปเปิลแต่ละใบหนัก 60 กรัม
 ขนาด B ซึ่งแอปเปิลแต่ละใบหนัก 50 กรัม
 และขนาด C ซึ่งแอปเปิลแต่ละใบหนัก 45 กรัม

เจ้ามิดวันหนึ่ง นายอินทร์ได้ทำการคัดแยกแอปเปิลไว้ 3 ขนาดเช่นทุกวัน แล้วบรรจุใส่กล่อง 14 กล่อง ซึ่งมีเบอร์ 1 ถึง 14 ติดอยู่ กล่องละ 20 ใบ แต่เจ้ากรรม... นายอินทร์กลับลืมไปแล้วว่า... กล่องใดบรรจุแอปเปิลขนาดใดอยู่บ้าง จำได้เพียงว่า...

มีแอปเปิลขนาด A - อยู่หนึ่งกล่อง และแอปเปิลขนาด C อยู่หนึ่งกล่อง ส่วนที่เหลือเป็นแอปเปิลขนาด B

นายอินทร์แก้ปัญหาโดยการเอาแอปเปิลจากกล่องเบอร์ 1 มา 1 ใบ กล่องเบอร์ 3 มา 3 ใบ... และกล่องเบอร์ 12 มา 12 ใบ ซึ่งก็คือ เอาแอปเปิลออกจากแต่ละกล่องเท่ากับจำนวนเบอร์ของกล่อง แล้วนำมาซึ่งรวมกัน ปากกฎว่าหนัก 5,300 กรัม

ด้วยวิธีแก้ปัญหานี้ข้างต้น อยากทราบว่า...

- (1) -กล่องเบอร์ไหนบ้างที่เราสามารถบอกได้แน่ๆ ว่ามีแอปเปิลขนาด B อยู่
- (2) -กล่องเบอร์ไหนบ้างที่มีแอปเปิลขนาด A และขนาด C อยู่ เพราะเหตุใด

FIGURE 21 CHAIWAT'S UNDERLINED PARTS ON THE APPLE PUZZLE PROBLEM

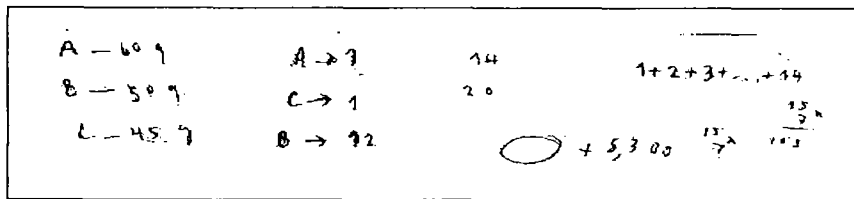


FIGURE 22 CHAIWAT'S RESTATE THE PROBLEM ON THE APPLE PUZZLE PROBLEM

Towards the end of the program, in Session 17 (*the magic traffic light problem*, see Appendix C), students showed further evidence of analyzing the problem. They not only underlined the important data, the conditions and the questions of the problem, but also drew diagrams or pictures to help them understand the problem. Figure 23 shows Kanda's diagram for visualizing the situation.

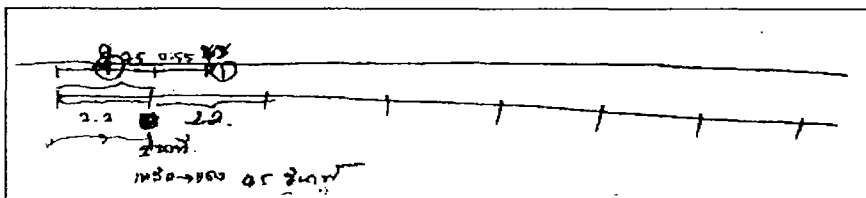


FIGURE 23 KANDA'S DIAGRAM ON THE MAGIC TRAFFIC LIGHT PROBLEM

In this same session, there were students (Nunya and Kanda) who analyzed the problem and thought about the key words. For example, for the word "velocity", Nunya, a target student, started to think about the relationship of distance and time. She wrote the formula of velocity as $v = \frac{s}{t}$ (see Figure 24). This indicated some students had been thinking beyond the words they saw in the problem.

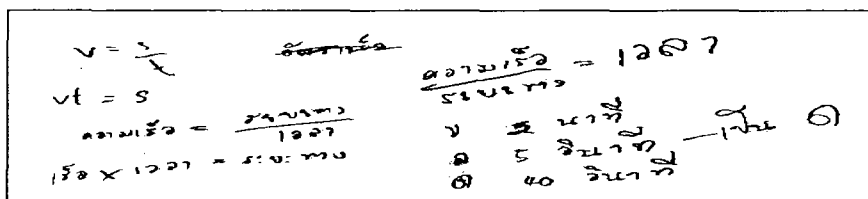


FIGURE 24 NUNYA'S USED FORMULA ON THE MAGIC TRAFFIC LIGHT PROBLEM

In summary, in the first session students underlined only one part of the problem. They underlined either the data or the question of the problem. In the later sessions, students recognized the importance of understanding the problem. This conclusion came from the evidence that they underlined important parts of the problem and drew diagrams. At the end of the program, students worked more on analyzing the problem for understanding.

Students spent more time in a group discussion on the problem and its concepts before attempting to solve it. As noted earlier, in the first session (*the digit 8 and + problem*, see Appendix C) students started working on the solution with little attention to analyze the problem. They spent only a short time for discussion. The discussion in the first session was limited to question-and-answer type to clarify as the following example shows.

Nunya: What is "the number representing only digit 8"?

Student 1: They are 8, 88, 888, 8888, ...

In Session 5, when students worked on *the locker puzzle problem* (see Appendix C), they started spending more time in a group discussion on the problem and concepts in the problem before attempting to solve it. They spent 5-6 minutes discussing the story of the problem. Chaiwat, a target student, did not only draw a diagram that helped him to understand the problem, but he also led a discussion in the group. The diagram illustrated the number of lockers that were opened or closed by the first, second, third, ... , and 6th sportsman respectively (see Figure 25).

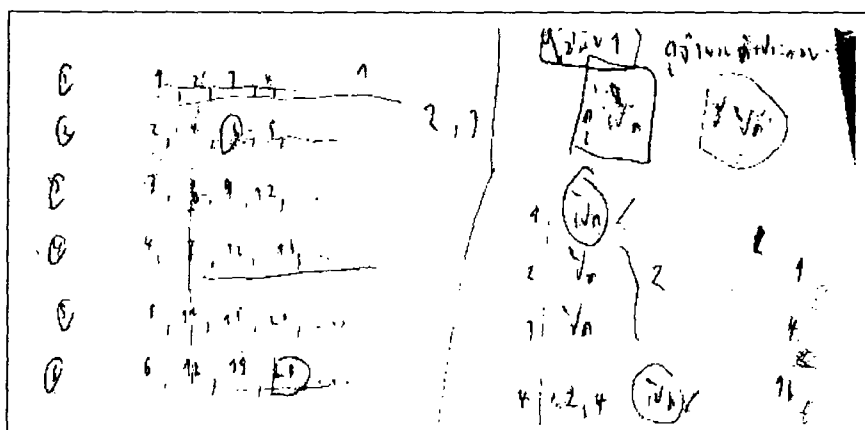


FIGURE 25 CHAIWAT'S DIAGRAM ON THE LOCKER PUZZLE PROBLEM

Chaiwat found a relationship between the locker number and the position of the locker door. He presented his idea and discussed it with other students in the group.

Chaiwat: I noticed a relationship between the locker number and its position.

Student 1: What is it?

Chaiwat: If the number of the factors of the locker number is odd, then that locker is opened. On the other hand, if such number is even, then that locker is closed.

Chaiwat listed the locker number and its factors, then talked to the group (see Figure 11).

Chaiwat: Number 1 has one factor ... odd This locker is opened.

Number 2 has two factors ... even... . This locker is closed.

Number 3 has two factors ... even This locker is closed.

Number 4 has three factors: 1, 2 and 4 ... odd This locker is opened.

Student 1: How about some other numbers?

Chaiwat: Shall we check?

Then they started working together. After the group discussion, Chaiwat and other students in the group expressed that they fully understood the problem and also found a relationship of the data before working on the solution.

From half way through the program, students consistently spent more time in a group discussion before attempting to solve it. In Session 17, students discussed the problem and its concepts before attempting to solve it. They spent 4-6 minutes to discuss in the group. As Nunya, a target student, worked on *the magic traffic light* problem (see Appendix C), she did not understand one of conditions of the problem. She discussed that condition with other students in the group.

Nunya: What is 'one period of the light signal' meant?

Student 1: Time that the light signal changes from green, to red, yellow and green again. That is, 120 seconds for green signal, 5 seconds for yellow signal and 40 second for red signal

Nunya: So one period of the light signal equals 165 seconds.

Student 1: Yes.

Student 2: We have to change unit of the time. I got $t = \frac{165}{3600} = 0.04583$ hour.

Student 1: I also got 0.45 hour.

The researcher noticed that as students worked on more problems, they began to recognize that group discussion helped them clarify the information in the problem.

Information from the above paragraphs suggested that as students worked on more problems, they spent more time on reading and analyzing the story of the problem, and showed more evidence of analyzing the problem to help them understand the problem. They also spent more time in a group discussion on the problem and its concepts before attempting to solve it. Additionally, students reflected in their reflection journal that they spend more time on understanding the story of the problem and its mathematical concepts and also more identify what are the data, conditions, relationships of the data, and questions. Kanda, a target student, reflected that "...my understanding was increased continuously...." One student also reflected that "my understanding the problem depends on its difficulty. Some problems are difficult to understand, but I had more understanding when I discussed about it with my friends in the group." These evidences indicated that students could develop their ability to understand the problem in this experimental problem-solving program. Ability to understand the problem was one factor that influenced the median score on problem-solving competence was higher for the experimental group.

Ability to adapt and apply appropriate strategies

For ability to adapt and apply appropriate strategies, the researcher looked at the strategies, the appropriateness of the strategies and the variety of strategies that students used. The strategies covered in this study were (a) accounting for all possibilities, (b) making a table, (c) making a drawing or diagram, (d) looking for a pattern, (e) guessing and checking, (f) working backward, (g) writing an equation, (h) using logical reasoning, (i) adapting a different point of view, (j) examining a simpler case or solving a simpler analogous problem, (k) using indirect reasoning, and (l) using a concrete model.

As noted earlier in the previous chapter, each problem in this program allowed a variety of appropriate strategies. Such strategies might either be used as the only strategy for solving the problem or be used with other strategies. There were four strategies that students in the experimental group often used for solving problems in this program. Those strategies that were used most often were "making a drawing or diagram", "accounting for all possibilities", "writing an equation", and "using logical reasoning." Strategies that students moderately used were "making a table", "looking for a pattern", "guessing and checking", "using a concrete model", "adapting a different point of view", and "examining a simpler case or solving a simpler analogous problem." Strategies that students rarely used

were “working backward” and “using indirect reasoning strategies.” Table 6 illustrates students’ responses to strategies that they used to solve the problem in each session of the program.

For the ability to adapt and apply appropriate strategies, four patterns were observed during classroom observation. These patterns based on students’ written work, checklists and field notes from observation. These patterns were (a) students made clearer drawings or diagrams; (b) students were more systematic on accounting for all possibilities; (c) students defined variables and set equations more appropriately; and (d) students used more logical reasoning in their arguments.

Students made clearer drawings or diagrams. In this program, “making a drawing or diagram” was the most frequency strategy that students used. Students used “making a drawing or diagram” in solving 9 out of 20 problems in the program. When looked at the frequency of using this strategy among the three groups, the researcher found that the total number of using was 20 (see Table 4). At the beginning of the program, in Session 2, students worked on *the green tea puzzle* problem, a story problem about a customer considered the promotion to get advantage to buy green tea. This promotion was “if a customer exchanges seven empty bottles at the POP store, he will get one free.” The question of this problem is that if a person who used this promotion found that he drank 348 bottles altogether, what is the smallest number of green tea bottles he bought (see Appendix C). This problem required for students to find the relationship among the number of bottles of green tea that the customer bought, drank, and got for free. In this problem, students attempted to draw diagrams to represent such relationship, but the diagram was not clear or complete. Chaiwat and the group paid attention to the number of green tea bottle that the customer bought and got free, and made a diagram that represented such information. The diagram was not clear or complete, it showed only that the customer bought 7 bottles in the first time, after he got one free then bought 6 more bottles in order to get another one free. He indicated that he did this 48 times which is not the right answer (see Figure 26).

TABLE 6 STRATEGIES STUDENTS USED

Strategy	Problem Number																				Number of problems	Frequency of using the strategy
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
1. Accounting for all possibilities	G1 G2 G3	G1+ G2+ G3+	G1+ G2+ G3+		G1+ T2 T3	T2 T3			G1+ G2 G3	G2 G3			G1 G2 G3+							T2	8	18
2. Making a table	G1 G2	G1+ G2				T3			G1 G3	G1+					G2		G1+				7	9
3. Writing an equation	G3	G2		G2			G1 G2 G3				G2+		G1+ G2+		G1+ G2 G3		G1+ G2 G3				8	15
4. Looking for a pattern		G1+ G3	G1 G3												G1+ G2 G3						3	7
5. Making a drawing or diagram		G1 G2+ G3+	G1+ G2+ G3+	G2 G3	G3+	T1 T2			G1 G2+ G3			G1+ G2+ G3					G1+ G2+ G3+	G1 G2			9	20
6. Working backward							G1 G2 G3														1	3
7. Guessing and checking								G1 G2 G3											T1 T2 T3		2	6
8. Using logical reasoning								G1 G2 G3	G1+ G3	G1+ G2 G3	T1 T2 T3	G1+ G3	G3+	G1 G3							7	15

G : Group (No.1, 2, 3)

T : Target student (No.1, 2, 3);

+ : Combining other strategies to solve the problem

TABLE 6 (CONTINUED)

Strategy	Problem Number																				Number of problems	Frequency of using the strategy
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
9. Solving simple case					G1+ G3+											T2+ T3+					2	4
10. Adapting different point of view														G1 G2 G3		T1 T2+ T3+					2	6
11. Using indirect reasoning																			G1 G2 G3		1	3
12. Using a concrete model														G1 G2 G3					G1 G2 G3		2	6

G : Group (No.1, 2, 3)

T : Target student (No.1, 2, 3);

+ : Combining other strategies to solve the problem

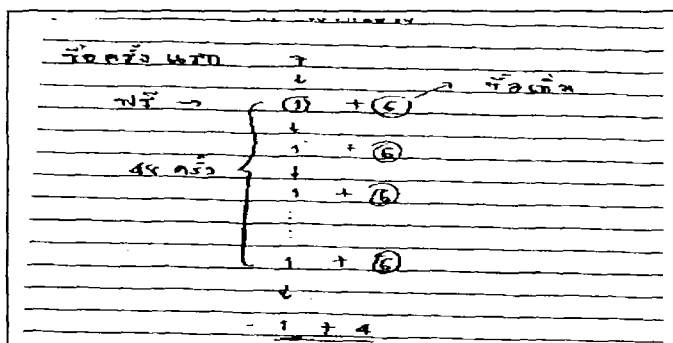


FIGURE 26 CHAIWAT GROUP'S DIAGRAM ON THE GREEN TEA PUZZLE PROBLEM

Later, in Session 9, as students worked on *the perfect square* problem, they began to draw clearer diagrams that reflected solution process of the problem. The story of this problem was about 18 person. Each person was given a card with a number, 1 to 18. This problem required students to pair two persons whose sum of the numbers on their cards was a perfect square (see Appendix C). In order to do this, one group drew a diagram that represented all possible cases of such matching. Because one person had to pair with only one person, thus, students crossed out some cases of the multiple pairs with the same person. For example, 18 was paired with 7, 17 was paired with 8 while 8 also paired with 1. Students crossed out the segment that indicated the pair with 8 and 1, since 8 was already paired with 17 (see Figure 27).

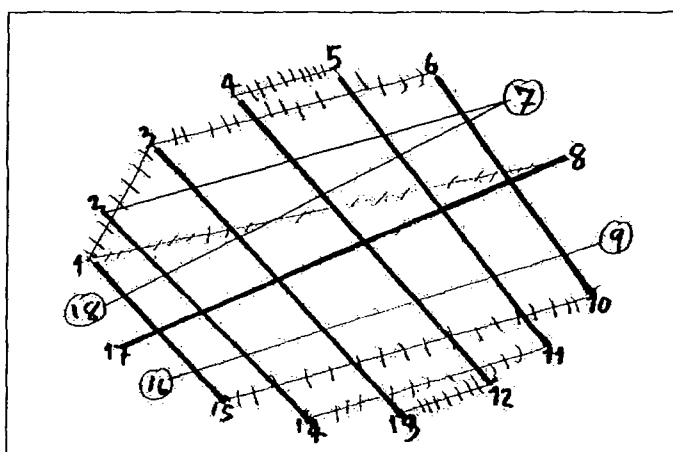


FIGURE 27 KANDA GROUP'S DRAWING ON THE PERFECT SQUARE PROBLEM

Toward the end of the program, in Session 17 (*the magic traffic light problem*) the researcher noticed that students began to recognize that clear drawings or diagrams help them solve the problem. They made a clear drawing and combined some other strategies to help them understand the problem and find the solution of the problem. This problem involved an actor driving a car on a straight road having traffic lights every 2.2 kilometers. The condition is that every traffic light change at the same time with a constant time for green, yellow, and red lights for 2 minutes, 5 seconds, and 40 seconds respectively (see Appendix C). In this problem, students were asked to find how fast he drives so that he would not stop by the traffic light. In order to understand the problem and its conditions, students made a drawing that represented the situation of the problem. Then they combined "writing an equation" and "using logical reasoning" to solve this problem. Nunya and the group made a drawing incorporated with using the formula of velocity as $v = \frac{s}{t}$. Then they also used logical reasoning to solve this problem (see Figure 28).

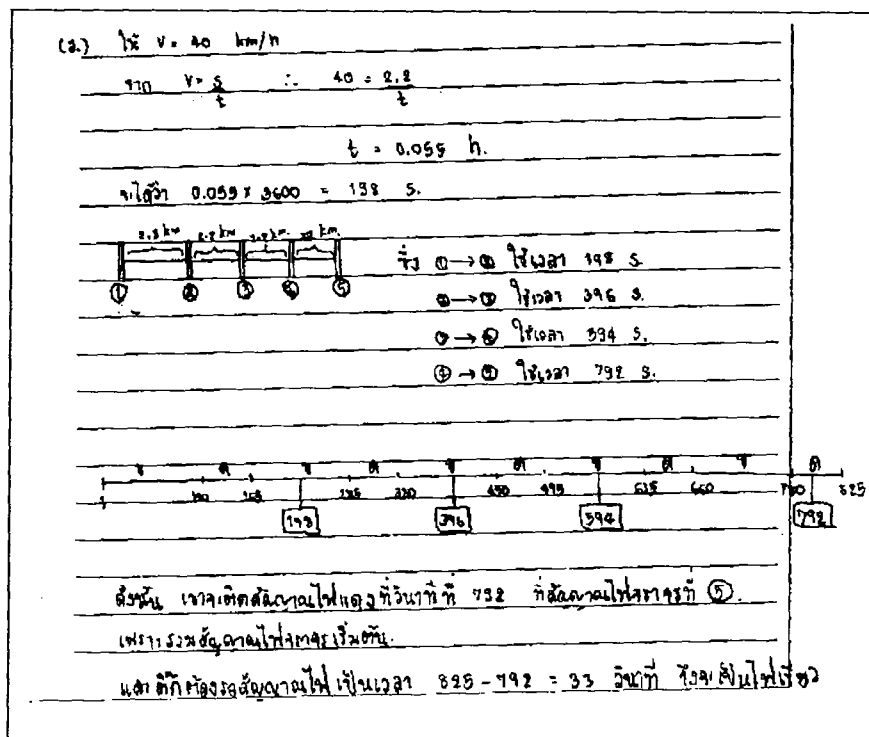


FIGURE 28 NUNYA GROUP'S DRAWING ON THE MAGIC TRAFFIC LIGHT PROBLEM

In summary, at the beginning of the program students drew unclear diagrams for visual representations to help them understand the problem and look for a relationship of

Later in the program, in Session 9 (*the perfect square* problem), when students used “accounting for all possibilities” strategy in listing all possible cases of matching two

numbers having the total score as a perfect square number (see Appendix C), they were more systematic. They also used some other strategies that helped them list all possibilities. Chaiwat, a target student, used tree diagrams to help him in listing all possible cases (see Figure 30).

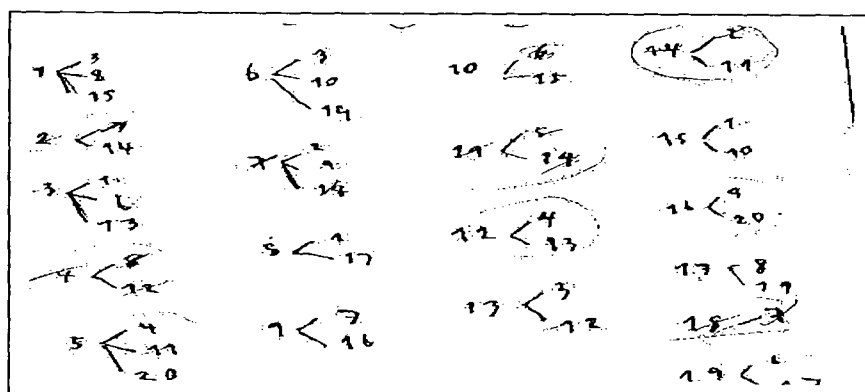


FIGURE 30 CHAIWAT'S DIAGRAM ON THE PERFECT SQUARE PROBLEM

As students were more than half way through the program, in Session 13 (*the apple puzzle problem*) the researcher noticed that they used a more organized and systematic procedure to account for all possible cases. This problem involved the farmer who categorized apples into three different sizes: A, B, and C. Then he put 20 apples for each size in each separately boxes labeled from 1 to 14 (since there were 14 boxes). Unfortunately, he forgot to mark the size of apples for each box. In order to determine the size of apples in each box, he picked up apples from each box equaled to the number on that box. Then, he weighed all apples that he picked up, which weighed 5300 grams with the condition is that the farmer knew that there was only 1 box for size A and only 1 box for size C (see Appendix C). Students combined this strategy with other strategies to solve the problem. The group that Chaiwat belonged to listed all possible box numbers that apples of size C were put in. In order to do this, students used logical reasoning to find out all answers of the problem (see Figure 31).

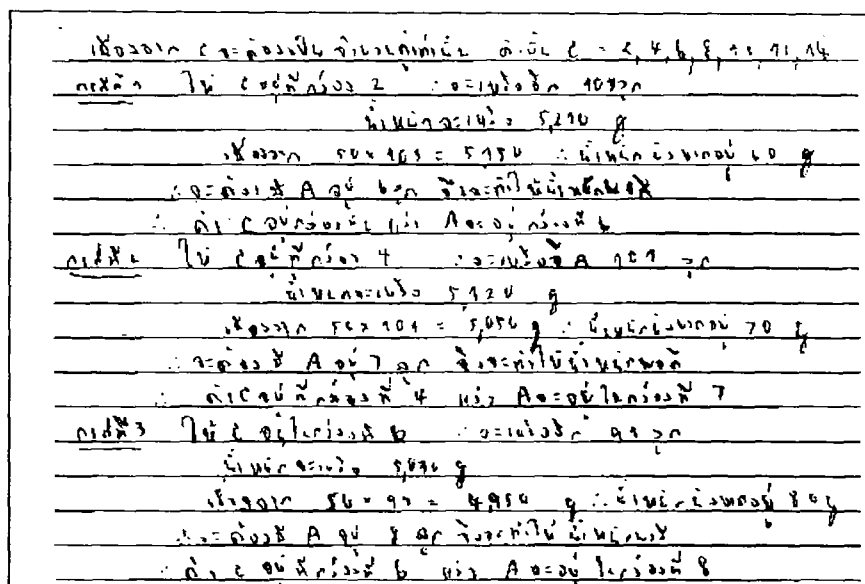


FIGURE 31 CHAIWAT GROUP'S SOLUTION ON THE APPLE PUZZLE PROBLEM

In summary, at the beginning of the program students did not use a systematic procedure to account for all cases. Later, students began to recognize the systematic procedure of accounting for all possibilities. They also combined this strategy with another strategy to solve the problem. This indicated that students were more systematic in accounting for all possibilities as the program proposed.

Students defined variables and set equations more appropriately. In this program, "writing an equation" was another strategy that students often used. They used "writing an equation" for solving 8 out of 20 problems through the program. The frequency of using this strategy among the three groups was 15 (see Table 4). In Session 2 (*the green tea puzzle problem*), this problem required students to find the relationship among the number of green tea bottles that the customer bought, drank, and got for free (see Appendix C). In using equation to solve this problem, students created variables, defined them, and set an equation. But, students did not define the variable and write an equation properly. Nunya and the group defined two variables, x and y and set an equation as follows:

x = the number of free green tea (times of getting for free);

y = the number of times buying green tea (not includes the first seven bottles)

Then, the number of drinking (bottles) = $7 + x + 6y$; $x = y + 1$ --- (1)

(see Figure 32).

$x = \text{จำนวนขวดชาเขียว (สีเขียวเข้ม)}$
 $y = \text{จำนวนขวดชาเขียว (สีเขียวอ่อน + ชาขาว)}$
 $\text{จำนวนขวด} = 7 + x + 6y \quad ; \quad x = y + 1 \rightarrow (1)$
 (ขวด)
 $\therefore 340 = 7 + x + 6y$
 $= 7 + (y+1) + 6y \quad (\because x = y+1)$
 $= 8 + 7y$
 $y = \frac{340 - 8}{7} = 48.57 \approx 49 \text{ ขวด}$
 $x = 49 \quad \text{จำนวน} \quad x = 49$

FIGURE 32 NUNYA GROUP'S WRITING EQUATIONS ON THE GREEN TEA PUZZLE PROBLEM

More than half way through the program, in Session 13, (*the apple puzzle problem*), students began to define appropriate variables and equations. They were more careful in defining variables and setting up equations (see Appendix C). Students defined three variables and set two equations, which were appropriate for solving this problem. Nunya and the group (see Figure 33) defined three variables, x , y and z as follows:

- x is the number of size A apples
- y is the number of size B apples
- z is the number of size C apples

They set two equations as follows:

$$x + y + z = 105 \quad \text{--- (1)}$$

$$60x + 50y + 45z = 5300 \quad \text{--- (2)}$$

$x = \text{ขนาดแอปเปิ้ล A}$
 $y = \text{ขนาดแอปเปิ้ล B}$
 $z = \text{ขนาดแอปเปิ้ล C}$
 $x + y + z = 105 \quad \text{--- (1)}$
 $60x + 50y + 45z = 5300 \quad \text{--- (2)}$
 $(60 + 10)x + 50y + (50 - 5)z = 5300$
 $50x + 10x + 50y + 50z - 5z = 5300$
 $50(x + y + z) + 10x - 5z = 5300$
 $50(105) + 10x - 5z = 5300$
 $5250 + 10x - 5z = 5300$
 $10x - 5z = 50$
 $2x - z = 10$

FIGURE 33 NUNYA GROUP'S WRITING EQUATIONS ON THE APPLE PUZZLE PROBLEM

This trend of defining variables and writing an equation clearly and appropriately was consistent till the last unit of the program.

In summary, at the beginning of the program students did not define variables properly and clearly. When students worked on more problems, they began to define variables properly as well as write equations better.

Students used more logical reasoning in their arguments. In this program, “using logical reasoning” strategy was one strategy that students often used. Students used “using logical reasoning” for solving 7 out of 20 problems through the program. The frequency of using this strategy among the three groups was 15 (see Table 4). In this program, students not only directly used logical reasoning to solve the problem but also used this strategy to check their work. The “using logical reasoning” always come in while working with other strategies like “guessing and checking,” “accounting for all possibilities,” and “making a drawing or diagram.” It was hard to separate logical reasoning from other strategies. At the beginning of this program, students used this strategy to justify their work. In Session 8, students worked on *the mystery number* problem. The mystery number is the largest 7-digit number that no two digits of this number are the same, and the mystery number is divisible by its own digits (see Appendix C). Students use logical reasoning to argue why three digits, 0, 5, and 4, were not the digits of the mystery number. Students selected to use logical reasoning to solve this problem. Kanda and her group tried to convince why digit 0 was not in the mystery number. They reason why they “drop ‘0’ out because it could not divide any number (see Figure 34).

0	1	2	3	4	5	6	7	8	9
0 หาร 0 ได้ 0									
5 หาร 5 ได้ 1									
2 หาร 2 ได้ 1									
4 หาร 4 ได้ 1									
6 หาร 6 ได้ 1									
8 หาร 8 ได้ 1									
9 หาร 9 ได้ 1									
1 หาร 1 ได้ 1									
3 หาร 3 ได้ 1									
7 หาร 7 ได้ 1									

FIGURE 34 KANDA GROUP'S REASONING ON THE MYSTERY NUMBER PROBLEM

Toward the end of the program, students consistently used more logical reasoning in their arguments. In session 16 students individually worked on *the segment in a circle* problem. This problem required students to find the length of the given segment in the

circle (see Appendix C). Students used logical reasoning to explain that the length of segment ST is equal to the radius of the circle (see Figure 35).

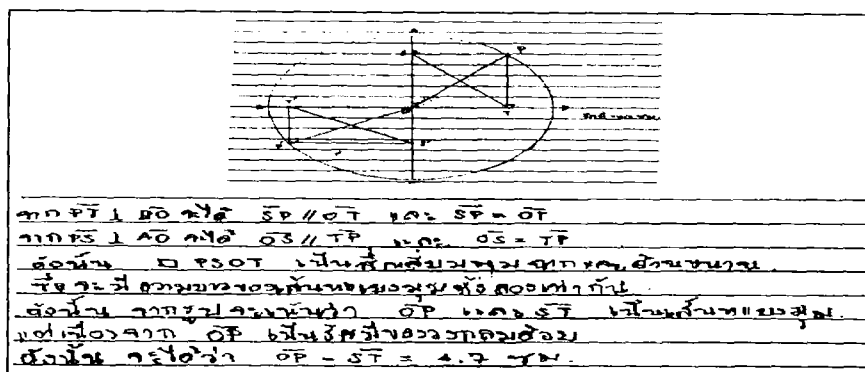


FIGURE 35 KANDA GROUP'S REASONING ON THE SEGMENT IN A CIRCLE PROBLEM

In summary, at the beginning of the program students used logical reasoning while working with other strategies. Later, they used logical reasoning directly for solving the problems. They also used logical reasoning to convince other group members.

Information from the above paragraphs suggested that strategies that students often used in this program were (a) making a drawing or diagram, (b) accounting for all possibilities, (c) writing an equation, and (d) using logical reasoning respectively. Strategies that students moderately used were (a) making a table, (b) looking for a pattern, (c) guessing and checking, (d) using a concrete model, (e) adapting a different point of view, and (f) examining a simpler case or solving a simpler analogous problem. Strategies that students rarely used were working backward, and using indirect reasoning strategies. As students worked on more problems, they made clearer drawings or diagrams and had more systematic on accounting for all possibilities. They defined variables and set equations more appropriately, and also used more logical reasoning in their arguments. Additionally, students reflected in their reflection journal that at the beginning of the program they used only one strategy, "accounting for all possibilities," to solve the given problems. After learning problem solving and having group discussion in this program, they acquired various ways of thinking and used various appropriate strategies to solve the given problems. For example, in Session 11, Nunya reflected that "...now I feel that I have increasing ways of thinking and various strategies, and know how to solve..." Chaiwat also reflected that "...this (program) makes me acquire various ways of thinking and various strategies for solving the problem..." These evidences showed that students could develop their ability to

adapt and apply appropriate strategies in this experimental problem-solving program. Ability to adapt and apply appropriate strategies was one factor that influenced the median score on problem-solving competence was higher for the experimental group.

Ability to find the correct answer to the problem with a clear explanation on solution process

In this study, the researcher looked at the ability to give a complete written explanation of the solution process used along with logical reasoning. Two performance patterns were discerned during classroom observation. These patterns were based on students' written works, checklists and field notes from observations. These patterns were: (a) students gave more reasons to support their written explanation of solution process; and (b) students had more systematic on solution process.

Students gave more reasons to support their written explanation of solution process. They gave logical arguments to support each procedure of solution process. At the beginning of this program, in Session 2 (the green tea puzzle problem, see Appendix C), students gave minimal written explanation of the solution process. Their written works were not clear and had no supporting reason. Students showed what was done but they did not explain how and why. Chaiwat and the group listed the numbers of bottles of green tea that a customer bought and got free, and the answer to the problem. They did not explain how they got such information (see Figure 36).

ถ้าซื้อ 7 ขวด

นม 1 ขวด

นม 1 ขวด

นม 1 ขวด

นม 1 ขวด

นม 1 ขวด

นม 1 ขวด

นม 1 ขวด

7 + (6 x 48) + (1 x 48) + 5 = 348

ถ้าซื้อ 6 ขวด

นม 1 ขวด

นม 1 ขวด

นม 1 ขวด

นม 1 ขวด

นม 1 ขวด

นม 1 ขวด

6 + (5 x 48) + (1 x 48) + 5 = 289

ถ้าซื้อ 5 ขวด

นม 1 ขวด

นม 1 ขวด

นม 1 ขวด

นม 1 ขวด

นม 1 ขวด

5 + (4 x 48) + (1 x 48) + 5 = 241

241 ขวด

FIGURE 36 CHAIWAT GROUP'S EXPLANATION ON THE GREEN TEA PUZZLE PROBLEM

As students studied more than half way of the program, in Session 13 (*the apple puzzle* problem, see Appendix C), students gave more written explanation of solution process along with logical reasoning. In this problem, students defined variables and explained how to use conditions of the problem to build equations. Such condition was "the farmer picks up apples from each box equals to the box number. The total weight of apples that he picked up equals 5300 grams." Their written explanation was clear with reason to support. Chaiwat and the group indicated their reasons before writing the equation (see Figure 38). Students gave the reason of why C (the number of size C apples that were picked up) should be even number.

บทสรุปที่ 1 :

ขนาดกล่อง	ขนาด A	น้ำหนักของกล่อง	60 กรัม
	B	"	50 กรัม
	C	"	45 กรัม

→ เวลาเอาแอปเปิ้ลออกจากกล่องแล้วจะเท่ากับจำนวนแอปเปิ้ลในกล่อง แล้วนำน้ำหนักของแอปเปิ้ลได้ 5300 กรัม

ถ้าให้ $60A + 50B + 45C = 5300$ — (1)

→ ถ้าเอาแอปเปิ้ลจากกล่องเบอร์ 1 มา 1 ใบ 2 มา 2 ใบ ... 14 มา 14 ใบ

∴ จำนวนแอปเปิ้ลที่รับมาคือ $1+2+3+...+14 = 105$ ผล

∴ $1+2+3+...+14 = A+B+C$

∴ $A+B+C = 105$ — (2)

จากสมการ (1) $60A + 50B + 45C = 5300$ — (1)

$A + B + C = 105$ — (2)

$50A + (2) = 50A + 50B + 50C = 5250$ — (3)

(1) - (3) ∴ $10A - 5C = 50$ — (4)

$2A = (4) ∴ 2A - C = 10$ — (5)

เนื่องจาก C จะต้องเป็นจำนวนคู่เท่านั้น เพราะว่าถ้า C เป็นจำนวนคี่ ผลรวมทั้งหมดจะออกมาเป็นเลขคี่ 5 คูณไว้จะได้เป็นเลขคี่ 5300 กรัม

ดังนั้น C ที่เป็นไปได้คือ 2, 4, 6, 8, 10, 12, 14

จาก (5) ถ้า $C = 2 \rightarrow A = 6$

$C = 4 \rightarrow A = 7$

$C = 6 \rightarrow A = 8$

$C = 8 \rightarrow A = 9$

FIGURE 38 CHAIWAT GROUP'S EXPLANATION ON THE APPLE PUZZLE PROBLEM

The researcher noticed that as students had experience in working on more problems, they indicated the reason in the written explanation of the solution process. They used logical reasoning to support their written work.

In summary, at the beginning of the program, students gave little written explanation of the solution process. Students' written works were not clear, and there was no supporting reason. Information from interviewing the target students suggested that they

125
 $\sqrt[5]{1000}$
 $\frac{1000}{8}$
 $\frac{20}{16}$
 $\frac{16}{40}$

$\sqrt{100}$



888 + 8 + 8

112
88

888 + 8 + 8

112
88

888 + 8 + 8

112
88

10. $\frac{888 + 8 + 8 + 1 + \dots + 8}{8 + 88 + 88 + 8}$

11. $\frac{888 + 8 + 8 + 1 + \dots + 8}{8 + 88 + 88 + 8}$

12. $\frac{888 + 8 + 8 + 1 + \dots + 8}{8 + 88 + 88 + 8}$

1. $\frac{888 + 8 + 8 + 1 + \dots + 8}{8 + 88 + 88 + 8} = 1000$

2. $\frac{888 + 8 + 8 + 1 + \dots + 8}{8 + 88 + 88 + 8} = 1000$

3. $\frac{888 + 8 + 8 + 1 + \dots + 8}{8 + 88 + 88 + 8} = 1000$

888 + 8 + 8 + 1 + \dots + 8 = 1000

888 + 8 + 8 + 1 + \dots + 8 = 1000

888 + 8 + 8 + 1 + \dots + 8 = 1000

Later, when students worked on more problems, they began to work increasing systematic on solution process. In Session 13 (*the apple puzzle* problem, see Appendix C), students adapted and applied “writing an equation” and “using logical reasoning” strategies for solving this problem. They defined variables and set equations appropriately. Then they solved the equation step by step. Their solution process was clear and systematic. For example, Nunya and the group defined three variables and two equations. Then they solved such equations systematically. After solving the equations, they used logical reasoning to explain all possible answers of the problem (see Figure 40).

$$\begin{aligned}
 & \text{If } x \text{ is the number of Apples and } y \text{ is the number of Bananas} \\
 & x + y = 100 \quad \text{--- (1)} \\
 & 50x + 20y + 10z = 3200 \quad \text{--- (2)} \\
 & (50 + 10)x + 20y + (100 - 5)x = 3200 \\
 & 50x + 10x + 20y + 1000 - 5x = 3200 \\
 & 50(x + y + 2) + 10x - 5x = 3200 \\
 & 50(100) + 10x - 5x = 3200 \\
 & 5200 + 10x - 5x = 3200 \\
 & 10x - 5x = 3200 - 5200 \\
 & 5x = -200 \\
 & x = -40
 \end{aligned}$$

$$\begin{aligned}
 & \text{If } x \text{ is the number of Apples and } y \text{ is the number of Bananas} \\
 & x + y = 100 \quad \text{--- (1)} \\
 & 50x + 20y + 10z = 3200 \quad \text{--- (2)} \\
 & (50 + 10)x + 20y + (100 - 5)x = 3200 \\
 & 50x + 10x + 20y + 1000 - 5x = 3200 \\
 & 50(x + y + 2) + 10x - 5x = 3200 \\
 & 50(100) + 10x - 5x = 3200 \\
 & 5200 + 10x - 5x = 3200 \\
 & 10x - 5x = 3200 - 5200 \\
 & 5x = -200 \\
 & x = -40
 \end{aligned}$$

$$\begin{aligned}
 & \text{If } x \text{ is the number of Apples and } y \text{ is the number of Bananas} \\
 & x + y = 100 \quad \text{--- (1)} \\
 & 50x + 20y + 10z = 3200 \quad \text{--- (2)} \\
 & (50 + 10)x + 20y + (100 - 5)x = 3200 \\
 & 50x + 10x + 20y + 1000 - 5x = 3200 \\
 & 50(x + y + 2) + 10x - 5x = 3200 \\
 & 50(100) + 10x - 5x = 3200 \\
 & 5200 + 10x - 5x = 3200 \\
 & 10x - 5x = 3200 - 5200 \\
 & 5x = -200 \\
 & x = -40
 \end{aligned}$$

FIGURE 40 NUNYA GROUP'S SOLUTION PROCESS ON THE APPLE PUZZLE PROBLEM

The pattern of having systematic on solution process was consistence till the last unit of the program.

In summary, at the beginning of the program students did not have an organized procedure on solution process. When students worked more on problems, they had increasing systematic on solution process.

Information from the above paragraphs suggested that as students worked on more problems, they gave more reasons to support their written explanation and had more systematic on solution process. Additionally, students reflected in their reflection journal that they had systematic on solution process and written explanation. For example, one student reflected that "If compared with the earlier sessions, now (in Session 16) I am more systematically in solving the problem..." These evidences suggested that students could develop their ability to find the correct answer to the problem with a better explanation on solution process. Ability to find the correct answer to the problem with a clear explanation on solution process was one factor that influenced the median score on problem-solving competence was higher for the experimental group.

Several evidences illustrated in this part suggested that students who had experience on problem solving in this program recognized the important of understanding the problem, they could develop their ability to understand the problem, to adapt and apply appropriate strategies, and to find the correct answer to the problem with a clear

explanation on solution process. These abilities supported the quantitative finding that the experimental group's competence in solving mathematical problems was higher than that of the control group.

Part 3 : Effects on Beliefs Related to Problem Solving

Comparison of Preservice Teachers' Beliefs Related to Problem Solving

In studying the effects on beliefs related to problem solving, the median scores on beliefs related to problem solving of the experimental group and that of the control group were compared. The Wilcoxon Signed Ranks Test was used. Table 7 illustrates results of comparing the median scores on appropriate beliefs related to problem solving of the experimental group and the control group.

TABLE 7 RESULTS OF COMPARING THE MEDIAN SCORES ON APPROPRIATE BELIEFS RELATED TO PROBLEM SOLVING OF THE EXPERIMENTAL GROUP AND THE CONTROL GROUP

		N	Mean Rank	Sum of Rank	z-score	p-value
Score of the experimental group – Score of the control group	Negative Ranks	5 ^b	4.70	23.50	1.538	p< 0.124
	Positive Ranks	8 ^a	8.44	67.50		
	Ties	0 ^c				
	Total	13				

a. Score of the experimental group < Score of the control group

b. Score of the experimental group > Score of the control group

c. Score of the experimental group = Score of the control group

It was found that the median score on appropriate beliefs related to problem solving of the experimental group was not significantly higher than that of the control group at the 0.05 level ($p < 0.124$). The hypothesis on appropriate beliefs related to problem solving of the experimental group was significantly higher than that of the control group was rejected.

In order to better understand appropriate beliefs related to problem solving of the experimental group and that of the control group, the researcher compared their median scores on appropriate beliefs in each aspect : beliefs about mathematics, beliefs about problem solving, and beliefs about teaching and learning mathematics. By using the Wilcoxon Signed Ranks Test, the comparison suggested that the median score on appropriate beliefs in each aspect of the experimental group was not significantly higher than that of the control group at the 0.05 level (see Appendix G Table c). This indicated that appropriate beliefs in each aspect of the experimental group and that of the control group were not different.

Comparison between Pre- and Post- Assessment of the Experimental Group

In addition to the research question, the researcher sought to know if the appropriate beliefs of the experimental groups had been changed between before and after the experiment. The researcher compared the median scores of pre- and post-assessment, by using the Wilcoxon Signed Ranks Test. It was found that the median score on appropriate beliefs related to problem solving of post-assessment was significantly higher than that of pre-assessment at the 0.05 level (see Appendix G Table d).

When looked at each aspect of beliefs, there were two out of three aspects that the median score on appropriate beliefs of post-assessment was significantly higher than that of pre-assessment at the 0.05 level (see Appendix G Table d). Those were beliefs about mathematics and beliefs about problem solving.

With respect to beliefs about mathematics, there were two out of five beliefs about mathematics that the median score on appropriate beliefs of post-assessment was significantly higher than that of pre-assessment at the 0.05 level (see Appendix G Table e). These beliefs were “in mathematics, solution is always accurate, certain, and static” and “mathematics is the study of mathematical operation.” Although they were inappropriate beliefs, students had positive change. For beliefs about problem solving, there were two out of five beliefs about problem solving that the median score on appropriate beliefs of post-assessment was significantly higher than that of pre-assessment at the 0.05 level (see Appendix G Table f). These beliefs were “finding the correct answer is not the most important purpose of solving mathematical problems” and “solving a variety of types of problems is better than solving only one type of problems all the time.”

Part 4 : Effects on Sociomathematical Norms

During the teaching experiment, sociomathematical norms were observed and analyzed from discussion of every presentation. The sociomathematical norms analyzed in this study were normative understanding of what counts as (1) *a reasonable solution process*, (2) *a sophisticated representation*, and (3) *an acceptable mathematical explanation and justification*. The researcher looked at sociomathematical norms that emerged in the experimental problem-solving class and focused on the application of the gained sociomathematical norms to analyzing and concluding about mathematical aspects of students' activities in the problem-solving classroom. The gained sociomathematical norms were negotiated in the small-group working and whole-class discussion that the students reasoned their own solution, presented their solution, asked questions, discussed their own ideas, listened to others ideas, and gave constructive criticism. The below paragraphs describe the emergence of the sociomathematical norms and the application of such sociomathematical norms.

What count as a reasonable solution process

In this study, the sociomathematical norm of what counts as a reasonable solution process emerged during discussion in Session 2. In this session, after students finished their working on *the green tea puzzle* problem, they presented their own solution on the board. After students' presentation, the researcher posted the question "what count as a reasonable solution process" and provided an opportunity for students to discuss and make a normative understanding of what counts as a reasonable solution process. Students discussed about this idea and gave ideas about what a reasonable solution process should be.

Student 1: The reasonable solution process should have facts, properties or theories to support each procedure.

Student 2: Between procedures, it should have a connection.

Researcher: Any thing else?

Student 3: It should also use correct mathematical principles.

Researcher: What else?

Every student: That's all.

After that, students used the gain sociomathematical norm as criteria to analyze the solution process of each group. Students selected a group work that was considered

as the best on “reasonable solution process” among the three groups. They gave the reason for their selection that such group expressed the reason on their solution process. At the end of this session, students agreed on a normative understanding of what counts as a reasonable solution process that emerged in the class. That was, “a mathematical solution process was called ‘a reasonable solution process’ if that solution process used correct mathematical principles; used facts, properties or theories to support each procedure; and had a continuous connection between procedure.”

Later, in Session 8 when students worked on *the mystery number* problem (see Appendix C), they added more normative understanding of what counts as a reasonable solution process. That was, “a solution process will be reasonable if each procedure followed logical reasoning.” During students used logical reasoning to explain why three digits, 0, 5 and 4, were not the digits of the mystery number, some students asked other students in the group a logical reason to support each procedure of the solution process.

Student 1: 4 is not a digit in the mystery number. But I don’t know how to explain. Is it necessary that we will explain it?

Student 2: Yes. Because the solution process will be reasonable if each procedure followed logical reasoning.

Students used the gain normative understanding of what counts as a reasonable solution process as criteria to analyze their own solution process and others’ solution process till the last session.

What counts as a sophisticated representation

The sociomathematical norm of what counts as a sophisticated representation also emerged during discussion in Session 2. In this session, after students gained the normative understanding of what counts as a reasonable solution process, the researcher continuously posted the question “what count as a sophisticated representation.” Then providing an opportunity for students to discuss and make a normative understanding of what counts as a sophisticated representation. Students discussed about this idea and proposed their ideas about what a sophisticated representation should be.

Student 1: It would use appropriate mathematical symbols or signs.

Student 2: The use of symbols must represent ideas precisely.

Student 1: Yes.

Researcher: Any thing else?

Student 3: It would be clear and systematic.

Student 1: The use of graphs, tables, or diagrams depicts mathematical ideas visually. Such as the table shown in Group A is work.

Researcher: Any thing else?

Other students: That's all.

Then students concluded that, "a mathematical representation was called '*a sophisticated representation*' if the mathematical representation used appropriate mathematical symbols or signs to represent ideas precisely; had clear and systematic representation, used graphs, tables, or diagrams to depict mathematical ideas visually (if needed). Students used this gain sociomathematical norm to analyze the mathematical representation of each group for the root of the program.

What counts as an acceptable mathematical explanation and justification

The sociomathematical norm of what counts as an acceptable mathematical explanation and justification was another norm that emerged during discussion after the representation of all groups in Session 2. After discussion on a sophisticated representation, the researcher posted the question "what count as an acceptable mathematical explanation and justification." Then providing an opportunity for students to discuss and make a normative understanding of what counts as an acceptable mathematical explanation and justification. One student suggested "at first, it should use the language of mathematics to express mathematical ideas precisely." Another added that "if any questions coming up, we will explain and justify them correctly."

After that, students had a normative understanding of what counts as an acceptable mathematical explanation and justification. That was, "A mathematical explanation was called '*an acceptable mathematical explanation and justification*' if the mathematical explanation used the language of mathematics to express mathematical ideas precisely, and explained or justified any question correctly.

The following table illustrates the summary of sociomathematical norms and normative understanding of the problem-solving classroom.

TABLE 13 SUMMARY OF SOCIOMATHEMATICAL NORMS EMERGED IN THE
PROBLEM-SOLVING CLASSROOM

Sociomathematical norms	Normative understanding of the problem-solving classroom
1. A mathematical solution process was called <i>"a reasonable solution process"</i>	If the mathematical solution process : ● used correct mathematical principles ● used facts, properties, or theories to support each procedure ● followed logical arguments ● had a continuous connection between procedures
2. A mathematical representation was called <i>"a sophisticated representation"</i>	If the mathematical representation : ● used appropriate mathematical symbols or signs; used symbols to represent ideas precisely ● had clear and systematic representation ● used graphs, tables, or diagrams to depict mathematical ideas visually (if need)
3. A mathematical explanation was called <i>"an acceptable mathematical explanation and justification"</i>	If the mathematical explanation : ● used the language of mathematics to express mathematical ideas precisely ● explained and justified any question correctly

CHAPTER 5

CONCLUSION AND DISCUSSION

Summary of the Study

Purposes of the Study

1. To study the effects of the problem-solving instructional program on preservice teachers' competence in solving mathematical problems.
2. To study the effects of the problem-solving instructional program on preservice teachers' beliefs related to problem solving.
3. To study preservice teachers' sociomathematical norms in the experimental problem-solving class.

Research Hypotheses

1. The median score on problem-solving competence of the experimental group was significantly higher than that of the control group.
2. Following instruction, the median score on appropriate beliefs related to problem solving of the experimental group was significantly higher than that of the control group.

Methodology

Selecting sample and research design. This study was an experimental research that used posttest-only control group design. Twenty-six second-year students in the mathematics teacher education program of Srinakharinwirot University were subjects for this study. Those students were paired and partitioned into two balanced groups: experimental group and control group. Each group consisted of 13 students. Three students of the experimental group are chosen as target students for case studying on problem-solving development and beliefs related to problem solving.

Developing the framework of the instructional program. This problem-solving instructional program was designed for second-year students in a mathematics teacher education program. This program consisted of twenty 2-hour class periods. The goals of this program were to develop students' competence in solving mathematical problems, to enhance appropriate beliefs related to problem solving, and to identify emerging

sociomathematical norms. In this program, students learned and experienced on using problem-solving process and strategies. To learn and experience solving problem, students were expected to solve various types of problems over a prolonged period of time. Non-routine problems and open-ended problems that allowed to solve more than one method or strategy were used in this program. Cooperative learning was also used in the program. During problem solving, students were encouraged to monitor and evaluate their own and group members' thinking by using metacognition, and also to reflect on the process of problem solving. Furthermore, students were motivated to have appropriate beliefs related to problem solving along with creating sociomathematical norms.

Developing instruments. There were two types of instruments used in the study: instruments for problem-solving instruction and instruments for evaluation. Instrument for problem-solving instruction consisted of twenty 2-hour lesson plans. While instrument for evaluation consisted of pretest on fundamental background, posttest on problem solving, questionnaire on belief related to problem solving, and observation forms. The pretest on fundamental background was used to assess mathematical knowledge and problem-solving ability that students brought to this experiment. The gain results were used to conjecture the learning process and formulate the lesson plans. The posttest on problem solving and questionnaire on beliefs related to problem solving were given to assess competence in solving mathematical problems and beliefs related to problem solving of the experimental group and that of the control group. The observation forms were constructed to assess competence in mathematical problem solving of target students and the groups as well as sociomathematical norms emerged in the classroom. Observation forms used in this program consisted of observation checklist and observation field notes. Three educational experts evaluated the lesson plans for content validity, appropriateness, and difficulty of the test questions. After revising, the instruments were tried out with twelve second-year students in a mathematics teacher education program at Suan Sununta Rajabhat University in order to examine the possibility and reliability of the test.

Experimenting and collecting data. One week before the teaching experiment started, the experimental group and the control group were assessed their mathematical knowledge, ability on problem solving, and beliefs related to problem solving. As the experimental group participated as students of the program, the control group took the same other courses as the experimental group. In this program, twenty 2-hour class periods were conducted, 3 times a week. The first session began with an introduction to the problem-solving process. Four-class periods involved group problem solving that

focused on 4-strategies. This was followed by one class period of individual problem solving that focused on learning one of the previous 4-strategies, and then students were asked to write their reflection journal about problem solving. During each problem solving activity, three target students were observed and analyzed with respect to their ability to understand the problem, adapt and apply appropriate strategies, and find the correct answer to the problem. The group's competence in solving mathematical problems was also observed and analyzed. After the teaching experiment, the posttest on problem solving and the questionnaire on beliefs related to problem solving were given to the experimental group and the control group.

Analyzing the data. There were three categories of data analyzed in this study: problem-solving competence, beliefs related to problem solving, and sociomathematical norms. With respect to problem-solving competence, the median scores on problem-solving competence of the experimental group and that of the control group were compared. Rubric scoring scale from 0 to 4 was used. The Wilcoxon Signed Ranks Test was used for hypothesis testing. Additionally, the group's written work along with information from the Observation Checklists and students' reflection journals were used to evaluate the group's competence in solving mathematical problems. Individual written work, interview data on problem-solving competence based on their individual written work, and the target students' reflection journal were used to evaluate the target students' problem-solving competence. According to beliefs related to problem solving, following instruction the median scores on appropriate beliefs related to problem solving of the experimental group and that of the control group were compared. Likert scaling was used. The Wilcoxon Signed Ranks Test was also used for hypothesis testing on beliefs. For sociomathematical norms, during the teaching experiment sociomathematical norms were observed and analyzed from discussion of every presentation.

Conclusion and Discussion

Effects on competence in solving mathematical problems

As the researcher compared competence in solving mathematical problems of the experimental group with the control group, it was found that students who had experience on problem solving in this program had significantly higher competence in solving mathematical problems than that of the control group. Additionally, evidence from the analysis of students' performance also indicated that during instruction the experimental

students developed their ability to understand the problem, adapt and apply appropriate strategies, and work out the correct answer of the problem with a clear explanation on solution process. Charles, Lester, and O'Daffer mentioned that ability to understand the problem, select and use problem-solving strategies, and find correct answers of the problem are important for developing students' problem-solving competence (Charles; Lester; & O'Daffer. 1987 : 7-13).

With respect to understanding the problem, the analysis demonstrated that when students worked on more problems, they recognized the importance of understanding the problem. They spent more time on trying to understand the problem, showed more evidence of analyzing the problem to help them understand the problem. They also spent more time in a group discussion on the problem and its concepts before attempting to solve it. Although some problems used in this program were difficult and complex, the cooperative learning and group discussion helped students for a better understanding the problem. As O'Connell stated that "cooperative learning strategies are valuable tools during problem-solving instruction, as they allow students to hear others' thoughts and help each students expand his or her repertoire of ideas" (O'Connell. 2000 : 9).

With respect to adapting and applying appropriate strategies, the analysis suggested that as students worked on more problems, they made clearer drawings or diagrams, and were increasing systematic on accounting for all possibilities. They defined variables and set equation increasing appropriately, and used increasing logical reasoning to convince their arguments. This program encouraged students to solve more than one strategy, and the problems to be solved in the program were problems that allowed a variety of appropriate strategies. Students learned and experienced on using various strategies to solve problems, they could develop their abilities to adapt and apply appropriate strategies. Kroll and Miller stated that "good problem solvers usually have a wide range of strategies at their disposal (strategies knowledge) and demonstrate facility in choosing which of these strategies to apply in various situations" (Kroll; & Miller. 1995 : 64).

With respect to finding the correct answer to the problem with a clear explanation on the solution process, the analysis also suggested that as students worked on more problems, they did better in giving more reasons to support their solution process, and working systematically on solution process as well. Since in this program, students were encouraged to show how they solved the problems, instead of just writing only the answer. This may help students to improve their problem-solving competence. According to the statement of Frank:

To get students to become better problem solvers, it should focus on entire solution, not just numerical answers. If a student uses a reasonable strategy in the solution to a problem, he or she should not harshly penalized for obtaining an incorrect answer. If the student did not obtain an answer at all, but can tell what has been learned from working at a solution, this response should also be encouraged (Frank. 1988 : 34).

These evidence demonstrated that the experimental students were significantly higher competence in solving mathematical problems than the control group.

Effects on beliefs related to problem solving

In studying the effects on beliefs related to problem solving, the researcher compared appropriate beliefs related to problem solving of the experimental students and the control group. The comparison demonstrated that appropriate beliefs related to problem solving of the experimental students were not significantly higher than that of the control group. When compared each aspect of appropriate beliefs, the comparison suggested that appropriate beliefs in each aspect of the experimental group were not significantly higher than that of the control group as well. Frank stated that "a person's mathematical beliefs do not develop overnight. They develop slowly, over a long period of mathematical encounters and experiences" (Frank. 1988 : 34). In the above comparison each aspect of beliefs, it was found that appropriate beliefs about problem solving of the experimental group was not significantly higher than that of the control group with $p < 0.054$ (see Appendix G Table c).

However, when looked at appropriate beliefs of the experimental group before and after the experiment, it was found that appropriate beliefs related to problem solving between pre- and post- assessments of the experimental students were significantly different. There were two out of three aspects that appropriate beliefs between pre- and post- assessment were significantly different. These aspects were beliefs about mathematics and beliefs about problem solving. Additionally, the comparison demonstrated that four out of fifteen beliefs between pre- and post- assessments of the experimental group were significantly different. These beliefs were: (1) in mathematics, solution is always accurate, certain, and static; (2) mathematics is the study of mathematical operation; (3) finding the correct answer is not the most important purpose of solving mathematical problems; and (4) solving a variety of types of problems is better than solving only one type of problems all the time. This finding suggested that this experimental program could change students' beliefs about mathematics. They began to realize that mathematics need not always accurate, certain, or static, and mathematics is not only the study of

mathematical operation. The experimental students also changed the beliefs about problem solving that finding the correct answer is not the most important purpose of solving mathematical problems; and solving a variety of types of problems is better than solving only one type of problems all the time. This finding was not surprising since the program was conducted for a short teaching experiment.

For beliefs about teaching and learning mathematics, this experimental program did not cover how to teach problem solving and mathematics. The experimental students' beliefs about teaching and learning mathematics did not change in this program.

Effects on Sociomathematical Norms

Studying the effects on sociomathematical norms was another purpose of this study. Sociomathematical norms emerged in this problem-solving classroom were normative understanding of what counts as a reasonable solution process, a sophisticated representation, and an acceptable mathematical explanation and justification. Students agreed on the normative understanding of what counts as a reasonable solution process that "a mathematical solution process was called '*a reasonable solution process*' if that solution process used correct mathematical principles; used facts, properties or theories to support each procedure; followed logical reasoning; and had a continuous connection between procedure. According to what counts as a sophisticated representation, students agreed on a normative understanding that "a mathematical representation was called '*a sophisticated representation*' if the mathematical representation used appropriate mathematical symbols or signs to represent ideas precisely; had clear and systematic representation and used graphs, tables, or diagrams to depict mathematical ideas visually (if needed). According to what counts as an acceptable mathematical explanation and justification, students made a normative understanding that "a mathematical explanation was called '*an acceptable mathematical explanation and justification*' if the mathematical explanation used the language of mathematics to express mathematical ideas precisely, and explained and justified any question correctly. Although the above sociomathematical norms were normative understanding for this classroom, they were important because they helped students and the teacher set forth a way of analyzing on mathematics aspect of their activities in the problem solving classroom.

Evidence from the above paragraphs indicated that the problem-solving instructional program developed by the researcher was effective for developing preservice teachers' competence in solving mathematical problems. The teacher education program

can adapt this problem-solving instructional program as a required course in the teacher education program for mathematics preservice teachers or integrate this program in mathematics courses.

Suggestions

Implication for Teaching and Teacher Education

As students who had experience on problem solving in this problem-solving instructional program had significantly competence in solving mathematical problems than the control group. This indicated that this program was effective for developing preservice teachers' competence in solving mathematical problems. Additionally, the current school curriculum reform in Thailand is focused on mathematical process, in particular, problem solving. Therefore, teacher education program should also focus on problem solving. The problem-solving instructional program developed by the researcher may be treated as a required course in the teacher education program for mathematics preservice teachers. Such program should have the preservice teachers experienced on problem solving with various types of problems, i.e., open-ended problems, non-routine problems, and real world problems. The problems to be solved should provide with a wide range for using a variety of strategies to solve the problems. Since this program focuses on the abilities to understand the problem, using appropriate strategies, and correct answers with a clear explanation on solution process which seems to develop the problem-solving competence, thus the researcher recommend that such course for the preservice teachers should focus on these aspects.

It is also recommended that the teacher education program may adapt this problem-solving instructional program by integrating problem solving in mathematics courses.

With respect to students' beliefs, the median score on appropriate beliefs about problem solving of the experimental group was not significantly higher than that of the control group with $p < 0.054$. This indicated that the experimental students' beliefs about problem solving was almost higher than that of the control group. Thus the program should add some activities or run for a longer period of time in order to have the effect on their beliefs.

Implication for Further Research

Although numerous studies have examined the problem-solving development of students, a few studies have examined that of preservice teachers. As this study provided information for developing a problem-solving instructional program, the further researcher may adapt this study to develop a problem-solving course by adding on "how to teach problem solving in schools" in the program. Further research may be conducted by extended developing two-semester problem-solving course with a combination of developing reasoning and communication. The researcher also recommends that for the further research to study the developing of competence on problem solving by integrating problem solving in mathematics courses.

Further research may study the result of developing competence in solving mathematical problems by integrating problem-solving experience in mathematics courses compare with the a separate course on problem solving.

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APPENDIX

APPENDIX A

NAME OF THE EXPERTS IN THIS STUDY

Name of the Experts in this Study

There are four experts to recommend lesson plans, pretest on fundamental background, posttest on problem solving, questionnaire on beliefs related to problem solving, and observation forms in this study as follows:

1. Assoc. Prof. Dr. Narong Punnim
Mathematics Department, Faculty of Science,
Srinakharinwirot University
2. Assoc. Prof. Dr. Preecha Nowyenphon
School of Educational Study,
Sukhothai Thammathirat Open University
3. Dr. Somdech Boonprajak
Mathematics and Applied Statistics Program
Phranakhon Rajabhat University
4. Dr. Chanya Pooudom
Donmuang Chaturachinda School

APPENDIX B
AN EXAMPLE OF LESSON PLAN



Lesson Plan # 2

Title ... Problem Solving _ The Green Tea Puzzle Problem

Course ... The Problem-Solving Instructional Program

Time 2 hours

“Problem solving _ The green tea puzzle problem” is the second session of the problem-solving instructional program. In this session, students are expected to have experience on group problem solving by following the guidance for group problem solving and performing in classroom that learn in the first session. As this program bases on the belief that students themselves should solve lots of problems and solve problems with more than one strategy. Students are expected to solve a non-routine problem and use more than one strategy for solving the problem. The problem used in this session is *the green tea puzzle* problem, a story problem about a customer considers the promotion to get advantage to buy green tea. This promotion is “if a customer exchanges seven empty bottles at the POP store, he will get one free.” The question of this problem is that “if a person who used this promotion found that he drank 348 bottles altogether, what is the smallest number of green tea bottles he bought.” This problem can be solved more than one method or strategies. These strategies are (a) accounting for all possibility, (b) making a table, (c) making a drawing or diagram, (d) writing an equation, and (e) looking for a pattern.

After solving the problem, each group of students is provided an opportunity to present their solutions and discuss their own ideas. After group presentation and discussion, understanding the concept of sociomathematical norms are introduced and discussed. These sociomathematical norms are normative understanding of what counts as (a) *a reasonable solution process*, (b) *a sophisticated representation*, and (c) *an acceptable mathematical explanation and justification*. Students are expected to participate in creating normative understanding of such norms. The gained sociomathematical norms are used as criteria to analyze the solution of each group.

Learning Objectives

1. ***According to competence in solving mathematical problems.*** Students are expected as follows:

- 1.1 To experience on group problem solving
- 1.2 To solve a non-routine problem, the green tea puzzle problem
- 1.3 To use various strategies to solve the given problem, such as,
 - accounting for all possibilities
 - making a table
 - making a drawing or diagram
 - writing an equation
 - looking for a pattern

2. ***According to beliefs related to problem solving.*** Students are expected as follows:

- 2.1 To recognize the belief that “many mathematics problems can be solved in more than one way”
- 2.2 To recognize the belief that “Mathematics is a creative and useful discipline in which they learn to think

3. ***According to sociomathematical norms in the classroom.*** Students are expected as follows:

- 3.1 To understand the guidance for group problem solving and performing in classroom
- 3.2 To understand the concept of sociomathematical norms
- 3.3 To begin to participate in creating normative understanding of what counts as a reasonable solution process, a sophisticated representation, and an acceptable mathematical explanation and justification

Learning Concepts

1. Solving a mathematical problem by using various strategies, i.e., accounting for all possibility, making a table, making a drawing or diagram, writing an equation, and looking for a pattern.

2. Solving a mathematical problem by using mathematical concepts on Algebra, i.e., pattern analysis and solving equations.

Materials

1. Problem #2 : The green tea puzzle problem
2. Work sheet #2 : The green tea puzzle problem
3. Observation checklist and observation field note

Instructional Activities

Two instructional activities are arranged in this session: Group problem solving, and evaluating students' problem solving.

1. ***According to group problem solving.*** This instructional activity spends 60 minutes on the following.

- 1.1 Arranging students as small groups with four students each. They are in the same group as in the first session.
- 1.2 Presenting the problem situation by giving the Problem #2 for all students and the Work sheet #2 for each group.
- 1.3 Providing 2-3 minutes students for reading and understanding the problem individually.
- 1.4 If students are stuck, the teacher posts a question for group discussion, for example,
 - What is this problem related to?
 - What brand of green tea do you know?
 - Explain me (the teacher), what does this problem concern? (restate the problem, not read it)
 - From the given problem, if a customer drank 14 bottles, what is the smallest number of green tea bottles he bought?
 - From the given problem, if a customer drank 22 bottles, what is the smallest number of green tea bottles he bought?
- 1.5 In solving the problem, the teacher supports students to use Polya's four step problem-solving process along with the dynamic problem-solving process. These processes are understanding the problem, devising a plan, carrying out the plan, looking back, and posing a problem.
- 1.6 Supporting students to participate in the group discussion and group problem solving as well.

1.7 During students are solving the problem, the teacher supports them to monitor or evaluate their thinking or problem solving by questioning themselves and other students in the group. For example,

- Do we clearly and correctly understand the problem?
- Is it possible to devise that plan in solving the problem?
- Does it work to carry out this plan? If not, there should be a change of thought?
- Is there another plan in solving this problem?
- What is the obstacle or factor helping in this problem solving?

1.8 During students are solving the problem, the teacher monitors and evaluates students' problem-solving performance. These performances are ability to understand the problem, to adapt and apply appropriate strategies, and to find the correct answers with a clear explanation on solution process. In order to examine their understanding the problem and its solution, the teacher provides students to pose or extend a problem that related to the given problem.

1.9 After they got the correct answer, supporting students to look back the solution and find out another strategy for solving the problem, or solve more than one strategy.

2. According to evaluating students' problem solving. This instructional activity spends 60 minutes on the following.

2.1 Providing each group to present their own solution.

2.2 Providing students to participate in whole class discussion on solution process and strategies of each group. Examples of topics for discussion are as follows:

- What content/ knowledge is essential to solve this problem?
- What methods/ strategies are used in solving problem?
- Which group's solution process is reasonable? Why?
- Which group's representation is sophisticate? Why?
- Which group' s explanation and justification is accepted? Why?
- From the given problem, what is relationship among the data?
- Is it possible to find the answer if changing the number of green tea bottles that an actor drank from 348 to 499?

- If changing the condition of promotion, such as, exchanging 14 empty bottles at the POP store, he will get two free, is there the difference between the new solution and the original solution? How?

2.3 The teacher and students participate in concluding the solution process, and appropriate strategies for solving this problem. Examples of topics for discussion are as follows:

- Mathematical concepts used for solving this problem are concepts on algebra, i.e., pattern analysis and solving equations.
- Strategies used for solving this problem are accounting for all possibility, making a table, making a drawing or diagram, writing an equation, and looking for a pattern.

2.4 Supporting students to pose or extend a problem.

Evaluation

Two data sources are used to evaluate students' performance as follows:

1. Students' solution in Written work #2 : The green tea puzzle problem
2. Evidences from observation on ability to understand the problem, to adapt and apply appropriate strategies, to find the correct answer with a clear explanation on solution process. These evidences are recorded in observation checklist and observation field note

Conjectured Learning Process

1. Students can solve the green tea puzzle problem.
2. Students use more than one strategy to solve the given problem. These strategies are accounting for all possibility, making a table, making a drawing or diagram, writing an equation, or looking for a pattern.

Notice after instruction

APPENDIX C
EXAMPLES OF PROBLEM

EXAMPLES OF PROBLEM

Session 1 : *The Digit 8 and + Problem.* Using digit “8” and sign “+” to make number “1000”, we can write the patterns as following examples:

$$\begin{aligned}8 + 8 + 8 + \dots &= 1000 \\8 + 88 + \dots &= 1000 \\8 + 888 + \dots &= 1000 \\8 + 88 + 888 + \dots &= 1000 \text{ etc.}\end{aligned}$$

If there is no limitation of number of “8s” and “+”, (1) What are all these patterns? (2) Which pattern has the smallest number of sign +?

Session 2: *The Green Tea Puzzle Problem.* Today, teenagers and working people like to drink green tea for maintaining their health. Green tea has ingredients of Poly-phenol, which is anti-oxidant naturally. Thus, drinking green tea regularly is not only being able for stopping thirst, but also for being healthy. The Rungfa Company produces green tea. The containers are both boxes and bottles. The brand name is Healthy Green Tea. In order to return the advantage or benefit to customers, the company arranges an option for sale promotion:

If you have 7 empty bottles of Healthy Green Tea, you can exchange for one bottle of green tea for free at every branch of POP store.

Moreover, the company advises a strategy of “Economical drinking” to customers that:

When you buy 13 bottles of Healthy Green Tea and drink the first 7 bottles. You can bring these 7 empty-bottles to get a new bottle at POP store. You have 7 bottles now. After drinking all, you can take the empty bottles to exchange one more bottle. In this case, you drank 15 bottles of green tea, but you paid only 13 bottles. Finally you still have one more empty-bottle.

An actor, Tik-Jedsada, is one of customers who drink this brand of green tea and use the promotion and strategy of “Economical drinking” according to the company advice above. During the last three months, he drank 348 bottles of green tea.

Question: What is the smallest number of bottles of green tea that he had to buy?

Session 5: The Locker Puzzle Problem. In the gymnasium, there are 100 lockers for 100 sportsmen. Each locker is labeled a sequential number, No.1-100. One day, every sportsman came in the gymnasium one at a time.

The 1st sportsman came in and opened the doors of every locker.

The 2nd sportsman came in and closed the doors of lockers, No.

2, 4, 6, 8, 10, ... , 100.

The 3rd sportsman came in and changed* the doors of the locker, No.

3, 6, 9, 12, 15, ...

* Note that the word “change” means

if the door is opened, the sportsman will close it.

If the door is closed, the sportsman will open it.

The 4th sportsman came in and changed the doors of lockers, No.

4, 8, 12, 16, 20, ...

The 5th sportsman came in and changed the doors of the lockers, No.

5, 10, 15, 20, 25, ...

Continuously, the 100th sportsman came in and changed the door.

Question: After the 100th sportsman came and changed the door, which lockers were opening?

Session 8: The Mystery Number Problem. Paradorn was in a game show of a TV program, ‘Champion of the Town.’ The host asked him to find the mystery number. This number was the largest 7-digit number and:

(1) no two digits of this number are the same,

(2) this number is divisible by its own digit.

Question: What is the mystery number? Explain your answer.

Session 9: The Perfect Square Problem. After graduated a Bachelor’s Degree in Mathematics, Miss Maikaew did not get in touch with her friends. Consequently, she had invited 17 friends for a dinner party at her place. She gave each friend an invited card, which was attached with one number from No. 2 to No. 18 (No.1 was with her). In order to have her friend using mathematics knowledge, she set a condition for her friends:

Could you come with a partner? You and your partner had to have sums of attached numbers in the cards be a “perfect square.”

In dinner party, her friends came by pairs and each pair had the sums of attached numbers on cards as the perfect square except one friend, who came as her partner.

Question: What number was attached in the card of the one coming as the partner of Miss Maikaew?

Session 13: The Apple Puzzle Problem. Every day a farmer picks up apples and sells them at a market. He categorizes apples into three different sizes, A, B and C.

Size A weighs 60 grams / each.

Size B weighs 50 grams / each.

Size C weighs 45 grams / each.

The same size apples were always loaded in the same boxes. One early morning, the farmer load apples in 14 boxes, 20 apples in each box. The boxes were labeled a sequential number 1-14. Unfortunately, the farmer did not label the size of apples on each box. He remembered that he loaded sized-A apples in one box, sized-C apples in one box, and other boxes for sized-B apples.

In order to determine the size of apples in each box, the farmer picked apples from each box. The number of apples he picked equal to the box number. He weighed all apples he picked from the 14 boxes. The total weight of all apples was 5300 grams.

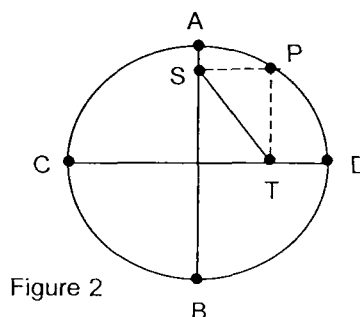
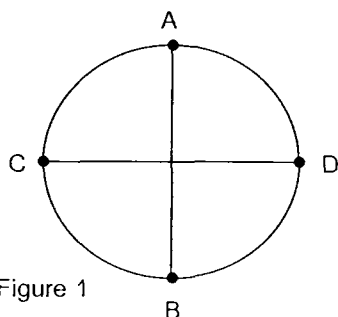
- Question:
- (1) Which boxes certainly contained sized-B apples?
 - (2) Which boxes contained sized-A apples?
 - (3) Which boxes contained sized-C apples? Explain your answer.

Session 16: The Segment in A Circle Problem.

Material: An A4 paper

A compass, a ruler, and a pen

- Processes:
1. Draw a circle on an A4 paper.
 2. Draw two perpendicular diameters AB and CD (see Figure 1).



3. Select any point P on an arc AD.
4. At P draw two segments, one segment is perpendicular to $\overline{CD}$ at T, the other perpendicular to $\overline{AB}$ at S (see Figure 2).

Questions: (1) Find the length of $\overline{ST}$ and explain your answer.
 (2) If we change the position of "P" to other position, will we get the same answer? Please explain your answer.

Session 17: The Magic Traffic Light Problem. In Bangkok, we have to have a good plan for a trip. Before going out, we should think about the short cut and the route that do not have many traffic lights. For one straight road, there are traffic lights for every 2.2 kilometers. Every traffic light changes at the same time with a constant time for green, yellow, and red lights for 2 minutes, 5 seconds, and 40 seconds respectively. One day, Mr. Tik-Jetsada, a Thai actor, must play a soap opera and traveled on that road. He drove out of the first traffic light after the red light has changed in to green.

Questions: (1) In order to pass through every traffic light without pausing, find the speeds of his car?
 (2) If Mr. Tik drive at 40 kms./ hrs., which traffic light will be his first get the red light and how long will he wait for the green light to come?

Explain your answer.

APPENDIX D
POSTTEST ON PROBLEM SOLVING

Posttest on Problem Solving

Problem 1 : Area Uncovered with Water

The yard of one public park is in round shape. The gardener sets 2 sprinkles at A and B position to water the yard everyday. Turning on the tap, the water will spray in circle. Because the efficiency of sprinkles is not equal, the water spreads out in different distances. Some areas are not covered with water (see Figure 1).

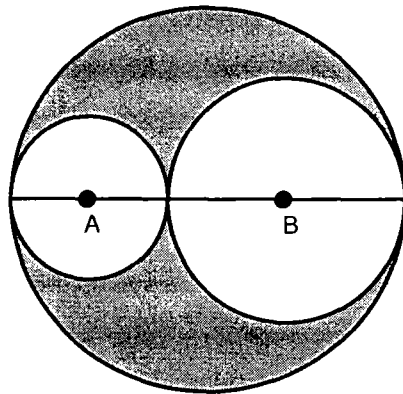


Figure 1

If the ratio of diameters of three circles is 2 : 3 : 5.

To determine what is the ratio between the areas uncovered with water and the area of whole yard? Explain your answer.

Posttest on Problem Solving**Problem 2 : Coffee Puzzle**

The Rungfa Company produces ice coffee in can. The brand name is "RF Ice Coffee." In order to return the advantage or benefit to customers, the company arranges 2 options for sale promotion:

Option 1: If you have 3 empty cans of RF Ice Coffee, you can exchange for one can of coffee for free at every branch of POP store.

Option 2: If you have 5 empty cans of RF Ice Coffee, you can exchange for two cans of coffee for free at every branch of POP store.

That is,

If you choose Option 1 and buy 3 cans of RF Ice Coffee. After drinking all, you can bring these 3 empty cans to get a new can at POP store. In this case, you drank 4 cans of ice coffee, you paid only 3 cans. Finally, you still have one more empty can.

If you choose Option 2 and buy 5 cans of RF Ice Coffee. After drinking all, you can bring these 5 empty cans to get two new cans at POP store. In this case, you drank 7 cans of ice coffee, you paid only 5 cans. Finally, you still have two more empty cans.

Karin is one of customers who drink this brand of ice coffee. He wants to drink 239 cans of ice coffee within six months and gets the most benefit of using these above promotions.

- (1) Which option should he choose?
- (2) By using such option, what is the smallest number of cans of ice coffee that he had to buy?

Posttest on Problem Solving**Problem 3 : Transportation Puzzle**

Nowadays, traveling from province to province is easy and convenient. You have not to wait for hours because there are many private bus transportation companies to support your travel. Moreover, each company holds the policy of “the best is customer services.”

For example: The Rungfa Transportation, a private transportation company, has a policy to have air-conditioned buses traveling round-trip between Bangkok and Khon Khan. These buses travel on the same route thorough 24 hours. Each bus leaves each province's bus-station in every 30 minutes.

Miss Paula, a passenger of this private transportation company, travels from Bangkok to Khon Khan. During traveling, she counts every bus of this company coming from Khon Khan along the way. If her bus leaves Bangkok bus-station, at 09.00 a.m. and arrives Khon Khan bus station at 15.00 p.m.

To determine...

- (1) What is the number of buses coming from Khon Khan counted by Miss Paula?
- (2) What time do these buses leave Khon Khan bus station?

Explain your answer.

Posttest on Problem Solving**Problem 4 : OTOP – Thai Product**

Nowadays, Thai Government supports and promotes people in each Tumbol to use local materials for producing products under the project called OTOP (One Tumbol One Product). People in Bongton Tumbol, Doikum Amphoe, Chiangmai Province have an idea of using northern local flowers such as Auang Luang, a very beautiful flower, to make ornamental flowers by being oven and paraffin them. These ornamental flowers are patented as "Ornamental flowers of Tumbol Bongton". In every OTOP Fair, Bongton villagers always demonstrate the production process of the ornamental flower and sell them.

In the Chiangmai Red Cross Fair 2004, Miss Soifa, Bongton's representative brought the ornamental flowers for sale at the price of 200 bahts/each, and everyday she also made new ornamental flowers before the booth was opened.

On the first day, after Miss Soifa made new ornamental flowers, the total number of the ornamental flowers that she originally brought and made was twice the number of the ornamental flowers she brought. Thorough that day, she can sell 30 flowers.

The second day is Saturday. Miss Soifa thought that today she has more customers than yesterday. So she increased to make the number of ornamental flowers before the booth was opened. The total number of the ornamental flowers was triple of the number of the ornamental flowers that left from yesterday. In the second day, she sold of the ornamental flowers and earned 10,800 bahts.

The third day is the end of the Chiangmai Red Cross Fair. Miss Soifa made the ornamental flowers four times of the number of the ornamental flowers that left from the day before. In this final day, she sold 72 ornamental flowers.

After the Chiangmai Red Cross Fair, Miss Soifa worked on daily account, check the number of the ornamental flowers that she made and sold. Unfortunately, she forgets the number of the ornamental flowers she brought for sale at the beginning. If there are 48 ornamental flowers left after selling.

- (1) How many ornamental flowers did Miss Soifa bring for sale at the beginning?
- (2) How many ornamental flowers did Miss Soifa make on each day?

APPENDIX E

QUESTIONNAIRE ON BELIEFS RELATED TO PROBLEM SOLVING

Questionnaire on Beliefs Related to Problem Solving

Description

The purpose of conducting this questionnaire was to explore and examine preservice mathematics teachers' beliefs related to problem solving:

1. Beliefs about mathematics
2. Beliefs about problem solving
3. Beliefs about teaching and learning mathematics

The questionnaire adapting from Mathematics Belief Scale of Fennema, Carpenter, and Peterson (1987), consisted of 30 items which these items were some of aspects of beliefs about mathematics, beliefs about problem solving, and beliefs about teaching and learning mathematics.

The questionnaire required the preservice teachers to present their ideas or beliefs about mathematics, about problem solving, and about teaching and learning mathematics. Their idea or belief was individual so that there was no right or wrong answer.

The preservice teachers understand each item, and then mark the symbol ✓ in one of the given cells that was the most response to the idea. For example,

Item	Question	Strongly Agree	Agree	Disagree	Strongly Disagree
1.	Mathematics is a creative and useful discipline in which they learn to think.		✓		

It showed that this preservice teacher agreed with the sentence "mathematics is a creative and useful discipline in which they learn to think".

Questionnaire on Beliefs Related to Problem Solving
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No.	Question	Strongly Agree	Agree	Disagree	Strongly Disagree
1.	Solutions in mathematics are flexible and adjustable.				
2.	Every mathematics problem has only one method for solving.				
3.	Solving the problem by own self is very important for learning the process and strategy in problem solving.				
4.	Arithmetical operation is only one basic component of mathematics.				
5.	Mathematical problems may have more than one solution.				
6.	Mathematics is a subject that encourages learners to have a reason, critical thinking, and systematic thinking.				
7.	A central point of promoting to learn problem solving is providing opportunities for learners to reflect their own thinking/ problem solving, and to discuss the ideas.				
8.	In teaching problem solving, problems should have varied (both easy and difficult) and can be solved by various methods or strategies.				
9.	Mathematics is the study of memorizing formula, following algorithm, and calculating.				
10.	A central point of teaching problem solving is teaching for learners discover and learn problem-solving strategies.				

Questionnaire on Beliefs Related to Problem Solving (cont.)
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No.	Question	Strongly Agree	Agree	Disagree	Strongly Disagree
11.	Mathematics is the study of mathematical operation.				
12.	A central point of learning problem-solving process and problem-solving strategies is learning by doing a lot of problems by themselves.				
13.	Problem solving should be a major goal of instruction and a principal activity in mathematics teaching and learning.				
14.	In teaching problem solving, the problems should be various --- both easy and difficult ---, take more or less time for solving, and can be solved by many methods.				
15.	Memorizing formula, following algorithm, and calculating are the most important thing in mathematics.				
16.	Mathematical problem must challenge the people to solve it and can not find the way to solve immediately.				
17.	Many mathematical problems can be solved in more than one way.				
18.	In mathematics, solution is always accurate, certain, and static.				
19.	Mathematical problem is a mathematical situation that challenges a person to solve, and for which the path to the solution is not immediately known				
20.	Teaching for learners discover and learn problem-solving strategies is the central point of teaching problem solving.				

Questionnaire on Beliefs Related to Problem Solving (cont.)
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No.	Question	Strongly Agree	Agree	Disagree	Strongly Disagree
21.	Solving problem is not the main objective and activity in learning and teaching Mathematics.				
22.	Each mathematical problem has only one answer.				
23.	New knowledge will be occurred all the time in mathematics class.				
24.	The correct answer is the most important thing in solving problem.				
25.	Solving only one type of problems for several times is more valuable than solving several types of problems.				
26.	Mathematics is a subject that always constructs new knowledge.				
27.	Solving several types of problems is more valuable than solving only one type of problems all the time.				
28.	Getting the correct answer is not the most important thing in solving mathematical problem.				
29.	Mathematics is a creative and useful discipline in which they learn to think.				
30.	In teaching problem solving, problems should have a variety of types and difficult levels, using both a short and along time to solve as well as using more than one strategy.				

APPENDIX F
OBSERVATION FORMS

Observation Check List : Mathematical Problem Solving

Name _____ Session No. _____

No.	Problem-Solving Performance	Yes	No	Note
1.	Understand the story of the problem			
2.	Understand all related mathematical concepts			
3.	Identify all questions of the problem			
4.	Identify all data of the problem			
5.	Identify relationships among the data			
6.	Restate the problem			
7.	Participate in group problem solving			
8.	Participate in group discussion			
9.	Participate in whole class discussion (if exists)			
10.	Devise a plan			
11.	Use appropriate strategies			
12.	Solve more than one strategies			
13.	Carry out the plan systematically			
14.	Make a clear drawing or diagram (if exists)			
15.	Use logical reasoning to support arguments			
16.	Give a clear explanation on solution process			
17.	Attempt to find another strategy when stuck			
18.	Monitor / evaluate the own thinking			
19.	Get the correct answers			
20.	Look back the solution			
21.	Extend / Pose a problem generally			
22.	Construct a created/ interesting problem			
23.	Discuss on sociomathematical norms			
24.	Have a reasonable solution process			
25.	Have a sophisticated representation			
26.	Have an acceptable mathematical explanation and justification			

Observation Field Note : Mathematical Problem Solving
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Name _____ Session No. _____

1. Understanding the problem

1.1 _____

2. Devising a plan

2.1 _____

3. Carrying out the plan

3.1 _____

4. Looking back the solution

4.1 _____

5. Extending / posing a problem

5.1 _____

6. Participating in group problem solving

6.1 _____

7. Discussing on sociomathematical norms

7.1 _____

8. Any thing else (if exists)

8.1 _____

APPENDIX G
DATA ANALYSES

TABLE a THE KOLOMOGOROV-SMIRNOV TEST AND SHAPIRO-WILK TEST FOR
SCORES ON PROBLEM-SOLVING COMPETENCE OF THE EXPERIMENTAL GROUP
AND THE CONTROL GROUP

Group	Kolomogorov-Smirnov ^a			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Experimental	0.233	13	0.051	0.900	13	0.180
Control	0.269	13	0.011	0.767	13	0.010**

** . This is an upper bound of the true significance.

a. Lilliefors Significance Correction

TABLE b THE KOLOMOGOROV-SMIRNOV TEST AND SHAPIRO-WILK TEST FOR
SCORES ON APPROPRIATE BELIEFS RELATED TO PROBLEM SOLVING OF THE
EXPERIMENTAL GROUP AND THE CONTROL GROUP

Group	Kolomogorov-Smirnov ^a			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Experimental	0.151	13	0.200*	0.911	13	0.256
Control	0.191	13	0.200*	0.898	13	0.168

*. This is a lower bound of the true significance.

a. Lilliefors Significance Correction

TABLE c RESULTS OF COMPARING THE MEDIAN SCORES ON APPROPRIATE BELIEFS IN EACH ASPECT OF THE EXPERIMENTAL GROUP AND THE CONTROL GROUP

		N	Mean Rank	Sum of Rank	Test Value
Scores on beliefs about mathematics of the experimental group – Scores on beliefs about mathematics of the control group	Negative Ranks	4 ^a	4.25	17.00	p< 0.154
	Positive Ranks	7 ^b	7.00	49.00	
	Ties	2 ^c			
	Total	13			
Scores on beliefs about problem solving of the experimental group – Scores on beliefs about problem solving of the control group	Negative Ranks	3 ^d	4.83	14.50	p< 0.054
	Positive Ranks	9 ^e	7.06	63.50	
	Ties	1 ^f			
	Total	13			
Scores on beliefs about teaching and learning mathematics of the experimental group – Scores on beliefs about teaching and learning mathematics of the control group	Negative Ranks	6 ^g	6.17	37.00	p< 0.875
	Positive Ranks	6 ^h	6.83	41.00	
	Ties	1 ⁱ			
	Total	13			

- a. Scores on beliefs about mathematics of the experimental group < Scores on beliefs about mathematics of the control group
- b. Scores on beliefs about mathematics of the experimental group > Scores on beliefs about mathematics of the control group
- c. Scores on beliefs about mathematics of the experimental group = Scores on beliefs about mathematics of the control group
- d. Scores on beliefs about problem solving of the experimental group < Scores on beliefs about problem solving of the control group
- e. Scores on beliefs about problem solving of the experimental group > Scores on beliefs about problem solving of the control group
- f. Scores on beliefs about problem solving of the experimental group = Scores on beliefs about problem solving of the control group
- g. Scores on beliefs about teaching and learning mathematics of the experimental group < Scores on beliefs about teaching and learning mathematics of the control group
- h. Scores on beliefs about teaching and learning mathematics of the experimental group > Scores on beliefs about teaching and learning mathematics of the control group
- i. Scores on beliefs about teaching and learning mathematics of the experimental group = Scores on beliefs about teaching and learning mathematics of the control group

TABLE d RESULTS OF COMPARING THE MEDIAN SCORES ON APPROPRIATE BELIEFS IN EACH ASPECT BETWEEN PRE- AND POST- ASSESSMENT OF THE EXPERIMENTAL GROUP

		N	Mean Rank	Sum of Rank	Test Value
Post-assessment scores on beliefs related to problem solving – Pre-assessment scores on beliefs related to problem solving	Negative Ranks	3 ^a	2.17	6.50	p< 0.011*
	Positive Ranks	9 ^b	7.94	71.50	
	Ties	1 ^c			
	Total	13			
Post-assessment scores on beliefs about mathematics – Pre-assessment scores on beliefs about mathematics	Negative Ranks	2 ^d	2.00	4.00	p< 0.028*
	Positive Ranks	7 ^e	5.86	41.00	
	Ties	4 ^f			
	Total	13			
Post-assessment scores on beliefs about problem solving – Pre-assessment scores on beliefs about problem solving	Negative Ranks	2 ^g	4.50	9.00	p< 0.018*
	Positive Ranks	10 ^h	6.90	69.00	
	Ties	1 ⁱ			
	Total	13			
Post-assessment scores on beliefs about teaching and learning mathematics – Pre-assessment scores on beliefs about teaching and learning mathematics	Negative Ranks	6 ^j	4.13	16.50	p< 0.138
	Positive Ranks	6 ^k	7.07	49.50	
	Ties	1 ^l			
	Total	13			

* Significant at the level 0.05

- a. Post-assessment scores on beliefs related to problem solving < Pre-assessment scores on beliefs related to problem solving
- b. Post-assessment scores on beliefs related to problem solving > Pre-assessment scores on beliefs related to problem solving
- c. Post-assessment scores on beliefs related to problem solving = Pre-assessment scores on beliefs related to problem solving
- d. Post-assessment scores on beliefs about mathematics < Pre-assessment scores on beliefs about mathematics
- e. Post-assessment scores on beliefs about mathematics > Pre-assessment scores on beliefs about mathematics
- f. Post-assessment scores on beliefs about mathematics = Pre-assessment scores on beliefs about mathematics
- g. Post-assessment scores on beliefs about problem solving < Pre-assessment scores on beliefs about problem solving group
- h. Post-assessment scores on beliefs about problem solving > Pre-assessment scores on beliefs about problem solving group
- i. Post-assessment scores on beliefs about problem solving = Pre-assessment scores on beliefs about problem solving group
- j. Post-assessment scores on beliefs about teaching and learning mathematics < Pre-assessment scores on beliefs about teaching and learning mathematics
- k. Post-assessment scores on beliefs about teaching and learning mathematics > Pre-assessment scores on beliefs about teaching and learning mathematics
- l. Post-assessment scores on beliefs about teaching and learning mathematics = Pre-assessment scores on beliefs about teaching and learning mathematics

TABLE e RESULTS OF COMPARING THE MEDIAN SCORES ON EACH BELIEF ABOUT MATHEMATICS BETWEEN PRE- AND POST- ASSESSMENT OF THE EXPERIMENTAL GROUP

		N	Mean Rank	Sum of Rank	Test Value
Post-assessment scores on B1 – Pre-assessment scores on B1	Negative Ranks	3 ^a	6.17	18.50	p< 0.624
	Positive Ranks	6 ^b	4.42	26.50	
	Ties	4 ^c			
	Total	13			
Post-assessment scores on B2 – Pre-assessment scores on B2	Negative Ranks	4 ^d	5.75	23.00	p< 0.627
	Positive Ranks	6 ^e	5.33	32.00	
	Ties	3 ^f			
	Total	13			
Post-assessment scores on B3 – Pre-assessment scores on B3	Negative Ranks	1 ^g	3.00	3.00	p< 0.018*
	Positive Ranks	8 ^h	5.25	42.00	
	Ties	4 ⁱ			
	Total	13			
Post-assessment scores on B4 – Pre-assessment scores on B4	Negative Ranks	1 ^j	3.00	3.00	p< 0.018*
	Positive Ranks	8 ^k	5.25	42.00	
	Ties	4 ^l			
	Total	13			
Post-assessment scores on B5 – Pre-assessment scores on B5	Negative Ranks	3 ^m	3.50	10.50	p< 1.000
	Positive Ranks	3 ⁿ	3.50	10.50	
	Ties	7 ^o			
	Total	13			

* Significant at the level 0.05

- a. Post-assessment scores on B1 < Pre-assessment scores on B1
- b. Post-assessment scores on B1 > Pre-assessment scores on B1
- c. Post-assessment scores on B1 = Pre-assessment scores on B1
- d. Post-assessment scores on B2 < Pre-assessment scores on B2
- e. Post-assessment scores on B2 > Pre-assessment scores on B2
- f. Post-assessment scores on B2 = Pre-assessment scores on B2
- g. Post-assessment scores on B3 < Pre-assessment scores on B3
- h. Post-assessment scores on B3 > Pre-assessment scores on B3
- i. Post-assessment scores on B3 = Pre-assessment scores on B3
- j. Post-assessment scores on B4 < Pre-assessment scores on B4
- k. Post-assessment scores on B4 > Pre-assessment scores on B4
- l. Post-assessment scores on B4 = Pre-assessment scores on B4
- m. Post-assessment scores on B5 < Pre-assessment scores on B5
- n. Post-assessment scores on B5 > Pre-assessment scores on B5
- o. Post-assessment scores on B5 = Pre-assessment scores on B5

TABLE f RESULTS OF COMPARING THE MEDIAN SCORES ON EACH BELIEF ABOUT
PROBLEM SOLVING BETWEEN PRE- AND POST- ASSESSMENT OF THE
EXPERIMENTAL GROUP

		N	Mean Rank	Sum of Rank	Test Value
Post-assessment scores on B6 – Pre-assessment scores on B6	Negative Ranks	2 ^a	2.50	5.00	p< 0.234
	Positive Ranks	4 ^b	4.00	16.00	
	Ties	7 ^c			
	Total	13			
Post-assessment scores on B7 – Pre-assessment scores on B7	Negative Ranks	0 ^d	0.00	0.00	p< 0.024*
	Positive Ranks	6 ^e	3.50	21.00	
	Ties	7 ^f			
	Total	13			
Post-assessment scores on B8 – Pre-assessment scores on B8	Negative Ranks	2 ^g	3.00	6.00	p< 0.084
	Positive Ranks	6 ^h	5.00	30.00	
	Ties	5 ⁱ			
	Total	13			
Post-assessment scores on B9 – Pre-assessment scores on B9	Negative Ranks	3 ^j	5.00	15.00	p< 0.317
	Positive Ranks	6 ^k	5.00	30.00	
	Ties	4 ^l			
	Total	13			
Post-assessment scores on B10 – Pre-assessment scores on B10	Negative Ranks	1 ^m	3.00	3.00	p< 0.030*
	Positive Ranks	7 ⁿ	4.71	33.00	
	Ties	5 ^o			
	Total	13			

* Significant at the level 0.05

- a. Post-assessment scores on B6 < Pre-assessment scores on B6
- b. Post-assessment scores on B6 > Pre-assessment scores on B6
- c. Post-assessment scores on B6 = Pre-assessment scores on B6
- d. Post-assessment scores on B7 < Pre-assessment scores on B7
- e. Post-assessment scores on B7 > Pre-assessment scores on B7
- f. Post-assessment scores on B7 = Pre-assessment scores on B7
- g. Post-assessment scores on B8 < Pre-assessment scores on B8
- h. Post-assessment scores on B8 > Pre-assessment scores on B8
- i. Post-assessment scores on B8 = Pre-assessment scores on B8
- j. Post-assessment scores on B9 < Pre-assessment scores on B9
- k. Post-assessment scores on B9 > Pre-assessment scores on B9
- l. Post-assessment scores on B9 = Pre-assessment scores on B9
- m. Post-assessment scores on B10 < Pre-assessment scores on B10
- n. Post-assessment scores on B10 > Pre-assessment scores on B10
- o. Post-assessment scores on B10 = Pre-assessment scores on B10

TABLE g RESULTS OF COMPARING THE MEDIAN SCORES ON EACH BELIEF ABOUT
TEACHING AND LEARNING MATHEMATICS BETWEEN PRE- AND POST-
ASSESSMENT OF THE EXPERIMENTAL GROUP

		N	Mean Rank	Sum of Rank	Test Value
Post-assessment scores on B11 – Pre-assessment scores on B11	Negative Ranks	3 ^a	3.50	10.50	p< 1.000
	Positive Ranks	3 ^b	3.50	10.50	
	Ties	7 ^c			
	Total	13			
Post-assessment scores on B12 – Pre-assessment scores on B12	Negative Ranks	5 ^d	5.00	25.00	p< 0.739
	Positive Ranks	4 ^e	5.00	20.00	
	Ties	4 ^f			
	Total	13			
Post-assessment scores on B13 – Pre-assessment scores on B13	Negative Ranks	3 ^g	3.00	9.00	p< 0.391
	Positive Ranks	4 ^h	4.75	19.00	
	Ties	6 ⁱ			
	Total	13			
Post-assessment scores on B14 – Pre-assessment scores on B14	Negative Ranks	3 ^j	3.50	10.50	p< 0.271
	Positive Ranks	5 ^k	5.10	25.50	
	Ties	5 ^l			
	Total	13			
Post-assessment scores on B15 – Pre-assessment scores on B15	Negative Ranks	1 ^m	2.50	2.50	p< 0.157
	Positive Ranks	4 ⁿ	3.13	12.50	
	Ties	8 ^o			
	Total	13			

a. Post-assessment scores on B11 < Pre-assessment scores on B11

b. Post-assessment scores on B11 > Pre-assessment scores on B11

c. Post-assessment scores on B11 = Pre-assessment scores on B11

d. Post-assessment scores on B12 < Pre-assessment scores on B12

e. Post-assessment scores on B12 > Pre-assessment scores on B12

f. Post-assessment scores on B12 = Pre-assessment scores on B12

g. Post-assessment scores on B13 < Pre-assessment scores on B13

h. Post-assessment scores on B13 > Pre-assessment scores on B13

i. Post-assessment scores on B13 = Pre-assessment scores on B13

j. Post-assessment scores on B14 < Pre-assessment scores on B14

k. Post-assessment scores on B14 > Pre-assessment scores on B14

l. Post-assessment scores on B14 = Pre-assessment scores on B14

m. Post-assessment scores on B15 < Pre-assessment scores on B15

n. Post-assessment scores on B15 > Pre-assessment scores on B15

o. Post-assessment scores on B15 = Pre-assessment scores on B15

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